Monte Carlo simulations of production of collimated panoramic X-rays using a conical anode-type X-ray tube
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Abstract
Monte Carlo simulations were used to study photon production in a panoramic X-ray tube with conical tungsten anode target to investigate the optimal characteristics of the target shape and electron beam configurations in order to maximize the intensity of the output X-rays. Several simulations were done for relevant NDT acceleration potentials of 250 kV, 300 kV and 500 kV and for different radii of electron beam and anode sizes. The angular distribution of the photon intensity was analyzed by numerical calculations for an X-ray tube and external collimator configuration to simulate a panoramic scanning system producing an X-ray pencil beam. The several important parameters and dependencies were studied: X-ray intensity of generated photons in the energy range of 250-500 keV, optimal target shape, the intensity of collimated photons exiting through the collimator, and the efficiency coefficient for energy transfer from an electron beam to generated X-ray flux.

Keywords: X-ray, Monte Carlo simulation, panoramic scanning, conical anode, bremsstrahlung radiation

1 Introduction
X-ray techniques are commonly used in the non-destructive testing [1] of industrial parts, material characterization and many medical applications [2, 3, 4]. X-ray tubes with a conical target [5, 6] have the simplest design for 360\textdegree panoramic scanning systems [7] and can be used, for example, to inspect oil wells and pipes. For scanning technology, maximising the output X-ray intensity in the collimated beam is important. Variations in the electron beam diameter, the shape of the conical target, and the inclination angle of the output collimated X-ray beam can significantly affect the output X-ray intensity in the collimated beam and limit the performance of the scanning system. However, the literature shows a lack of detailed information published about the effect of the geometrical parameters of an electron beam and a conical target on the intensity of the angular distribution of collimated X-rays in the 250-500 kV range of accelerating potentials (which is interesting for industrial applications in which the panoramic scanning technique is used). The general purpose of this study was to investigate, using Monte Carlo simulations, the X-ray generation process in an X-ray tube with a conical tungsten anode and cylindrical symmetry. We determined the following parameters for use in practical applications: the intensity of generated X-rays in the energy range of 250-500 keV, the optimal target shape, the angular distribution of the intensity of the generated X-rays and collimated through a cylindrical hole from an external collimator, and the efficiency of X-ray generation.
2 Description of the Monte Carlo simulations

The Monte Carlo (MC) simulations were generated with the software suite Xenos by Field Precision [8]. The particular program to generate the Monte Carlo is called GamBet. GamBet combines Field Precision’s technology for finite-element codes with the package PPENELOPE [9]. PENELOPE is a public program for MC simulations of electron-photon transport in various materials. The solution volume for the object mesh had different ranges for the different sizes of the tungsten conical anode (it depends on the apex angle of the conical anode $2\gamma$, Figure 1) and for the different radii of the electron beam ($r_e$). MC simulations were done for the geometries presented in table 1 and Figure 1, when the electron energy was 250 keV, 300 keV and 500 keV. The number of high-energy electrons was the same for all simulations: $N_e = 4.613 \cdot 10^5$.

![Figure 1: A geometrical model of the conical tungsten target and electron beam that were used for the MC simulations.](image)

The radial distribution of electrons in an electron beam was modelled as a Gaussian function of the diameter of the electron beam ($D_e$):

$$f_e(x, y, D_e) = \frac{1}{2\pi \cdot \left(\frac{D_e}{4}\right)^2} \cdot \exp \left(-\frac{x^2 + y^2}{2 \cdot \left(\frac{D_e}{4}\right)^2}\right)$$

This electron distribution corresponds to a 90% probability of any electrons occurring within an electron beam with a diameter $D_e$.

<table>
<thead>
<tr>
<th>$r_e$, mm</th>
<th>$2\gamma$, degrees</th>
<th>X-axis, mm</th>
<th>Y-axis, mm</th>
<th>Z-axis, mm</th>
<th>Resolution, $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10</td>
<td>[0; 6]</td>
<td>[-0.6; 0.6]</td>
<td>[0; 1.0]</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
<td></td>
<td>[0; 3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td></td>
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<td>[0; 1.5]</td>
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<td></td>
<td>90</td>
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<td>[0; 1.0]</td>
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<td></td>
<td>110</td>
<td></td>
<td></td>
<td>[0; 1.0]</td>
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</tr>
</tbody>
</table>
3 The energy distribution of generated photons

X-ray spectra were generated using an MC generator with electron energies equal to 250 keV, 300 keV, and 500 keV. These spectra were created to investigate the effect of the shape of the conical anode (target) and the electron beam size on the efficiency of X-ray generation and on the angular distribution of the generated and collimated photons. The energy distributions of all generated X-rays (which were emitted in different directions after being generated but with the same energy within an energy bin of 2 keV) obtained when the electron energy was equal to 250 keV and 500 keV are shown in Figures 2 and 3.

Figure 2: The energy distributions of the photons generated at an electron energy of 250 kV and at different values of the apex angle of the conical anode. $r_e = 2$ mm.

<table>
<thead>
<tr>
<th>2.0</th>
<th>10 \ 30 \ 60 \ 90 \ 110</th>
<th>[-2.2; 2.2] \ [-2.2; 2.2]</th>
<th>[0; 23] \ [0; 10] \ [0; 5] \ [0; 3] \ [0; 3]</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>10 \ 30 \ 60 \ 90 \ 110</td>
<td>[-5.1; 5.1] \ [-5.1; 5.1]</td>
<td>[0; 60] \ [0; 20] \ [0; 10] \ [0; 10] \ [0; 5]</td>
<td>35</td>
</tr>
</tbody>
</table>
It was observed that the shapes of the X-ray spectra of all X-ray photons generated in $4\pi$ solid angle were the same for different electron beam radii. There are some differences in the intensity of the X-ray spectra in the range of energies up to approximately 50 keV for the different types of conical anodes, which showed different apex angle values. These differences occurred because the photons had a larger attenuation depth for larger values of the apex angle $2\gamma$. This effect may also depend on the angular distribution of the generated photons, and it had a small influence on the total intensity of the generated photons within 1.0-1.2% for all simulations at different electron energies.

4 The angular distribution of the photon intensity in collimated X-ray beam

To simplify the MC simulations, which require a large amount of computation time, the following simple numerical method was used to simulate an external cylindrical collimator with any thickness $\Delta R$ and any radius. For this task, it is necessary to know the spatial distribution of the photons on the first surface of the collimator (with inner radius $R_c$) and on the second surface of the collimator (with outer radius $R_c + \Delta R$). This spatial distribution can be calculated using two translations of the spatial positions of the photons on the surface with radius $R_c$ and then to the surface with new radius $R_c + \Delta R$; moreover, it is necessary to estimate the geometrical position of the cylindrical hole inside the collimator. Only photons whose coordinates lie on the hole on the first collimator surface and on the hole on the second collimator surface will be able to pass through the collimator hole. To minimise the statistical deviations in the calculated number of collimated photons ($\Delta N$), it is possible to simulate the circular slit instead of the hole. This estimate is acceptable because a cylindrical geometry is used, which is symmetrical about the Z-axis in any direction. The number of photons will be greater with a circular slit and depend on the radius of the collimator hole ($r_c$) and on the external radius of the collimator:

$$\frac{\Delta N_{\text{slit}}}{\Delta N_{\text{hole}}} \approx \frac{2\pi (R_c + \Delta R) 2r_c}{\pi r_c^2} = \frac{4(R_c + \Delta R)}{r_c}$$

(2)
A simplified schematic of the measurement of the number of photons that can pass through the collimator slit (or hole) in the proposed model is shown in Figure 4.

Figure 4: A schematic (side view) of the measurement of the number of photons (generated by MC) that can pass through the collimator slit.

The spatial positions of the generated photons \((x_i, y_i, z_i)\) after the MC simulation were distributed on an external rectangular surface (the conical anode was located inside this cubic solution volume), which was determined by programming a mesh for the volume of simulated objects. To investigate the angular distribution of the photons in cylindrical geometry (which is more useful for the panoramic scanning technique), it is necessary to translate the initial spatial positions of the photons on the surface of the external cylinder into a radius \(R_c\) (the value of 10 mm was used). This translation can be performed in the following way. First, we form a ray directed from the initial position of a photon on the rectangular surface to a second position on the cylindrical surface using the following linear coordinate transformation:

\[
\begin{align*}
  x_{2i} &= x_i + x_{di}t_i \\
  y_{2i} &= y_i + y_{di}t_i \\
  z_{2i} &= z_i + z_{di}t_i
\end{align*}
\]  

(3)

where the direction vector \((x_{di}, y_{di}, z_{di})\) is formed by the normalised components of the momentum vector \((p_{xi}, p_{yi}, p_{zi})\). Then, we substitute the ray into the parametric equation for a cylinder:

\[
x_{2i}^2 + y_{2i}^2 = R_c^2
\]  

(4)

Next, we solve for \(t\):

\[
t_i = \frac{x_{2i}x_{di} + y_{2i}y_{di}}{x_{di}^2 + y_{di}^2} \left( \sqrt{1 - \left( \frac{x_{di}^2 + y_{di}^2}{x_{2i}^2 + y_{2i}^2} \right) \left( x_{2i}^2 + y_{2i}^2 - R_c^2 \right)} - 1 \right)
\]  

(5)

Lastly, we substitute the value of \(t\) back into the equations in Formula (3), which give the new coordinates \((x_{2i}, y_{2i}, z_{2i})\) of the photons on the surface of the external cylinder (as it is shown in Figure 4). The translation of the photon coordinates on the second cylindrical surface of the external collimator was performed by using Formulae (3) - (5). As shown in Figure 4, the direct projection of the output X-ray beam on Z-axis gives the height of the collimation slit as follows: \(\Delta Z_c(\alpha) = 2r_c/\sin(\alpha)\).
Therefore, the following logical conditions were used to select the photons with a momentum vector directed through the collimator slit. These logical conditions can be used only for Z-coordinates because for each photon located on the cylindrical surface with radius \( R_c \), the following relation is satisfied: \( x_j^2 + y_j^2 = R_c^2 \). The condition for the first cylindrical surface within collimation slit is as follows:

\[
Z_c + \frac{R_c}{\tan(\alpha)} < Z_{2i} < Z_c + \frac{R_c}{\tan(\alpha)} + \frac{2r_c}{\sin(\alpha)},
\]

where \( Z_c \) is the coordinate of the top of the cone and \( Z_{2i} \) is the coordinate of the photon with index \( i \) after the first coordinate translation onto the first cylindrical surface (from the initial square surface). The condition for the second cylindrical surface within collimation slit is similar:

\[
Z_c + \frac{R_c + \Delta R}{\tan(\alpha)} < Z_{3i} < Z_c + \frac{R_c + \Delta R}{\tan(\alpha)} + \frac{2r_c}{\sin(\alpha)},
\]

where \( Z_{3i} \) is the Z-coordinate of the photon with index \( i \) after the second translation onto the second cylindrical surface with radius \( R_c + \Delta R \). The following geometrical parameters of the external collimator were used for all simulations: \( R_c = 10 \) mm, \( \Delta R = 3.5 \) mm and \( r_c = 0.45 \) mm. Typical results for the simulations of the beam intensity after collimation when the radius of the electron beam is 2.0 mm and the electron energy is 250 keV are shown in Figure 5, with the electrical power of the electron beam normalised to 1.0 W.

It was observed that the shape of the function of the angular distribution of the photon intensity after collimation is different for different electron energies, radii of the electron beam and shapes of the conical anode. The maximum value of the photon intensity in the collimated X-ray beam (at an inclination angle \( \alpha_{jm} \)) is one of the important parameters in this case. The relationships between these parameters are shown in Figures 6-8 for different electron energies and geometries. A spline

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**Figure 5:** The angular distributions of the intensity of collimated X-ray photons (recalculated for a collimator hole with radius \( r_c \)) in cylindrical geometry at different values of the top angle of the anode when the electron energy is equal to 250 keV and the electron beam radius is equal to 2.0 mm.
interpolation within the obtained statistical errors was used to connect the points on the presented figures.

Figure 6: The dependences of the inclination angle, associated with the maximum value of the intensity of collimated X-rays (upper plot), and of the maximum value of the intensity of collimated X-rays (lower plot) on the value of the apex angle of the conical anode at an electron energy equal to 250 keV and electron beam radii equal to 0.5 mm (1), 2.0 mm (2) and 5.0 mm (3).

Figure 7: The dependences of the inclination angle, associated with the maximum value of the intensity of collimated X-rays (upper plot), and of the maximum value of the intensity of collimated X-rays (lower plot) on the value of the apex angle of the conical anode at an electron energy equal to 300 keV and electron beam radii equal to 0.5 mm (1), 2.0 mm (2) and 5.0 mm (3).
Figure 8: The dependences of the inclination angle, associated with the maximal value of the intensity of collimated X-rays (upper plot), and of the maximal value of the intensity of collimated X-rays (lower plot) on the value of the apex angle of the conical anode at an electron energy equal to 500 keV and electron beam radii equal to 0.5 mm (1), 2.0 mm (2) and 5.0 mm (3).

The maximum errors in the inclination angles \( \alpha_{jm} \) were estimated to be \( 2\Delta\alpha = 6^\circ \), and a step \( \Delta\alpha \) was used to calculate the number of X-ray photons in the output beam after the collimation (using Formulae 6 and 7). The errors in the maximum values of the photon intensity \( I_{xm} \) at an inclination angle \( \alpha_{jm} \) were calculated as probable statistical deviations of the number of photons \( \Delta N_j \) that were passed through the collimation slit using following Formula [10]:

\[
\Delta I_{xm}(\alpha_{jm}) = \frac{I_{xm}(\alpha_{jm})}{\sqrt{\Delta N_j(\alpha_{jm})}}. \tag{8}
\]

The results, presented in Figures 6-8, allow for the selection of the optimal value of the anode apex angle \( \gamma \) or the inclination angle \( \alpha \) that maximises the intensity of the output X-ray beam if one of these parameters is known. The results (except for the precise value of the intensity after the collimation) hold for any collimator with the same ratios between geometrical parameters such as distance from the anode, the collimator thickness and the diameter of the collimator hole. An acceleration potential in the range of 250-500 kV does not significantly affect the maximum flux of the generated X-rays at the same electrical power and at the same radial distance from the symmetry axis of the conical anode. The probable variations are located within the range of the estimated numerical errors.

5 The efficiency of X-ray generation

High-energy electrons have different stopping probabilities for different thicknesses of the tungsten target and therefore have different probabilities of the generation of bremsstrahlung radiation. To determine the efficiency of X-ray generation, several simulations were carried out for electron energies equal to 250 keV, 300 keV and 500 keV. The X-ray spectra for the acceleration potentials equal to 250
kV and 500 kV are shown in Figures 2 and 3. The efficiency coefficient for the energy transfer from an electron beam to the flux of generated X-rays for all directions over a solid angle $4\pi$ was calculated using the following relation:

$$\eta_X \approx \frac{\sum_j \left( \frac{\partial N_{xi}}{\partial E} \cdot \delta E \cdot E_j \right)}{N_{e0} \cdot E_{e0}}, \quad (9)$$

where $\delta E = 2$ keV is the energy step of the simulated X-ray spectrum, which has energy distribution $\partial N_{xi}/\partial E$, and $N_{e0}$ is the initial number of high-energy electrons with energy $E_{e0}$. The error in the efficiency coefficient was calculated using the following relation [10]:

$$\delta \eta_X = \frac{1}{N_{e0} E_{e0}} \sqrt{\sum_j \left( \frac{\partial N_{xi}}{\partial E} \cdot \delta E \cdot E_j \right)^2} \quad (10)$$

The calculated efficiency coefficients of the energy transfer from the electron beam to the flux of generated X-rays are shown in Figures 9-10 for electron energies equal to 250 keV and 500 keV and for different electron beam radii and apex angles of the conical anode. The theoretical efficiency coefficient was calculated and plotted as well to compare with the efficiency of the output flux of generated photons to estimate how the different shapes of the conical anode influence the additional attenuation of the generated photons. This efficiency can be estimated for bremsstrahlung radiation using the following relation [15]:

$$\eta_X \approx (8 \pm 2) \cdot 10^{-10} Z E_e, \quad (11)$$

where the electron energy $E_e$ has units of “eV”.

Figure 9: The dependences of the efficiency coefficients of the energy transfer from the electron beam to the flux of generated photons on the apex angle of the conical anode for different electron beam radii and at an electron energy equal to 250 keV.
Figure 10: The dependences of the efficiency coefficients of the energy transfer from the electron beam to the flux of generated photons on the apex angle of the conical anode for different electron beam radii and at an electron energy equal to 500 keV.

As shown in Figures 9 and 10, an X-ray tube with a conical target shows relatively low X-ray generation efficiency (approximately 6 times less) compare to the total efficiency of the generation of bremsstrahlung radiation [12]. Also X-ray generation efficiency is better for the narrow electron beam. The results obtained for different electron energies and for different geometries of the electron beam and target demonstrate that the additional attenuation inside the target is the main reason for this effect, which clearly depends on the range of electron energies in the target medium [13].

6 Conclusions
From Monte Carlo simulations of the X-ray generation process in an X-ray tube with a conical target (anode), several important parameters and dependences were determined: the X-ray spectra in the acceleration potential range of 250-500 kV, the optimal target shape, the angular distribution of the intensity of X-rays collimated through the cylindrical hole or slit of an external collimator and the efficiency coefficient for energy transfer from an electron beam to a generated X-ray flux. It was demonstrated that the angular distribution of the intensity in a collimated X-ray beam depends on the radius of the electron beam and on the shape of the conical target and that these dependences are different at different acceleration potentials of the X-ray tube. The optimal values of the geometrical parameters of a scanning system based on an X-ray tube integrated with an external collimator were observed to maximise the output X-ray intensity in the beam. These results and the proposed methods for the numerical modeling of the external collimators can be used in practical applications to design scanning systems based on the pencil-beam technique and X-ray tubes with massive conical targets.
References


