

CT geometry determination using individual radiographs of calibrated multi-sphere standards

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Abstract

Determining the absolute CT machine geometry is crucial for performing accurate measurements. A common method to achieve this is recording a set of radiographs of calibrated multi-sphere standards under different angles of rotation and subsequently minimising reprojection errors of the sphere positions. Here, we tested methods using individual radiographs and sets of radiographs under accurately measured displacements. We focused on source-object and source-detector distances that, if not accurately known, introduce scale errors. However, it is emphasised that all geometrical parameters (e.g. detector and rotation axis angular deviations) were determined. Using individual radiographs enabled stage error motions and drifts to be determined. The sensitivities of the proposed method were evaluated by displacing the CT's motion axes on specified trajectories, and were below one micrometre for displacements parallel to the detector and about 10^{-4} for deviations in the source-detector distance. Using sets of radiographs recorded during one revolution enabled average parameters of the CT geometry to be determined and increased the accuracy of the method. First, the dependence on the number of radiographs, i.e. sphere positions, was investigated. Next, sets of radiographs of the multi-sphere standard were recorded at different source-object distances, the relative displacements of which were accurately measured by interferometry. Using this information improved the relative accuracy to determine displacements in the source-object distance to below 10^{-5} .

Keywords: CT machine geometry, correction, multi-sphere standard, metrology

1 Introduction

CT machine geometry is a key influence factor on the accuracy of CT data [1]. Therefore, it is critical to determine and correct deviations from the ideal geometry. Scanning a calibrated multi-sphere standard is a common and straightforward method to determine the geometry parameters of a CT (see e.g. [2,3]). Because the X-ray focal spot and the active detector surface lack mechanical reference points, their positions have to be determined in an indirect manner from radiographs. Subsequently, they can be related to the encoder index positions of the motion axes or to the index of the interferometric geometry measurement system of METAS-CT [4] that measures displacements relative to a metrology frame.

In this paper, the implementation of a method to determine the full CT geometry from individual radiographs and sets of radiographs is described. This enables the correction of errors originating from the rotary or translation axes or the characterisation of drift in the CT system. The method was experimentally evaluated by introducing specific deviations, which enabled the sensitivity for certain CT geometry parameters to be estimated. Furthermore, the systematic geometry errors were determined from a set of radiographs consisting of images of the multi-sphere standard recorded during one revolution. The dependence on the number of spheres was investigated by evaluating sets with different numbers of radiographs.

2 METAS-CT metrology system and rotary stage correction

2.1 CT geometry measurement system

In the following, METAS-CT is briefly described (see Figure 1, left; for more details refer to Reference [4]). The CT system consists of a high-resolution transmission X-ray tube and a high-resolution flat-panel detector (4000×4000 pixel). The positioning system comprises air-bearing rotary and linear axes. In addition to the rotary and linear stage encoders used for motion control, METAS-CT is equipped with a unique metrology system that enables continuous monitoring of the CT geometry. The metrology system consists of eight laser interferometers, five laser straightness sensors, and three CMOS based position sensors. Displacements are measured along the laser beams by the interferometers and perpendicular to them by the straightness sensors. The metrology system enables the position of the X-ray tube target (3 degrees of freedom; DoF), the rotary stage (6 DoF) and the flat-panel detector (6 DoF) to be measured relative to a metrology frame.

2.2 Correction of the rotary stage

The rotary stage (RT150, LAB Motion Systems) is a key element of the CT system and was, therefore, thoroughly characterised for its guideway and angular positioning errors [5]. To determine the CT geometry using a multi-sphere standard, the angular positioning was assumed to be ideal and therefore corrected as follows. The positioning accuracy was measured using a calibrated



36-sided optical polygon (10° angular step) and an autocollimator (Möller-Wedel ELCOMAT 3000). The measured deviations, obtained from two polygon orientations, were stored in a look-up table with 5° angular increments. Subsequently, the positioning was corrected using spline interpolation. To evaluate the accuracy of the interpolation, the optical polygon was rotated by fractions (5° , 3° , -1°) of its angular step (10°). As shown in Figure 1, right, the positions were within $\pm 5 \mu\text{rad}$, which corresponds to 100 nm peak-peak deviation 10 mm from the rotary axis. This is considered sufficient since it is on the order of the measurement uncertainty of the sphere position calibration (see Section 3.1).

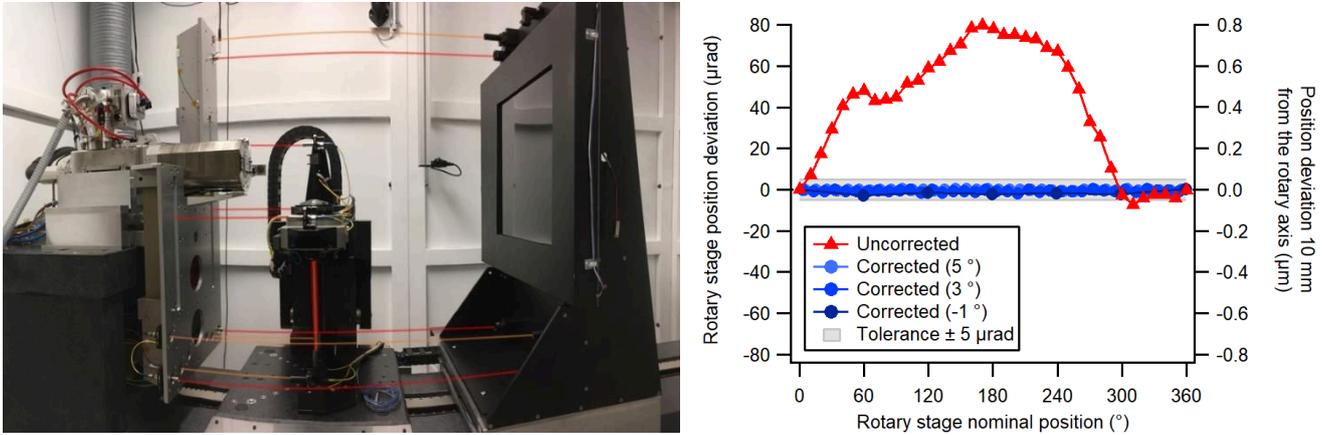


Figure 1: Left: Photograph of METAS-CT. The laser beams of the geometry measurement system are indicated in red (interferometers) and orange (straightness sensors). All measurements are made relative to the metrology frame on the left. Right: Positioning error of the rotary stage before (red triangles) and after correction (blue circles). The optical polygon was rotated by a fraction (5° , 3° , -1°) of its angular step (10°), to evaluate the interpolation accuracy of the correction.

3 Determination of the CT machine geometry using a calibrated multi-sphere standard

3.1 Design and calibration of the multi-sphere standards

Two multi-sphere standards were developed: One based on a hollow aluminium cylinder (\varnothing 22 mm, MSS-Alu, Figure 2, left) and another on a hollow carbon-fibre reinforced polymer cylinder (\varnothing 23.8 mm, MSS-CFRP), both fitted with 14 steel spheres (\varnothing 1 mm) and three reference spheres (\varnothing 1.5 mm). Under usual cone-beam opening angles, the 14 spheres on top and bottom of the cylinders, do not overlap in the radiographs and are therefore used for the CT geometry determination. The three additional reference spheres are used to identify a unique orientation and to link the tactile calibration measurements made from two sides. Tactile calibration of the sphere centre positions was performed using the METAS μCMM (see Reference [6]) with a measurement uncertainty of $0.1 \mu\text{m}$. The dominant uncertainty contribution was the form deviation of the spheres of about $0.2 \mu\text{m}$.

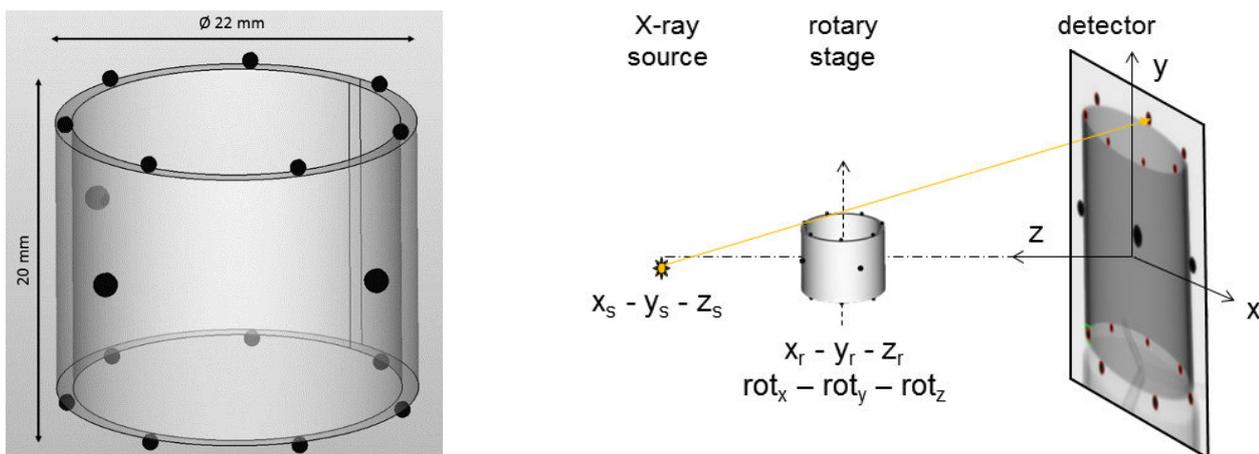


Figure 2: Left: CAD of the multi-sphere standard (MSS-Alu) consisting of a hollow aluminium cylinder and 17 steel spheres. Right: CT machine geometry determination using the multi-sphere standard: The ideal sphere positions are determined by forward projecting the calibrated sphere positions onto the detector plane (indicated by the yellow arrow). Comparison to the position determined in the radiographs, enables the CT geometry to be determined. The origin of the coordinate system is set in the detector centre, the X-ray source has 3 translational degrees of freedom, and the rotary stage (object) has 3 translational and 3 angular degrees of freedom.

3.2 CT geometry determination procedure

The CT geometry was defined as follows (see Figure 2, right): The origin of the coordinate system was defined in the centre of the flat-panel detector with the z-axis perpendicular to it, and the x- and y-axes parallel to the detector rows and columns, respectively. The X-ray source spot was assigned three degrees of freedom (DoF): Two deviations from the magnification axis (x_s, y_s) and the source-detector distance ($z_s = SDD$). The rotation stage, i.e. sample position, has three translational (x_r, y_r, z_r) and three rotational DoFs (rot_x, rot_y, rot_z). The object-detector distance z_r is not directly parametrised, but instead linked to the magnification M by $z_r = SDD(1 - 1/M)$. It is emphasised that the often used source-object distance (SOD) is related as follows: $SOD = SDD/M$. Assigning six DoFs to the rotation axis, instead of the commonly used four describing an ideal axis [3], enables rotation stage positioning errors as well as axial runout to be parametrised.

The following procedure was used to determine the CT machine geometry, i.e. the arrangement of the X-ray source spot, the rotary axis, and the detector plane (Figure 2, right): The calibrated sphere centre positions were forward projected (see Reference [7]) from the object onto the detector plane using an initial guess of the geometry. In parallel, the actual sphere centre positions on the detector were determined by applying a differentiation edge detection algorithm and fitting a circle. Subsequently, the correction proposed by Deng et al. [8] was applied to the sphere positions to account for elliptical distortions caused by the cone-beam projection. The nominal-actual deviations of the sphere centre positions were minimised by varying the CT geometry parameters using a nonlinear Levenberg-Marquardt algorithm in LabVIEW. First, the object position relative to the rotary stage is globally adjusted (6 DoF) and then held constant. Subsequently, the CT geometry (3 DoF for the X-ray source and 6 DoF for the rotary stage) was either determined for every radiograph separately, resulting in a CT geometry as a function of the nominal rotation angle (Figure 3a), or globally for a set of radiographs providing average parameters of the CT geometry (Figure 3b and c).

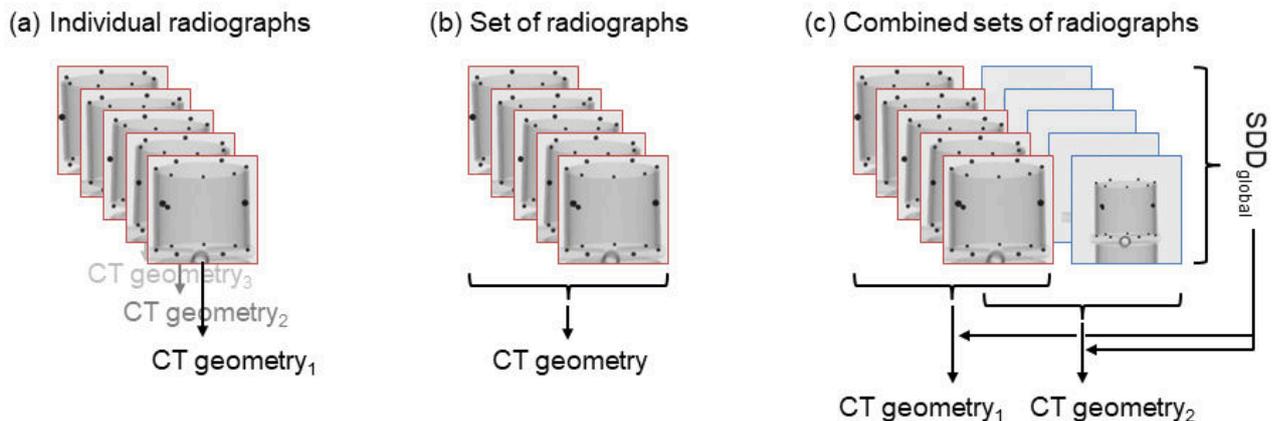


Figure 3: Different analyses were tested: (a) Individual radiographs resulting in a CT geometry for each projection angle (Section 4.1), (b) sets of radiographs from one revolution resulting in average parameters of the CT geometry (Section 4.2), (c) combined sets of radiographs recorded at accurately realised displacements in the source-object distance to determine the common source-detector distance (SDD_{global} , Section 4.3).

4 Measurements and results

4.1 Analysis of individual radiographs

To validate the method and to determine its sensitivity, 36 radiographs in steps of 10° were recorded on METAS-CT with a nominal SOD of 46.6 mm and SDD of 648 mm (nominal magnification: 13.9). The following geometry deviations were simultaneously introduced to the motion axes during one revolution: The rotary stage was displaced vertically in a sinusoidal pattern (frequency: 3 per revolution, amplitude: $10 \mu\text{m}$), the rotary stage angular positioning was modulated in a cosine pattern (frequency: 1 per revolution, amplitude 0.5°), and the source-detector distance was linearly altered over a range of 1 mm by displacing the detector. Even though exaggerated here, such errors could arise from rotary stage errors or drift. The built-in metrology system was used to monitor the simultaneously introduced deviations [4].

Figure 4 shows the results of the CT geometry evaluation method. The sensitivities, derived from the deviations between the actual values and the ones determined based on individual radiographs, strongly depended on the measured geometry parameters and how they influence the sphere positions in the radiographs: The sensitivity to axial rotary stage runouts was below $1 \mu\text{m}$, whereas it was below 0.1 mm for the source-detector distance, resulting in a relative sensitivity in the order of 10^{-4} . This difference in sensitivity is due to the fact, that displacements parallel to the detector plane (e.g. rotary stage axial errors) directly translate into displacements in the radiograph enlarged by the magnification. In contrast, changes perpendicular to the detector plane (e.g. SDD) cause much smaller displacements in the radiographs and depend on the cone-beam angle at the corresponding sample position.

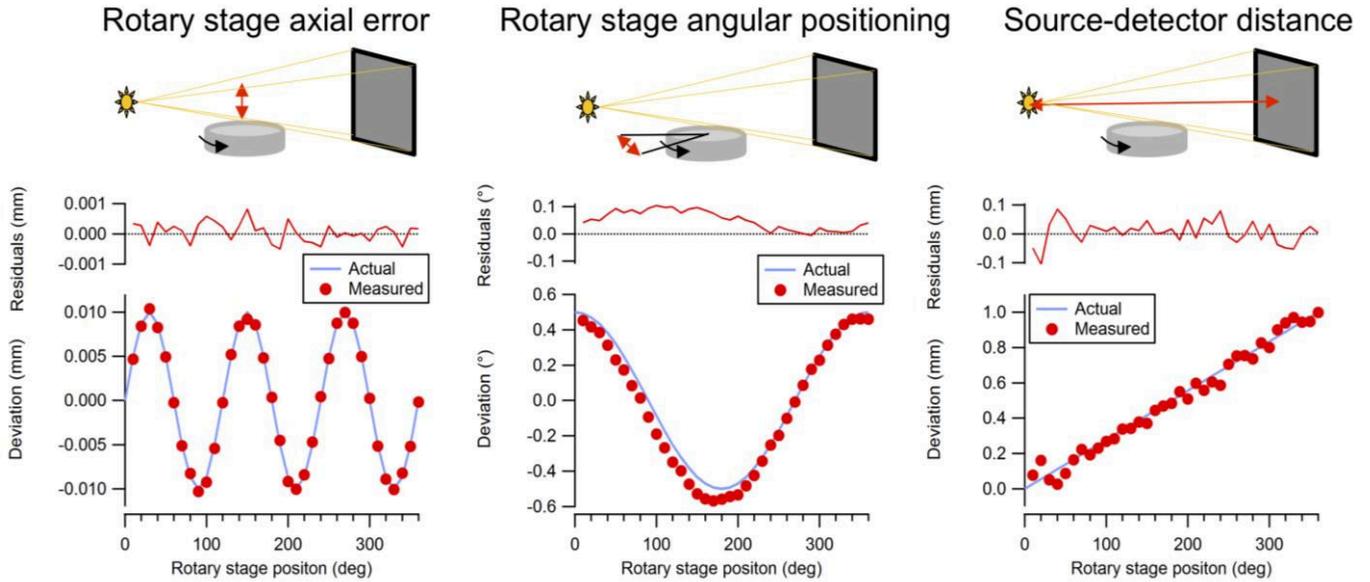


Figure 4: Experimental evaluation of the method to determine the CT geometry based on individual radiographs. The actual deviations (lines) were realised by displacing three motion axes simultaneously. The values (circles) were determined by analysing 36 individual radiographs of the calibrated multi-sphere standard. Deviations between the actual and measured values are shown in the top graphs (residuals) and were used to determine the sensitivity.

4.2 Analysis of sets of radiographs

Next, the above CT geometry determination procedure was applied to sets of radiographs recorded during one revolution (360°). The sensitivity was investigated as a function of the number of radiographs, i.e. number of sphere positions (14 spheres per radiograph), recorded and used for analysis. Therefore, the nominal SDD was kept constant at 1000 mm and sets of 2, 5, 10, 18, 36 and 72 radiographs were recorded for two different nominal SODs: 65 mm and 100 mm (nominal magnifications: 15.4 and 10.0). Because the magnification M , SOD and SDD are related ($M = SDD/SOD$), only two variables were determined during the analysis (SDD and M). As shown in Figure 5, the magnifications converged rapidly when using more radiographs. Compared to the magnification determined from 72 radiographs, the relative deviations were below 10^{-5} when using more than 18 radiographs (252 sphere positions). However, the SDD that remained stable during all measurements within $0.3 \mu\text{m}$ as determined by the metrology system, was not in good agreement between the individual measurements: It varied between 999.986 mm and 1000.29 mm (relative $3 \cdot 10^{-4}$). The magnification, i.e. the ratio of SDD/SOD, was accurately determined because it directly scales the sphere positions in the radiographs. In contrast, at constant magnification the absolute value of the SDD only influences the cone-beam angle. This correlation between SOD and SDD was previously reported to limit the accuracy of the method [3]. The cone-beam angle shifts off-centre spheres closer to the X-ray tube outwards and spheres closer to the detector inwards in the radiographs. That means spheres that lie below or above the central detector row, move up and down during one revolution and thereby describe an ellipse. The vertical axis of this ellipse is a function of the cone-beam angle. However, the changes are very subtle for the employed geometric arrangement: Changing the SDD (1000 mm) by 10^{-5} at constant magnification causes a sphere position, lying in the top centre of the radiograph, to shift by about $0.4 \mu\text{m}$ (or 0.004 pixels). In contrast, a 10^{-5} magnification change causes a shift of roughly $3 \mu\text{m}$ (or 0.03 pixels). Thus, it is concluded that the sensitivity of the method to SDD changes is about 8-fold lower than to magnification changes for the employed geometry.

4.3 Analysis of combined sets of radiographs at different source-object distances

To improve the determination of the SOD and SDD, we suggest a method, where the multi-sphere standard is scanned at different SODs (see Figure 6, left). It is an extension of the approach in Reference [9] where SOD and SDD are determined by recording radiographs of grids at different SODs. Whereas the absolute SOD is unknown, the shifts between them were accurately measured using the metrology system on METAS-CT (measurement uncertainty $0.1 \mu\text{m}$). After analysing the individual datasets, the SOD shifts (interferometric measurement) were plotted against the magnification (from the multi-sphere standard radiograph analysis) as shown in Figure 6, right. The following equation was then fitted to the data points using a least-squares algorithm:

$$M = \frac{SDD_{global}}{SOD_0 + \Delta SOD} \quad (1)$$

where M is the magnification determined from each set consisting of 72 radiographs, ΔSOD the independent variable from the interferometric displacement measurements, and SOD_0 and SDD_{global} are the fit coefficients and are, respectively, equal to the SOD the interferometric measurements are referred to and the SDD that was constant for all measurements. The relative deviations between equation (1) and the data points were below about 10^{-6} , indicating sufficiently accurate agreement. Interestingly, SDD_{global} was determined to be 999.278 mm and, thus, significantly below the values estimated using single sets of radiographs. This deviation is most likely due to the correlation between SOD and SDD discussed in Section 4.2. The additional information from the SOD displacements (ΔSOD) enabled the determination of a more accurate solution for the common SDD. Subsequently, the SOD of the individual multi-sphere standard measurements can be calculated based on SDD_{global} and the (more accurate) magnification M : $SOD = SDD_{global}/M$.

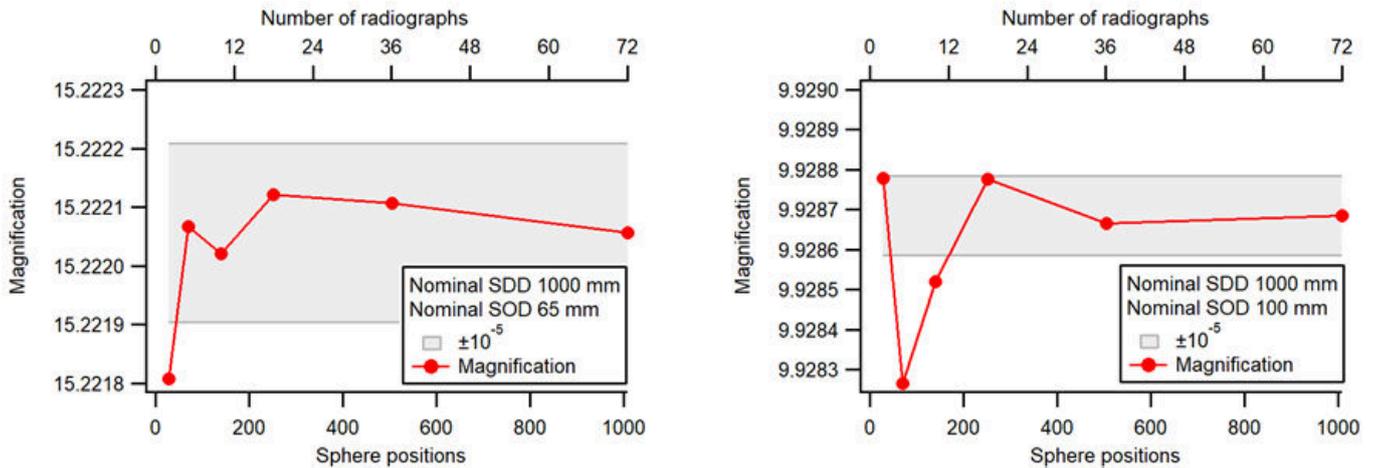


Figure 5: Determination of the magnification using sets consisting of different numbers of radiographs (14 calibrated spheres per radiograph were analysed). The measurements were performed with a nominal SOD of 65 mm (left) and 100 mm (right) and a nominal SDD of 1000 mm. The grey area indicates a zone describing relative deviations of $\pm 10^{-5}$ with regard to the value determined from 72 radiographs.

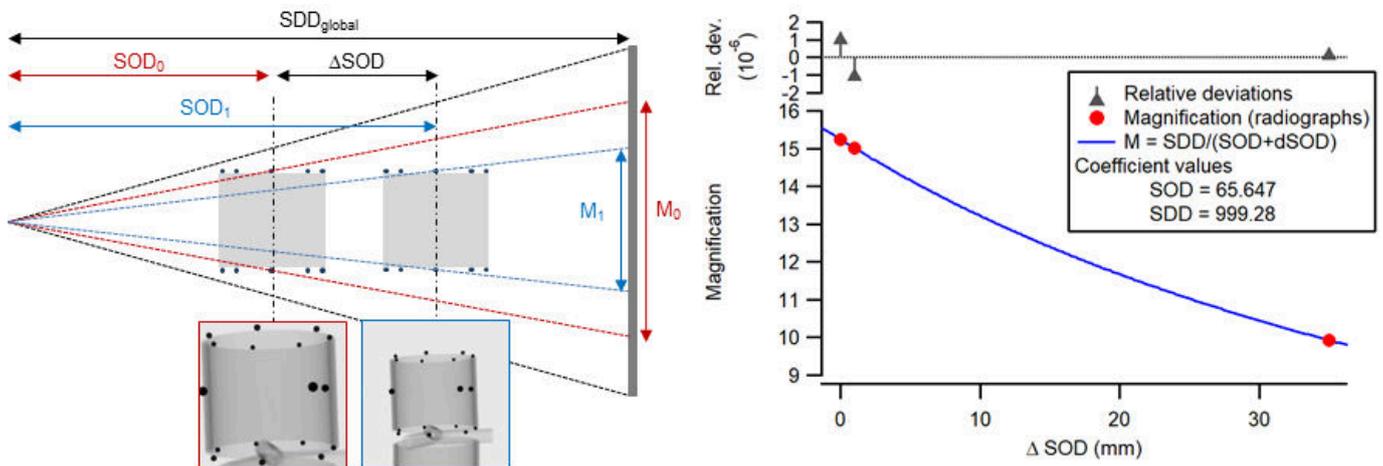


Figure 6: Positioning the multi-sphere standard at multiple SODs enabled a more accurate determination of the absolute CT geometry. Left: The shifts ΔSOD were measured using the METAS-CT interferometers and the magnifications M can be extracted from radiographs (insets). Right: The relation between the shifts in SOD and the magnification can be used to determine the absolute SDD_{global} by fitting equation (1). The relative deviations between the data points and the fitted curve are displayed in the top graph.

In the following, the method is validated by comparing values recorded at three different nominal SODs of 65 mm, 66 mm, and 100 mm. Because of the absence of a method to determine traceable absolute values for SOD and SDD, the differences between the results of the three measurements ($\Delta SOD = SOD_{66} - SOD_{65}$ and $\Delta SOD = SOD_{100} - SOD_{65}$) based on sets of radiographs (Section 4.2) and combined sets of radiographs (Section 4.3) were compared to the displacements measured by the interferometers ($\Delta SOD_{interferometer}$). Since the measurement uncertainty of the interferometric displacement measurements is about $0.1 \mu\text{m}$, it was

considered as reference. The results are shown in Figure 7 for sets consisting of different numbers of radiographs: The small deviations in the interferometrically measured SOD displacements $\Delta\text{SOD}_{\text{interferometer}}$ are due to drifts during the measurements. The blue circles correspond to ΔSOD determined solely on individual sets of radiographs (Section 4.2). Small shifts in SOD of 1 mm (Figure 7, left) were determined with relative deviations below 10^{-5} when using 18 or more radiographs. However, for the larger shift of 35 mm (Figure 7, right) the method failed to reach a stable value within the relative tolerance of 10^{-5} . Employing $\text{SDD}_{\text{global}}$ (Section 4.3), derived from measurements at different SODs, and the corresponding magnification to determine the shift in SOD led to more stable results (Figure 7, red triangles). Using 18 or more radiographs (252 sphere positions), led to deviations well below 10^{-5} in all cases.

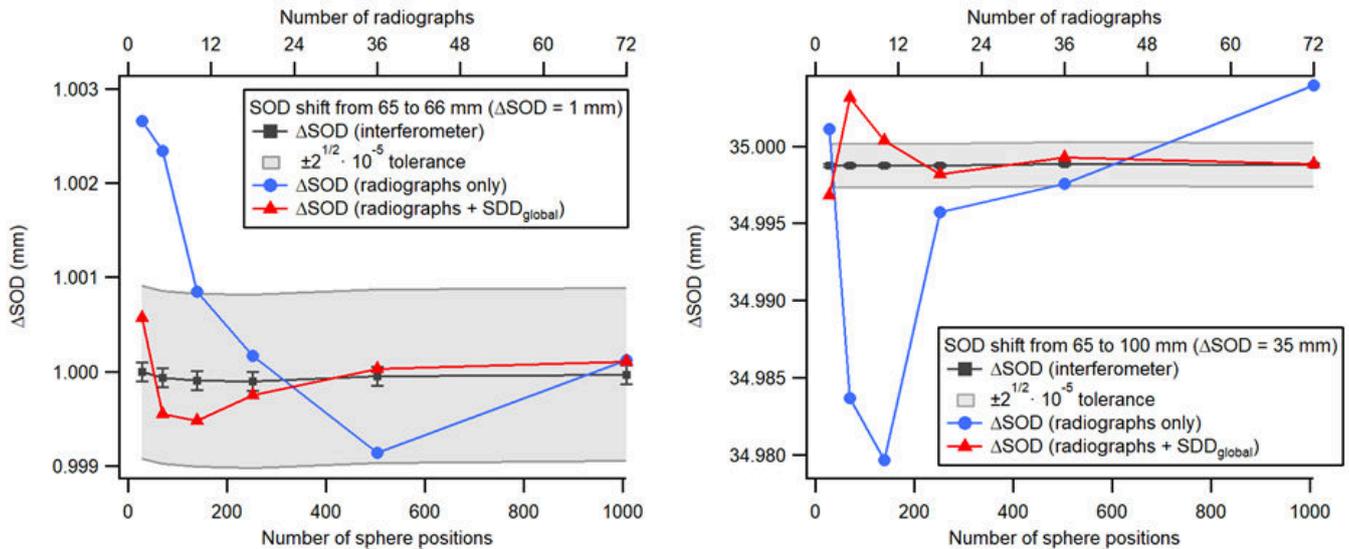


Figure 7: Comparison between determining the SOD based on one set of radiographs of a multi-sphere standard (blue circles, Section 4.2) or based on one set of radiographs with a predetermined $\text{SDD}_{\text{global}}$ (red triangles, see Figure 6, Section 4.3). To evaluate each method, the analysis was performed at three SODs, the displacement between which was measured by interferometry. Results for SOD shifts of 1 mm (left) and 35 mm (right) are shown.

5 Conclusions

In conclusion, the METAS-CT metrology system enables radiography-based CT geometry calibration methods to be validated and forms the basis for measurement uncertainty estimations. Experimental evaluation of the method, based on analyses of individual radiographs of a calibrated multi-sphere standard, showed that stage errors and drifts can be investigated with a relative sensitivity of about 10^{-4} . To improve the accuracy, the CT geometry parameters were determined globally by analysing sets of radiographs recorded during one revolution of the multi-sphere standard. This reduced the relative deviations in the magnification to below 10^{-5} . The determination of the absolute value of the source-detector distance (SDD) however remained limited to about $3 \cdot 10^{-4}$. This is accounted to the correlation between source-object distance (SOD) and SDD [3], i.e. they cause no change in magnification if their ratio remains identical. To improve the accuracy, the multi-sphere standard was scanned at different SODs, with interferometrically measured shifts in between. This enabled the correlation between the SOD and SDD to be disentangled. Subsequently, displacements in the SOD were detected well within the target tolerance of 10^{-5} . In the future, we plan to study further CT geometry parameters, such as angular deviations and the influence of the measurement uncertainty of the tactile reference calibration.

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