Accurate surface extraction on CT volume using analytical gradient of FDK formula

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Abstract
In this paper, we propose an accurate surface extraction algorithm from X-ray CT volumes for industrial applications. Isosurfacing is well known and the most widely used surface extraction, but varying the isovalue may cause deformation of the extracted shape and the resulting surface is not very suitable to some industrial applications. Another type of surface is the gradient maximal surface that is the set of the maxima of the gradient norm of CT value. The gradient maximal surface gives more accurate surface in the sense of being less affected by inhomogeneity of the CT value than the isosurface. For computing the gradient, there are two ways. The discrete gradient first integrates the projection values to obtain the CT value, and then differentiates the CT value. In contrast, the analytical gradient first differentiate the projection values to obtain the CT value, and then integrates the obtained gradient by the filtered back projection (FDK). In this paper, we investigate the influence of the gradient computation ways on the accuracy of their extracted surfaces. We found that the analytical gradient contributes more than the discrete gradient to the accuracy of the extracted gradient maximal surface, and show this fact with some experimental results and comparisons of them.

Keywords: surface extraction, CT volume, maxima of gradient norm, analytical differentials, FDK

1 Introduction
1.1 Background
Defining a surface of an object on its CT volume is a necessary process in measuring the shape of the object. Since the CT value generally blurs around the boundary of an object because of the discretization error and the optical factors as the focal spot size and scattering radiation, the CT value varies smoothly across the boundary of the object and the exact boundary position becomes unclear as in Figure ?? (a). For blurring boundaries, thresholding the CT value extracts different shapes with different threshold values as shown in Figure ?? (b) and (c).

To obtain a more accurate surface, the maxima of the norm of the CT gradient is used [? ]. Here we call this surface as gradient maximal surface. This algorithm defines the shape of the object as the points with the maxima of the CT gradient norm and works robustly to the blurred boundaries as observed by comparing the resultant extraction (d) with (b) and (c) in Figure ??.

It is easily understandable that the quality of the gradient of the CT value has a large influence on the result of shape extraction. It is widely recognized the advantage of the gradient maximal surface over the traditional isosurface, however, how the ways of computing the gradient affect the accuracy of the gradient maximal surface has not been analyzed.

The CT volume consists of points equipped with CT values arranged on an orthogonal grid. The gradient of the CT value at a grid point is generally computed by the central difference of the CT values on the grid. The gradient at an arbitrary point is obtained by interpolating the gradients at the neighboring grid points. Many algorithms use the differential values computed in this way, but it is worthy to notice that the differential values can also be computed by analytically differentiating the formula of CT value computation. We refer the gradient obtained by computation on the voxel grid as discrete gradient, and the gradients computed by differentiating the formula for computing the CT value as analytical gradient. The analytical gradient is expected to be less affected by the discretization and give more accurate surfaces than the discrete gradient. In this paper, we propose to define surfaces by the analytical gradient maximal surface and show the advantages of this proposal over the discrete gradient maximal surface.

Figure 1: Unclear boundary problem caused by the blur of CT value. (a) a cross-section of a CT volume, (b) and (c) results of thresholding CT values with two different threshold values, and (d) a result of selection by the gradient maxima of the CT value.
1.2 Related work

There are plenty of region extraction algorithms as segmentation in image processing field, but here we only focus on the ones which can be used for industrial applications, especially for general objects. It means that our target objects do not include microstructures as fibers and pores which can be more efficiently extracted using prior information of their shape.

The simplest shape extraction algorithms is CT value-based thresholdings (isosurfacing). ISO-50%’s threshold is the mean value of the peak of the object’s CT value and the peak of the background’s CT value. This is very intuitive and most commonly used for visualization, checking, and extraction of the measured object’s shape, but it may cause some troubles to a blurring CT volume as illustrated in Figure 2. Otsu’s method gives the threshold value which maximizes the separation metrics of the CT value histogram. It gives much better results than ISO-50%. Unfortunately general CT volumes include artifacts which cause partial changes of CT value. It changes the histogram too, so Otsu’s method cannot avoid the influence of them.

Shammaa et al. proposed to use region growing and global optimization for more smart segmentation of CT datasets. It overcame the limitation of thresholding methods to the inhomogeneity of CT value, but it’s a voxel-accurary optimization and extracts a surface based on the CT value.

As a differential-based surface extraction, Canny edge detector is widely used and implemented in some commercial softwares. Canny edge detector essentially extracts the points with the local maxima of the gradient norm in the direction of the gradient of the CT value. Since the gradient is less influenced by the change of the CT value caused by artifacts than CT value thresholding, this gradient-based method is considered more suitable for accurate shape extraction and becomes the basement of our gradient maximal surface.

For the gradient itself, Yamakawa et al. proposed to define the surface using the analytical gradient for the first time. They only implicitly used this differential value for estimating the normals of surfaces and has not extracted surfaces by the analytical gradient maximal surface. Applying the analytical gradient to the gradient maximal surface and investigating the potential of it is the aim of our research.

1.3 Paper organization

This section was devoted for describing the research background and introducing the existing papers regarding accurate surface extraction from CT volumes. Section 2 will be for explaining the algorithms for computing the CT value gradient and for the proposing a surface extraction algorithm. In Section 3, the results obtained by the proposing algorithm will be shown and compared with results by other methods. The accuracy will be evaluated. Section 4 will conclude this paper and show the future work.

2 Algorithm

This time we assume to reconstruct CT volumes by the FDK algorithm which is the most commonly used for reconstructing cone beam CT datasets, but the same argument can hold for other CT reconstruction algorithms.

In this section, we first introduce the computation of the CT value by FDK algorithm, and then show the discrete gradient and the analytical gradient. Lastly, we will explain our algorithm for computing the gradient maximal surface.
2.1 FDK algorithm

Let us consider the CT-scanning setting with the coordinate systems shown in Figure 2.1. The CT value \( f(p) \) at a 3-dimensional point \( p = (x, y, z) \) in the cylindrical reconstruction region is computed by the following equation [1]:

\[
f(p) = \frac{1}{2} \int_0^{2\pi} \alpha(\theta, p)^2 S_{\text{filtered}}(\theta, p) d\theta.
\]  

The value \( \alpha(\theta, p) \) is defined by \( d_{\text{rod}}/(d_{\text{rod}} + d_z) \) where \( d_{\text{rod}} \) is the source-to-object distance and \( d_z \) is the \( z \) coordinate of \( p \) after a rotation by an angle of \( \theta \). The value \( S_{\text{filtered}}(\theta, p) \) is the filtered projection value of the X-ray passing through \( p \) rotated by \( \theta \). For deconvolution, Shepp-Logan filter [2] was applied in this paper.

2.2 Gradient of CT value

The gradient of CT value has important roles on extracting object shape from CT volumes and it is easily understandable that the gradient’s accuracy severely affects the accuracy of the extracted shape. The gradient is generally computed on a tomogram which is a discrete dataset of CT values sampled at grid points. Thus the gradient computed on the grid must include some discretization error. If the gradient can be computed without discretizaion by the grid, it is expected to improve the accuracy.

For FDK algorithm’s case, the analytical gradient can be computed on the projection image before the discrete reconstruction of the CT volume. This is different from the discrete gradient computation for which the filtered back projection is first conducted, and then the gradient is computed on the obtained CT volume.

**Discrete gradient.** The computation of the discrete gradient \( \nabla \text{dis} f(p) \) of the CT value at a grid point is obtained by the central difference of the neighboring CT values. If FDK algorithm is used for computing the CT value, the gradient is expressed as

\[
\nabla \text{dis} f(p) = \frac{\partial}{\partial(x,y,z)} f(p) = \frac{1}{2} \int_0^{2\pi} \alpha(\theta, p)^2 S_{\text{filtered}}(\theta, p) d\theta.
\]  

The content of the bracket on the right hand side means the CT value computed by FDK.

The discrete gradient \( \nabla \text{dis} f_{i,j,k} \) at the \((i, j, k)\)-th grid point (voxel) can be obtained from the CT values at the neighboring voxels distance of \( \delta \) away of the \((i, j, k)\)-th voxel.

\[
\nabla \text{dis} f_{i,j,k} = \left( \frac{f_{i+1,j,k} - f_{i-1,j,k}}{2\delta}, \frac{f_{i,j+1,k} - f_{i,j-1,k}}{2\delta}, \frac{f_{i,j,k+1} - f_{i,j,k-1}}{2\delta} \right)^T.
\]  

The discrete gradient at an arbitrary point is realized by the trilinear interpolation of the gradients at the neighboring grid points computed by Equation (??).

**Analytical gradient.** Yamanaka et al. [2] showed that the gradient of the CT value could also be simply computed by differentiating the FDK formula by \( u \) and \( v \) (the detector coordinates). Under the assumption that \( \alpha \) is constant (free from \( p \) for each \( \theta \)), the analytical gradient \( \nabla \text{ana} f(p) \) is computed by

\[
\nabla \text{ana} f(p) = \frac{1}{2} \int_0^{2\pi} \alpha(\theta, p)^2 R_{-\theta} \left( S_{\text{filtered}}(\theta, p), 0 \right)^T d\theta
\]  

where \( R_{-\theta} \) is a 3D rotation matrix by \(-\theta\). The operator \( \partial/\partial(u,v) \) means the differential conducted in the detector coordinate system.

As the discrete differential, \( \frac{\partial}{\partial(u,v)} S_{\text{filtered}}(\theta, p) \) at an arbitrary point can be computed by the bilinear interpolation of the central differences of \( S_{\text{filtered}}(\theta, p) \) at the neighboring pixels, \( \{S_{i,j,k}\} \). The neighboring pixels for interpolation are partially illustrated in Figure 2.2 (b). The differentials after rotation will be

\[
R_{-\theta} \begin{pmatrix} \frac{\partial}{\partial u} S_{\text{filtered}}(\theta, p) \\ \frac{\partial}{\partial v} S_{\text{filtered}}(\theta, p) \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial u} S_{\text{filtered}}(\theta, p) \\ \frac{\partial}{\partial v} S_{\text{filtered}}(\theta, p) \\ 0 \end{pmatrix}
\]  

\[
= \begin{pmatrix} \cos \theta \cdot \frac{\partial}{\partial u} S_{\text{filtered}}(\theta, p) + \sin \theta \cdot \frac{\partial}{\partial v} S_{\text{filtered}}(\theta, p) \\ -\sin \theta \cdot \frac{\partial}{\partial u} S_{\text{filtered}}(\theta, p) + \cos \theta \cdot \frac{\partial}{\partial v} S_{\text{filtered}}(\theta, p) \\ 0 \end{pmatrix}
\]
2.3 Gradient maximal surface

Here we show the flow of the gradient maximal surfaces computation on a CT volume. The data flow is also shown in Figure ??.

The orange arrows show the data flow for the proposing algorithm. The blue arrows mean the data flow of the existing algorithms which take a CT volume as input, extract an isosurface and move its vertices or directly extract the gradient maximal surface from the tomogram.

For the proposed algorithm, the measured object is supposed to be made of a single material. The algorithm takes its sinogram as input, and outputs a triangulated gradient maximal surface. The algorithm is below.

1. Extract an isosurface mesh from the CT volume.
2. For each vertex of the mesh, sample the gradient values in the direction of the gradient of the CT value.
3. Move each vertex to a point with the largest gradient norm.

Step 1 can be conducted by an existing grid-based polygonization algorithm, such as Marching cubes [? ]. The gradient computed in Step 2 and 3 are the analytical gradient in Equation (??). In Step 2, the sampling pitch is 0.2 of the voxel size and the number of samples is 20 in each of ±g directions in our experiments. In Step 3, the vertex is moved to the sample with the largest gradient norm. This vertex relocation is illustrated in Figure ??.

3 Results and discussion

We applied the algorithm proposed in the previous section to real datasets. The CT volumes here were acquired with METROTOM 1500 by Carl Zeiss. We implemented a CT reconstruction algorithm by our selves. Distortion of the detector panel is not taken into account to the implementation, thus large error values are observed in some areas.

The first example is for an aluminum step cylinder. The input CT volume has 1000 × 1000 × 1009 voxels with the voxel size of 121.7µm. The CT value has inhomogeneity caused by the beam hardening as in the cross-section in Figure ?? (a).

From this CT volume and its sinogram, we extracted the maximal gradient surfaces with three different ways: VGSTUDIO MAX 3.2 [? ], the proposed algorithm with the discrete gradient \( \nabla_{\text{dis}} f(p) \) in Equation (??), and the proposed algorithm with the analytical gradient \( \nabla_{\text{ana}} f(p) \) in Equation (??). For computing the gradient maximal surface with the discrete gradient, we used the algorithm proposed in section ?? replacing the gradients from the analytical one to the discrete one.
Figure 4: Vertex relocation of the proposed algorithm. Left: the initial state and right: after the relocation. Each vertex \( \nu \) is moved to the sample point with the largest gradient norm.

Figure 5: Results for an aluminum step cylinder. (a) a cross-section of the CT volume. (b) the discrete gradient maximal surface, and (c) the analytical gradient maximal surface. On the surfaces in (b) and (c), the deviations from the fitted cylinders were color-coded.

Figure ?? (b) and (c) show the surfaces obtained by the discrete gradient maximal surface and the analytical gradient maximal surface respectively. The deviation from the best-fit cylinder at each step was color-coded with red for outward deviation and blue for inward deviation. The graph in Figure ?? indicates the standard deviations of the distances from the mesh vertices to the fitting cylinders. These images and the graph show that the analytical gradient maximal surface has smaller deviation than the discrete gradient maximal surface.

Figure ?? is the surfaces obtained by VGSTUDIO MAX and the analytical gradient maximal surface. Table ?? shows the comparison of the accuracy of these two surfaces. We checked the radius errors from CMM measurement and the standard deviations at each step. The radius error is the difference of the best-fit cylinder’s radius from the radius measured by CMM. The uncertainty of the CMM was 0.9 \( \mu \)m. The analytical gradient surface showed a better accuracy on the first and the second steps, while VGSTUDIO MAX gave better results on the other steps.

The next example is an aluminum machine component with more complex geometry. The number of voxels is 1000 \( \times \) 1000 \( \times \) 896 and the voxel size is 121.6\( \mu \)m. The discrete and the analytical maximal gradient surfaces were compared with a surface acquired by an optical scanner, ATOS Core 135 by GOM. The results were indicated with color in Figure ??, it is clearly observed that the analytical gradient maximal surface gave a more accurate surface with less artifact than the discrete one, see the areas surrounded by the pink squares.

Another experiment is for a forest gauge with 27 ruby spheres. The diameter of a sphere is measured as 4.997mm by CMM. The CT volume has 1024 \( \times \) 1024 \( \times \) 516 voxels with the voxel size of 118.7 \( \mu \)m. Figure ?? shows a gauge’s photo and extracted surfaces by isosurfacing, the discrete gradient maximal surface, and the analytical gradient maximal surface. Table ?? shows the diameters of the extracted spheres. Average diameter means the average of the radii of the 27 fitting spheres and average of diameter differences is the difference of the average diameter from the diameter measured by CMM, 4.997mm. In this case, the isosurfacing gave surfaces smooth but larger than the real spheres. The discrete gradient maximal surface showed a better radius values but were suffered from the grid artifact. The analytical gradient maximal surface generated comparatively smooth surfaces with the highest accuracy.
4 Conclusion and Futurework

In this paper, we proposed the analytical gradient maximal surface, which is an accurate surface extraction algorithm from a sinogram using the analytical gradient of the CT value. This algorithm’s effectiveness was shown with some experiments, but there is still room for improvement of the accuracy.

Now we are planning to give a theoretical support for this empirical observation and investigate more the ability of the analytical gradient maximal surface. One issue to be solved is the extracted surface’s roughness as in Figure 7 (d). A possible solution is to determine the new position of vertex using another method which can give a higher smoothness of the surface, for instance, an interpolation with a higher degree. Handling artifacts as beam hardening is also necessary for generating a practically high quality surface.

References

Figure 8: Results for a machine component in the left photos. The discrete gradient maximal surface (middle) and the analytical gradient maximal surface (right) are shown. The magnified images of the highlighted regions are also indicated. The deviations from the optical scanned surface were color-coded.

Figure 9: Results for an forest gauge and a comparison of the accuracy of the extracted surfaces. (a) a photo of the forest gauge, (b) to (d) closeup images of the ruby sphere surrounded by the pink squares in (e) to (g). Note that the value range of (b) (isosurface) is three times larger than (c) and (d) for a better visualization. (e) an isosurface, (f) the discrete gradient maximal surface, and (g) the analytical gradient maximal surface. The value range of (e), (f) and (g) is common and indicated on the right of (g). On the surfaces, the deviations from the fitted spheres with the diameter of the true value (4.997 µm) were color-coded.

Table 1: Comparison of the gradient maximal surfaces by VGSTUDIO MAX and the analytical gradient maxima to the CMM measurement. Radius error is the difference from the radius of the best-fit cylinder of an gradient maximal surface to the result of CMM. SD means the standard deviation of the distances of the mesh vertices to the fitted cylinder.

<table>
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<th>Step</th>
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<th>CMM</th>
<th>Gradient maximal surface</th>
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<td>SD [µm]</td>
<td>Radius [mm]</td>
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Table 2: Evaluation of the diameters of the extracted spheres. Average diameter means the average of the radii of the 27 fitting spheres. Average of diameter differences is the difference of the average diameter from the diameter measured by CMM.

<table>
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<tr>
<th></th>
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<th>Analytical grad. max. surf.</th>
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</thead>
<tbody>
<tr>
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