Uncertainty for uncorrected measurement results in X-ray computed tomography

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Abstract

In a recent paper [1], a mathematical formalism for the uncertainty estimation of dimensional measurements obtained from X-ray computed tomography (CT) data was outlined. The formalism includes the treatment of both ‘corrected’—when the final result is corrected for bias—and ‘uncorrected’ measurement results. Such formalism is mainly based on the ISO 15530 series and the VDI/VDE 2630-2.1 guidelines. However, the treatment of uncertainty in the ‘uncorrected’ case—not compensated for bias—is limited to the use of the root-sum-of-squares of standard uncertainties (RSSu) approach. The present paper expands to other possibilities for the uncertainty estimation of ‘uncorrected’ results that could be applied to CT measurements, namely the root-sum-of-squares of expanded uncertainties (RSSU), the algebraic sum of expanded uncertainty with the signed bias (SUMU), the enlargement of the expanded uncertainty by adding the absolute value of the bias (SUMU$_{\text{MAX}}$), and the so-called U$_g$ method that sums the expanded uncertainty with the absolute value of the bias scaled by a factor $\varepsilon$ assigned for a 95% distribution coverage. In addition, the alternative of using a maximum permissible error (MPE) statement—typically specified by the manufacturer of the CT instrument—to get a rough estimate of the expanded uncertainties of CT measurements is considered. Through a concrete example, by using dimensional X-ray CT data extracted from a metallic artifact that has internal features, these possibilities are analysed. From all the possibilities investigated, the RSSu method seems to be the most conservative for the estimation of expanded uncertainties associated with CT dimensional measurement that are not compensated for bias. On the other hand, uncertainty bounds estimated with the MPE-based approach vary little from a constant value, and, therefore, risk creating significant under- or over-estimation of the uncertainty intervals.

Keywords: X-ray computed tomography, CMM, bias, uncertainty, RSSU, SUMU, MPE (maximum permissible error).
Uncertainty for uncorrected measurement results in X-ray computed tomography

‘Error’ or bias: \( \Delta = \bar{x}_{CT} - x_{ref} \)

Uncertainty range: \( (\bar{x}_{CT} - U'_{CT}, \bar{x}_{CT} + U'_{CT}) \)

\( U_{ref} \ll U'_{CT} \)

‘Operative’ reference value or ‘conventional true’ value

Measured value

Measurand: \( X \)

By: H. Villarraga-Gómez and S. T. Smith
ON A NEW KIND OF RAYS

By W. C. RÖNTGEN

1. A discharge from a large induction coil is passed through a Hittorf's vacuum tube, or through a well-exhausted Crookes' or Lenard's tube. The tube is surrounded by a fairly close-fitting shield of black paper; it is then possible to see, in a completely darkened room, that paper covered on one side with barium platinocyanide lights up with brilliant fluorescence when brought into the neighborhood of the tube, whether the painted side or the other be turned towards the tube. The fluorescence is still visible at two metres distance. It is easy to show that the origin of the fluorescence lies within the vacuum tube.

2. It is seen, therefore, that some agent is capable of penetrating black cardboard which is quite opaque to ultra-violet light, sunlight, or arc-light. It is therefore of interest to investigate how far other bodies can be penetrated by the same agent. It is readily shown that all bodies possess this same transparency, but in varying degrees. For example, paper is very transparent; the fluorescent screen will light up when placed behind a book of a thousand pages.

X-rays discovery

November 8, 1895
History (medical diagnoses)

First X-ray CT

1927

1895

1971

First X-ray CT
History (non-destructive evaluation)
History (dimensional metrology)

First X-ray CT

1895

A.M. Cormack
G.N. Hounsfield

1927

1971

1980s

1990s

2000s

2011

2013

2016

Thin wall
Concave wall
Dimensional Metrology

150 μm
Dimensional metrology with X-ray CT

X-ray CT scan of a water pump
Workflow for Dimensional X-ray CT:
Workflow for Dimensional X-ray CT:

1. Scanning & X-ray image acquisition
2. Reconstruction from 2D images to 3D volume
3. Surface determination
4. Geometry fitting & Dimensional measurements

Local adaptive thresholding

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Volume 51, pp 291-307, 2018
Industrial X-ray CT
Metrology X-ray CT
What's up with the CAT?
CAT = Computed Axial (or Aided) Tomography

Thanks Lily!
Dimensional metrology with CAT Scan
(or ‘X-ray CT’)

A measurement result is incomplete until coupled with an associated uncertainty.

'Error' or bias: \[ \Delta = \bar{x}_{CT} - x_{ref} \]

'Measurand': Measured value

'Operative' reference value or 'conventional true' value

The compatibility/incompatibility of 'error-based' and 'uncertainty-based' modeling of measurement is still being debated.
A measurement result is incomplete until coupled with an associated uncertainty. The compatibility/incompatibility of 'error-based' and 'uncertainty-based' modeling of measurement is still being debated.

CT Measurement

\[ X_{uncorr} = \bar{x}_{CT} \pm U'_{CT}, \]

\[ X_{corr} = \bar{x}_{CT} - b \pm U_{CT}, \]

\[ U'_{CT} \approx \sqrt{U_p^2 + U_w^2 + U_b^2 + \Delta^2} \]

\[ U_{CT} \approx \sqrt{U_p^2 + U_w^2 + U_b^2} \]

'Error' or bias: \[ \Delta = \bar{x}_{CT} - x_{ref} \]

Uncertainty range: \( (\bar{x}_{CT} - U'_{CT}, \bar{x}_{CT} + U'_{CT}) \)

CT Measurement = Result ± Uncertainty

\[ U_{ref} \ll U'_{CT} \]

'Measured value'

'Measurand: X'

'The compatibility/incompatibility of 'error-based' and 'uncertainty-based' modeling of measurement is still being debated.'
Uncertainties in CT measurements

\[ X_{uncorr} = \bar{x}_{CT} \pm U'_{CT}, \quad U'_{CT} \approx \sqrt{U_p^2 + U_w^2 + U_b^2 + \Delta^2} \]

\[ X_{corr} = \bar{x}_{CT} - b \pm U_{CT}, \quad U_{CT} \approx \sqrt{U_p^2 + U_w^2 + U_b^2} \]

CT Measurement

Table 1

<table>
<thead>
<tr>
<th>Mathematical form</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ u_p = t_{n-1,\nu}(s_x/\sqrt{n}), ] \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (3)</td>
<td>\textit{Uncertainty sources}: repeatability variances dependent on the measurement procedure for the measurand ( x ) on the workpiece (measured feature, measurement strategy, and/or calibration condition of the measurement system).</td>
</tr>
<tr>
<td>[ u_w \approx \sqrt{u_p^2 + u_T^2}, ] \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (4)</td>
<td>\textit{Remarks}: may strongly vary under different measurement parameters and conditions (e.g., radiation generation, temperature gradients, focal spot drift, scale resolution, geometrical errors, imaging, clamping, handling, dirt, workpiece’s temperature, attenuation properties, probing system’s errors, etc.).</td>
</tr>
<tr>
<td>[ u_b = (R_{\text{mean}}/2)/\sqrt{3}, ]</td>
<td></td>
</tr>
<tr>
<td>[ u_T = L\left( \Delta T \left( \frac{50^2}{2} + \beta \left( \frac{50^2}{2} \right) \right) \right)/\sqrt{3}, ]</td>
<td></td>
</tr>
<tr>
<td>[ \Delta T = (T - 20^\circ \text{C}). ]</td>
<td></td>
</tr>
<tr>
<td>[ u_b \approx \sqrt{(\delta_b/\sqrt{n})^2 + u_{\text{ref}}^2}, ] \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (5)</td>
<td>\textit{Uncertainty sources}: variations associated to mechanical and thermal changes in the measured workpieces (e.g., changes in form or shape, surface texture, thermal expansion, elasticity and plasticity, etc.).</td>
</tr>
<tr>
<td>[ u_{\text{ref}} \approx \sqrt{u_p^2 + u_{\text{err}}^2 + u_{\text{cal}}^2}, ] \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (6)</td>
<td>\textit{Remarks}: the factor ( 1/\sqrt{3} ) comes from assuming the error margins of a rectangular distribution (e.g., all values of error within the band ( \pm \delta_b/2 ) are equally probable). When neglecting changes in the workpiece’s form, elasticity, or roughness, the only significant uncertainty contribution is ( u_T ).</td>
</tr>
<tr>
<td>[ u_{\text{cal}} = U_{\text{cal}}/2, ]</td>
<td></td>
</tr>
<tr>
<td>[ u_{\text{err}}; \text{correspond to the same form of } u_p \text{ and } u_w \text{ defined above but when the measurement procedure is carried out on the reference workpiece with the instrument used for its measurement.} ]</td>
<td></td>
</tr>
<tr>
<td>[ u_{CT} \approx \sqrt{u_p^2 + u_w^2 + u_b^2} ] \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7)</td>
<td>\textit{Uncertainty sources}: uncertainty of bias. It includes the variance of the systematic deviation between CT measurements and reference measurement.</td>
</tr>
<tr>
<td></td>
<td>\textit{Remarks}: the sub-index ( r ) refers to parameters associated to the reference workpiece material or from the reference measurement. When the uncalibrated workpiece is identical with the reference workpiece, then ( \delta_{\beta_r} = \delta_{\beta} ). Note that ( u_{\text{ref}} ) could be determined by a tactile CMM, in which case ( U_{\text{cal}} ) corresponds to the expanded uncertainty of calibration of a master artifact or working standard for the CMM system, generally provided in a calibration certificate.</td>
</tr>
<tr>
<td></td>
<td>\textit{Generalized estimation of standard uncertainty of the mean value of} ( n ) \textit{measurements.}</td>
</tr>
</tbody>
</table>
Uncertainties in CT measurements

CT Measurement

\[
\begin{align*}
X_{\text{uncorr}} &= \bar{x}_{\text{CT}} \pm U'_{\text{CT}}, \\
X_{\text{corr}} &= \bar{x}_{\text{CT}} - b \pm U_{\text{CT}}, \\
U'_{\text{CT}} &\approx \sqrt{U_p^2 + U_w^2 + U_b^2 + \bar{\Delta}^2} \\
U_{\text{CT}} &\approx \sqrt{U_p^2 + U_w^2 + U_b^2} \\
&= k \sqrt{u_p^2 + u_w^2 + u_b^2}
\end{align*}
\]

Uncertainty \( U' \)

(in the presence of bias)

<table>
<thead>
<tr>
<th>Mathematical form</th>
<th>Name</th>
</tr>
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<tbody>
<tr>
<td>( U' = \sqrt{(k u_c)^2 + b^2} )</td>
<td>RSSU</td>
</tr>
<tr>
<td>( U' = k \sqrt{u_c^2 + b^2} )</td>
<td>RSSu</td>
</tr>
<tr>
<td>( U'_+ = \max (0, ku_c - b) )</td>
<td>SUMU</td>
</tr>
<tr>
<td>( U'_- = \max (0, ku_c + b) )</td>
<td>SUMU_{\text{MAX}}</td>
</tr>
<tr>
<td>( U' = ku_c +</td>
<td>b</td>
</tr>
<tr>
<td>( U' = ku_c + \varepsilon</td>
<td>b</td>
</tr>
</tbody>
</table>
Example: the NIST “Frustum artefact”
Example: the NIST “Frustum artefact”

Gleaned from other sectional planes:
Example: the NIST "Frustum artefact"

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</table>

$u_c \approx \sqrt{u_p^2 + u_w^2 + u_b^2}$,

$U'_{CT} \approx k \cdot \sqrt{u_w^2 + \left(\frac{MPE_{CT}}{\sqrt{3}}\right)^2}$,

$MPE_{CT} = \left(8 + \frac{L}{100}\right) \mu$m.

$\bar{x}_{CT} = 9.9753 \text{ mm}$,

$x_{ref} = 9.9790 \text{ mm}$,

$u_{ref} = 0.07 \mu$m,

$b \approx \bar{x}_{CT} - x_{ref} = -3.7 \mu$m

<table>
<thead>
<tr>
<th>Uncertainty ($k = 2$)</th>
<th>Symbol</th>
<th>Value ((\mu m))</th>
</tr>
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<tbody>
<tr>
<td>$U'_{CT}$</td>
<td>$U'_{CT}$</td>
<td>3.26</td>
</tr>
<tr>
<td>RSSU</td>
<td>$U'_{CT}$</td>
<td>4.94</td>
</tr>
<tr>
<td>RSSu</td>
<td>$U'_{CT}$</td>
<td>8.09</td>
</tr>
<tr>
<td>SUMU</td>
<td>$-U'_-$</td>
<td>0.00</td>
</tr>
<tr>
<td>SUMUMAX</td>
<td>$+U'_+$</td>
<td>6.97</td>
</tr>
<tr>
<td>SUMUMAX</td>
<td>$U'_+$</td>
<td>6.97</td>
</tr>
<tr>
<td>U'_\varepsilon</td>
<td>$U'_{CT}$</td>
<td>6.40</td>
</tr>
<tr>
<td>U'(MPE)</td>
<td>$U'_{CT}$</td>
<td>9.35</td>
</tr>
</tbody>
</table>

Diagram:
- Cone 1
- Cone 2
- F1
- F2

Villarraga-Gómez, 2018 PhD Dissertation, UNCC
Example: the NIST "Frustum artefact"
Summary & Conclusions
Imagine… you can see inside!

Casting void analysis

Additive Manufacturing: Void and dimensional analysis

Electronics Inspection & Assembly Analysis

Fiber Analysis

Dimensional Inspection

Dimensional Metrology
Measurement phases with X-ray CT

Phase 1: Scan planning and setup

Scan parameters

Phase 2: Scanning

2D projections

Reconstruction and surface determination

Phase 3: Strategy for measurements

3D volume

Measurements
## Uncertainties in CT measurements

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$u_c \approx \sqrt{u_p^2 + u_w^2 + u_b^2}$

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ISO 15530 series & VDI/VDE 2630-2.1 guidelines!

---

**Bias**: $\bar{x}_{CT} - x_{ref}$

**Uncertainty range**: $(\bar{x}_{CT} - U'_{CT}, \bar{x}_{CT} + U'_{CT})$

---

**Operative** reference value or **conventional true** value

Measured value
Uncertainties in CT measurements
Uncertainties in CT measurements

Villarraga-Gómez, 2018
PhD Dissertation, UNCC
Acknowledgments

Standing on the shoulders of giants!!

Thanks (Grazie)!
Thanks for coming (Grazie)!!

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Enjoy il giorno (14 febbraio):
La festa di san Valentino