THE INFLUENCE OF THE INITIAL STRESSES ON STRESS INTENSITY FACTOR OF MODE II AT THE CRACK TIPS IN A COMPOSITE STRIP

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ABSTRACT: The influence of the initial finite stretching or compressing of the strip containing a single crack on the Energy Release Rate (ERR) and on the SIF of mode II at the crack tips is studied by the use of the Three-Dimensional Linearized Theory of Elasticity. In the present paper these investigations are developed for a strip made from composite material. It is assumed that the ends of the strip are simply supported and this strip contains a crack whose edges are parallel to the face planes. It is supposed that the strip is stretched or compressed along the crack edges and after this stretching (compressing) the edges of the crack are loaded by additional uniformly distributed tangential (sliding) forces. Corresponding Boundary Value Problems were formulated with the use of the Three-Dimensional Linearized Theory of Elasticity. All investigations are carried out numerically by employing the FEM. The numerical results on the influence of the initial stresses on the values of the ERR and of the SIF of mode II are presented. In particular, it is established that the values of the ERR and of the SIF of mode II decrease (increase) monotonically with an increase (decrease) in the initial stretching (compression).

Key words: Crack, Energy Release Rate, composite, Stress Intensity Factor of mode II, strip.

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1. INTRODUCTION

A topical problem of fracture mechanics is the determination of the Energy Release Rate (ERR) or of the Stress Intensity Factors (SIF) at the tips of cracks occurring in structural members. Up to now a lot of investigations had been made in this field and the corresponding results were tabulated in many references (hand-books) such as [1]. At present, numerical methods based on the domain-integral formulation for ERR and SIF computations are being developed intensively. The review of these investigations is given in [2].

It should be noted that the above-mentioned and other huge number of investigations regarding to SIF for brittle and quasi-brittle materials had been made within the framework of the linear theory of elasticity. It is known that this theory cannot take into account the influence of the initial stretching or compression along the cracks on the SIF. For this purpose it is necessary to use the Three-Dimensional Linearized Theory of Elasticity (TDLTE) detailed in [3]. The TDLTE for brittle fracture problems was developed in [4,5]. Recently, review of the investigations on the brittle fracture mechanics of pre-stressed materials is given in [6]. It follows from [4-6] that, up to now the concrete results regarding to the influence of the initial stresses on the SIF are obtained for an infinite body. Consequently, up to now the investigations on the influence of the initial stretching or compression along the cracks contained by the finite regions on the SIF have not been made. The importance and actuality of such investigations are evident.

The first attempt in this field was made in the paper [11] and the investigations were carried out for a simply supported and initially stretched (compressed) strip containing a crack on the edges of which additional uniformly distributed normal forces operate. A plane-strain state is considered and the material of the strip was assumed orthotropic with normalized mechanical properties. In the present paper the investigation carried out in [11] is developed for the case where on the edges of the crack the uniformly distributed tangential (sliding) forces operate.

The numerical results on the influence of the initial stresses on the ERR and on the SIF of mode II are presented and analysed. The corresponding boundary-value problem is solved by employing FEM.

2. FORMULATION OF THE PROBLEM

We consider the strip which occupies the region $\{0 \le x_2 \le h;\ 0 \le x_1 \le \ell, -\infty < x_3 < +\infty\}$ and associate the Lagrange coordinates which in the initial state coincide with the Cartesian coordinates $Ox_1x_2x_3$ (Fig.1). In the initial state, assume that the uniformly distributed normal forces with intensity q act at the ends of the strip and the strip contains a crack which is located at $\{x_2 = h_{U\pm 0}, \ell/2 - \ell_0/2 < x_1 < \ell/2 - \ell_0/2\}$ (Fig.1). On the edges of the crack act the additional uniformly distributed tangential (sliding) forces with intensity p. It is also assumed that $q >> p$.

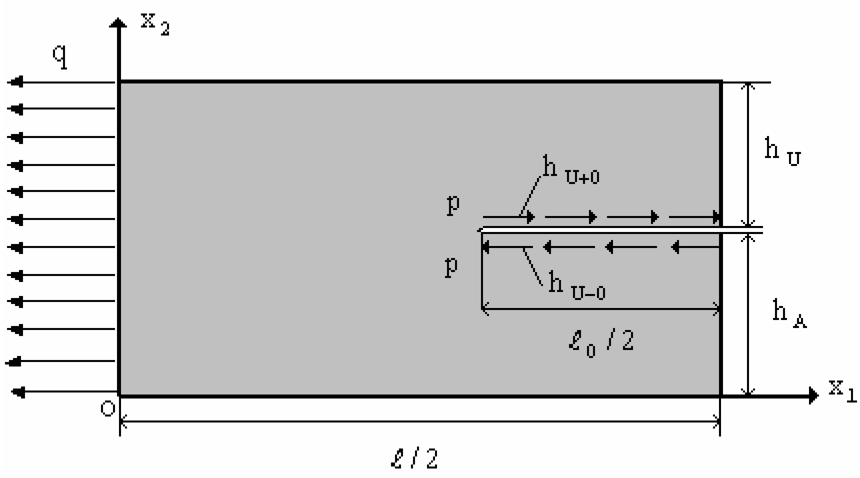

Fig. 1. The geometry of the considered strip

In the present paper we attempt to investigate the influence of the initial stretching (or compressing) of the strip on the ERR and on the SIF of mode II arising as a result of the action of the tangential (sliding) forces p. For this purpose we use the TDLTE, the equation that can be written for the considered case as follows.

$$\frac{\partial}{\partial x_i}\left[\sigma_{ij} + \sigma_{in}^{(0)}\frac{\partial u_j}{\partial x_n}\right] = 0,\ \ \sigma_{11} = A_{11}\varepsilon_{11} + A_{12}\varepsilon_{12},\ \ \sigma_{22} = A_{12}\varepsilon_{11} + A_{22}\varepsilon_{22},$$

$$\sigma_{12} = 2A_{66}\varepsilon_{12},\ \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad i;j=1,2 \qquad (1)$$

For the equations (1) we have the following boundary conditions:

$$u_2\Big|_{x_1=0;\ell}=0\,,\ \sigma_{1i}\Big|_{x_1=0;\ell}=0\,,\ \sigma_{i2}\Big|_{\substack{x_2=h_U\pm 0\\ x_1\in(\ell/2-\ell_0/2,\ell/2+\ell_0/2)}}=-p\delta_i^1\,,\ \sigma_{i2}\Big|_{\substack{x_2=0;h\\ x_1\in(0,\ell)}}=0\,. \qquad (2)$$

Note that in (1) and (2) the conventional notation is used and for the repeated indices the summation rule holds.

The values in equation (1) indicated by upper indices (0) regard to the initial state and are determined from the solution to the following boundary-value problem:

$$\frac{\partial\sigma_{ij}^{(0)}}{\partial x_j}=0\,,\ \sigma_{11}^{(0)}=A_{11}\varepsilon_{11}^{(0)}+A_{12}\varepsilon_{22}^{(0)}\,;\ \sigma_{22}^{(0)}=A_{12}\varepsilon_{11}^{(0)}+A_{22}\varepsilon_{22}^{(0)}\,;\ \sigma_{12}^{(0)}=2A_{66}\varepsilon_{12}^{(0)}$$

$$\varepsilon_{ij}^{(0)}=\frac{1}{2}\left(\frac{\partial u_i^{(0)}}{\partial x_j}+\frac{\partial u_j^{(0)}}{\partial x_i}\right) \qquad (3)$$

$$\sigma_{11}^{(0)}\Big|_{x_1=0}=\sigma_{11}^{(0)}\Big|_{x_1=\ell}=q\,,\ \sigma_{i2}^{(0)}\Big|_{\substack{x_2=0;h\\ x_1\in(0,\ell)}}=0\,,\ \ u_2^{(0)}\Big|_{x_1=0;\ell}=0\,,\ i;j=1,2 \qquad (4)$$

It is evident that for the considered case the solution to the problem (3), (4) is

$$\sigma_{11}^{(0)}=q,\ \ \sigma_{ij}^{(0)}=0\ \text{ for } ij\neq 11 \qquad (5)$$

Thus, the formulation of the problem is exhausted.

3. THE FEM MODELLING

As the analytical solution to the problem (1) and (2) is impossible, we will attempt to investigate this problem by employing the FEM. According to [3], for this purpose the following functional is introduced:

$$\Pi=\frac{1}{2}\iint_{\Omega-\Omega'}\left(\left(\sigma_{11}+q\frac{\partial u_1}{\partial x_1}\right)\frac{\partial u_1}{\partial x_1}+\left(\sigma_{12}+q\frac{\partial u_2}{\partial x_1}\right)\frac{\partial u_2}{\partial x_1}+\sigma_{12}\frac{\partial u_1}{\partial x_2}+\sigma_{22}\frac{\partial u_2}{\partial x_2}\right)dx_1dx_2+$$

$$\int_{(\ell-\ell_0)/2}^{(\ell+\ell_0)/2}pu_2\Big|_{x_2=h_U-0}dx_1-\int_{(\ell-\ell_0)/2}^{(\ell+\ell_0)/2}pu_2\Big|_{x_2=h_U+0}dx_1\,. \qquad (6)$$

where,

$$\Omega'=\{x_2=h_U+0,(\ell-\ell_0)/2<x_1<(\ell+\ell_0)/2\}\cup$$
$$\{x_2=h_U-0,(\ell-\ell_0)/2<x_1<(\ell+\ell_0)/2\} \qquad (7)$$

Taking the expressions (5) into account, the first two equations in (1) can be written as follows:

$$\frac{\partial\sigma_{11}}{\partial x_1}+q\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial\sigma_{12}}{\partial x_2}=0\ \ ,\quad \frac{\partial\sigma_{12}}{\partial x_1}+q\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial\sigma_{22}}{\partial x_2}=0\,. \qquad (8)$$

We introduce the following notation:

$$\omega_{1111} = A_{11} + q\,,\ \omega_{1122} = A_{12}\,,\ \omega_{1212} = A_{66}\,,\ \omega_{1221} = A_{66} + q\,,\ \omega_{2112} = A_{66}\,,$$

$$\omega_{2121} = A_{66}\,,\ \omega_{2211} = A_{12}\,,\ \omega_{2222} = A_{22}\,. \qquad (9)$$

Using (9), we can write the following relations from (1), (8), (9);

$$\sigma_{ij} + \sigma_{in}^{(0)} \frac{\partial u_j}{\partial x_n} = \omega_{ijkm} \frac{\partial u_k}{\partial x_m} \qquad (10)$$

By direct verification we prove that $\omega_{ijnm} = \omega_{mnji}$ and using the presentations (10), by the usual manner it is proven that the equations (8) and the boundary conditions (2) are the Euler equations of the functional (6).

So, we prove the validity of the functional (6) for FEM modelling of the problem in question.

4. NUMERICAL RESULTS AND DISCUSSIONS

We assume that the plate-strip is fabricated from the multilayered composite material consisting of the two alternating isotropic layers which lie in the planes $x_2 = \text{const}$. The material of the reinforcing (matrix) layers is supposed to be elastic with mechanical characteristics $E_2(E_1)$ (Young's modulus), $\nu_2(\nu_1)$ (Poisson coefficient). It is known that in the continuum approach this is a transversally-isotropic material with effective mechanical properties whose isotropy axis lies on the Ox_2 axis. According to [7], these effective mechanical properties are calculated by the following formulae:

$$A_{66} = \frac{\mu_1\mu_2}{\mu_1\eta_2 + \mu_2\eta_1}\,,\ A_{12} = \lambda_1\eta_1 + \lambda_2\eta_2 - \eta_1\eta_2(\lambda_1 - \lambda_2)\frac{(\lambda_1 + 2\mu_1) - (\lambda_2 + 2\mu_2)}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_1}\,,$$

$$A_{11} = (\lambda_1 + 2\mu_1)\eta_1 + (\lambda_2 + 2\mu_2)\eta_2 - \eta_1\eta_2 \frac{(\lambda_1 - \lambda_2)^2}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_1}\,,$$

$$A_{22} = (\lambda_1 + 2\mu_1)\eta_1 + (\lambda_2 + 2\mu_2)\eta_2 - \eta_1\eta_2 \frac{(\lambda_1 + 2\mu_1) - (\lambda_2 + 2\mu_2)}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_1} \qquad (11)$$

where

$$\eta_1 = 1 - \eta_2\,,\ \lambda_k = \frac{E_k\nu_k}{(1+\nu_k)(1-2\nu_k)}\,,\ \mu_k = \frac{E_k}{2(1+\nu_k)}\,. \qquad (12)$$

In (11) and (12), η_2 (η_1) shows the concentration of the reinforcing (matrix) layers in the composite material.

The concrete numerical investigations are made for $\nu_1 = \nu_2 = 0.3,\ \eta_2 = 0.5,\ h/\ell = 0.2$. Using the symmetry of the problem considered with respect to $x_1 = \ell/2$, under finite element division of the region Ω we consider only the half of this region. In this case, the path around the crack tip is modelled by singular triangular finite elements [8, 9] (Fig.2). But, out of this part standard quadratic Lagrange family rectangular finite elements (Fig.2) are used.

Now we consider numerical results obtained within the framework of the above-stated modelling and we first analyse the results through which we attempt to prove the validity of the algorithm used and the programmes which are written by the authors in FTN77. The regarding results are given in Table 1. Note that under calculation procedure the parameter

$$\tilde{q} = q/(E_1\eta_1 + E_2\eta_2) \qquad (13)$$

is introduced and the data given in these tables are attained for $\tilde{q} = 0$. In this case Table 1 shows the values of $K_I^{(s)}/K_{I\infty}$, $K_I^{(f)}/K_{I\infty}$ and $K_I^{(E)}/K_{I\infty}$ for various ℓ_0/ℓ , ℓ_0/h under $E_2/E_1 = 1,\ h_u = h/2$. Here, $K_{I\infty} = p\sqrt{\pi\ell_0}$, $K_I^{(s)}$ and $K_I^{(f)}$ are the values of the SIF for the mode I calculated by the use of exact solution in the infinite plate and by the approximate analytical series formulae given in [1] and by the present approach through the values of the nodal displacement of the singular triangular finite elements shown in Fig.2 respectively.

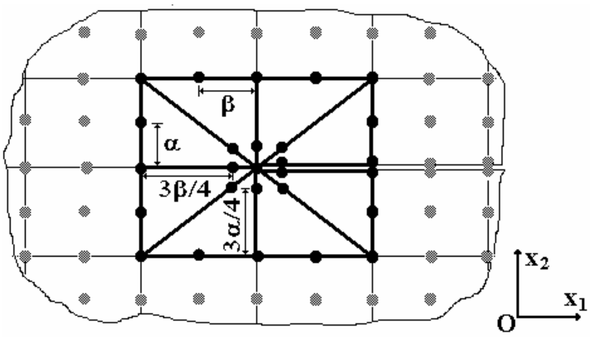

Fig. 2. Finite element modelling at the crack tip.

But, $K_I^{(E)}$ shows the values of the SIF for the mode I calculated through the values of $\partial U/\partial\ell_0$ via the well-known expression. The agreement of the corresponding results given in Table 1 testifies the proposed numerical approach. Moreover, these numerical results agree with the mechanical consideration, according to which, the values of $K_I^{(s)}/K_{I\infty}$, $K_I^{(f)}/K_{I\infty}$ and $K_I^{(E)}/K_{I\infty}$ must approach simultaneously 1 with decreasing ℓ_0/ℓ and ℓ_0/h. This situation is also observed in Table 1.

Table 1. The values of SIF of mode I for the $E_2/E_1 = 1$, $h/\ell = 0.20$, $h_u/\ell = h_A/\ell$.

ℓ_0/ℓ	ℓ_0/h	$K_I^{(s)}/K_{I\infty}$	$K_I^{(f)}/K_{I\infty}$	$K_I^{(E)}/K_{I\infty}$
0.080	0.80	1.2406	1.2406	1.1930
0.075	0.75	1.2009	1.2009	1.1716
0.060	0.60	1.1444	1.1444	1.1108
0.050	0.50	1.0936	1.0931	1.0729
0.040	0.40	1.0473	1.0473	1.0409

Numerous concrete investigations show that in the sense of the convergence of the numerical results with respect to the numbers of finite elements the energy release rate method (i.e. the calculation of K_I through $\partial U/\partial \ell_0$) is more reliable [11] .

Note that under obtaining the results given in Table 1 it is assumed that the edges of the crack are loaded by the uniformly distributed normal forces with intensity p.

Now we consider the numerical results which are obtained by the use of the programmes the validity of which is proved above and show the influence of the parameter $\tilde{q}$ (13) (i.e. of the initial stretching or compressing) on the values of $K_{II}/K_{II\infty}$. Here, $K_{II\infty} = p\sqrt{\pi\ell_0}$, is the value of the SIF for the mode II calculated by the use of exact solution in the infinite plate. Note that according to [4, 5], this initial stress does not influence the values of K_{II} for the crack contained by the infinite body. However, the numerical results obtained in the considered case, show that for the finite domains containing a crack initial stretching or compression along the crack can significantly change the values of $K_{II}/K_{II\infty}$. This conclusion is proven by the numerical results given in Table 2, which show the values of $K_{II}/K_{II\infty}$ for various $\tilde{q}$, E_2/E_1 and ℓ_0/ℓ under $h/\ell = 0.20$, $h_U = h/2$. In this table the upper (lower) row corresponds to the initial stretching (compressing) of the considered strip. Note that, here and below under consideration of the compressed initial stresses the values of the $\tilde{q}$ are taken significantly smaller than $\tilde{q}_{cr}$ which corresponds the stability loss of the strip containing a crack [10].

Thus, it follows from the results given in the Table 2 that the initial stretching (compressing) along the crack decreases (increases) monotonically with $|\tilde{q}|$. This decreasing or increasing becomes more considerable with increasing crack length, i.e. with ℓ_0/ℓ.

Moreover, the numerical results show that the influence of the initial stretching or compressing of the considered strip on the values of $(\partial U/\partial \ell_0)/p\ell$ depends significantly on the location of the crack along the thickness of the plate-strip, i.e. on the values of h_U/ℓ.

The results shown in Table 3 which are obtained for $\ell_0/\ell = 0.24$ respectively under various h_U/ℓ, E_2/E_1 and $|\tilde{q}|$ prove this conclusion.

Thus, it follows from these results that the values of $K_{II}/K_{II\infty}$ decrease (increase) monotonically with the crack getting nearer the free face plane of the strip under initial stretching (compressing).

Table 2. The influence of the initial stretching (numerator)/compression (denominator) on the $K_{II}/K_{II\infty}$ for, $h/\ell = 0.20$, $h_U/\ell = h_A/\ell$.

$\ell_0/2\ell$	E_2/E_1	$\tilde{q} = q/(E_1\eta_1 + E_2\eta_2)$: 0.000	$\frac{+0.001}{-0.001}$	$\frac{+0.01}{-0.01}$	$\frac{+0.02}{-0.02}$
0.08	1	1.0948	$\frac{1.0940}{1.0956}$	$\frac{1.0874}{1.1033}$	$\frac{1.0809}{1.1133}$
	5	1.0788	$\frac{1.0780}{1.0797}$	$\frac{1.0708}{1.0886}$	$\frac{1.0641}{1.1007}$
	10	1.0729	$\frac{1.0719}{1.0739}$	$\frac{1.0642}{1.0843}$	$\frac{1.0573}{1.1000}$
	20	1.0694	$\frac{1.0683}{1.0706}$	$\frac{1.0598}{1.0839}$	$\frac{1.0528}{1.0086}$
0.16	1	1.2634	$\frac{1.2607}{1.2662}$	$\frac{1.2389}{1.2935}$	$\frac{1.2183}{1.3313}$
	5	1.1968	$\frac{1.1941}{1.1968}$	$\frac{1.1725}{1.2284}$	$\frac{1.1531}{1.2712}$
	10	1.1572	$\frac{1.1543}{1.1601}$	$\frac{1.1326}{1.1918}$	$\frac{1.1143}{1.2444}$
	20	1.1274	$\frac{1.1244}{1.1306}$	$\frac{1.1025}{1.1687}$	$\frac{1.0858}{1.2110}$
0.24	1	1.4240	$\frac{1.4187}{1.4296}$	$\frac{1.3761}{1.4873}$	$\frac{1.3375}{1.5739}$
	5	1.3182	$\frac{1.3129}{1.3236}$	$\frac{1.2723}{1.3819}$	$\frac{12377}{1.4768}$
	10	1.2489	$\frac{1.2436}{1.2543}$	$\frac{1.2045}{1.3161}$	$\frac{1.1731}{1.4302}$
	20	1.1912	$\frac{1.1858}{1.1968}$	$\frac{1.1484}{1.2681}$	$\frac{1.1212}{1.3876}$
0.40	1	1.6785	$\frac{1.6666}{1.6910}$	$\frac{1.5768}{1.8313}$	$\frac{1.5042}{2.0862}$
	5	1.5246	$\frac{1.5135}{1.5363}$	$\frac{1.4322}{1.6723}$	$\frac{1.1368}{1.9454}$
	10	1.4158	$\frac{1.4052}{1.4269}$	$\frac{1.3306}{1.5648}$	$\frac{1.2754}{1.8907}$
	20	1.3167	$\frac{1.3066}{1.3276}$	$\frac{1.2395}{1.4784}$	$\frac{1.1943}{1.7708}$

Table 3 The influence of the initial stretching (numerator)/compression (denominator) on the $(\partial U/\partial \ell_0)/p\ell$ for, $h/\ell = 0.20$, $h_U/\ell = h_A/\ell$, $\ell_0/2\ell = 0.24$.

h_U/ℓ	E_2/E_1	$\tilde{q} = q/(E_1\eta_1 + E_2\eta_2)$			
		0.000	$\frac{+0.001}{-0.001}$	$\frac{+0.01}{-0.01}$	$\frac{+0.02}{-0.02}$
0.10	1	0.6958	$\frac{0.6900}{0.7017}$	$\frac{0.6447}{0.7649}$	$\frac{0.6046}{0.8636}$
	5	0.2601	$\frac{0.2579}{0.2625}$	$\frac{0.2404}{0.2883}$	$\frac{0.2258}{0.3322}$
	10	0.1616	$\frac{0.1601}{0.1632}$	$\frac{0.1491}{0.1811}$	$\frac{0.1403}{0.2159}$
	20	0.1016	$\frac{0.1006}{0.1027}$	$\frac{0.0936}{0.1163}$	$\frac{0.0885}{0.1533}$
0.0833	1	0.6877	$\frac{0.6822}{0.6934}$	$\frac{0.6389}{0.7531}$	$\frac{0.6008}{0.8455}$
	5	0.2584	$\frac{0.2562}{0.2606}$	$\frac{0.2394}{0.2853}$	$\frac{0.2252}{0.3270}$
	10	0.1610	$\frac{0.1596}{0.1626}$	$\frac{0.1489}{0.1799}$	$\frac{0.1403}{0.2135}$
	20	0.1015	$\frac{0.1005}{0.1026}$	$\frac{0.0936}{0.1159}$	$\frac{0.0886}{0.1521}$
0.0666	1	0.6621	$\frac{0.6574}{0.6669}$	$\frac{0.6203}{0.7170}$	$\frac{0.5873}{0.7929}$
	5	0.2519	$\frac{0.2500}{0.2526}$	$\frac{0.2351}{0.2589}$	$\frac{0.2224}{0.2666}$
	10	0.1586	$\frac{0.1573}{0.1588}$	$\frac{0.1474}{0.1611}$	$\frac{0.1395}{0.1639}$
	20	0.1008	$\frac{0.0999}{0.1009}$	$\frac{0.0933}{0.1017}$	$\frac{0.0886}{0.1027}$
0.0500	1	0.6332	$\frac{0.6297}{0.6369}$	$\frac{0.6008}{0.6749}$	$\frac{0.5747}{0.7310}$
	5	0.2443	$\frac{0.2427}{0.2459}$	$\frac{0.2306}{0.2631}$	$\frac{0.2200}{0.2707}$
	10	0.1555	$\frac{0.1544}{0.1567}$	$\frac{0.1460}{0.1698}$	$\frac{0.1391}{0.1940}$
	20	0.0999	$\frac{0.0990}{0.1008}$	$\frac{0.0931}{0.1119}$	$\frac{0.0888}{0.1409}$

5. CONCLUSION

Thus, in the present paper within the framework of the Three-Dimensional Linearized Theory of Elasticity the influence of the initial stretching and compressing of the simply supported plate-strip containing a crack on the values of SIF of the Mode II is investigated. It is assumed that the initial stresses act along the crack whose edges are parallel to the free face planes of the strip. The investigations are made by employing the FEM. According to the obtained numerical results, it is established that:

- The values of the ERR and of the SIF of the mode II decrease (increase) monotonically with the values of the initial stretching (compressing),
- The influence of the initial stresses on the values of the ERR and of the SIF of the mode II increases monotonically with the increase of the length of the crack,
- The influence of the initial stresses on the values of the ERR increases monotonically with the crack location getting nearer the free face-plane of the strip.

6. REFERENCES

1. Sih G., Handbook of Stress Intensity Factors-Lehigh University, 1973.

2. Gosz M., Dolbow J. and Moran B., Domain integral formulation for stress intensity factors computations along curved three-dimensional interface cracks // Int. J. Solids and Structures- 1998-P.1763-1783.

3. Guz A.N., Fundamentals of the Three-Dimensional Theory of Stability of Deformable, Bodies- Berlin, Heidelberg: Springer-Verlag, 1999.

4. Guz A.N., Brittle Fracture Mechanics of Prestressed Materials –Kiev: Naukova Dumka, 1983 (in Russian).

5. Guz A.N., Brittle Fracture of Prestressed Materials- Vol.2 of the four-volume five-book series Guz, A.N (General Editor), Nonclassical Problems of Fracture Mechanics- Kiev: Naukova Dumka, 1983 (in Russian).

6. Guz A.N. and Guz I.A., On Publications on the Brittle Fracture Mechanics of Prestessed Materials// Int. Appl. Mech.- 2003- Vol. 39, No.7-P. 797-801.

7. Cristensen R.M., Mechanics of Composite Materials- New York: Wiley, 1979.

8. Tan C.L. and Gao Y.L., Treatment of bimaterial interface crack problems using the boundary element method // Eng. Fracture Mechanics-1990-Vol. 36, No. 6- P. 919-932.

9. Zienkiewicz,0.C and Taylor R.L., The Finite Element Method- 4th Ed. Vol. 1, Basic Formulation and Linear Problems-London: McGraw-Hill Book Company, 1989.

10. Akbarov S.D. and Rzayev O.G., Delamination of unidirectional viscoelastic composite materials// Mechanics of Composite Materials-2002-Vol. 38, No.1- P.17-24.

11. Akbarov S.D., Yahnioğlu, N and Turan, A., Influence of Initial Stresses on Stress Intensity Factors at Crack Tips in a Composite Strip// Mech. Compos. Mater.-2004-Vol. 40, No.6- P. 299-308.