Revisited the Mathematical Derivation Wall Thickness Measurement of Pipe for Radiography

Ab. R. Hamzah[1] and Siti M. M. Amir[1]
[1] Non Destructive Testing(NDT) Group, Industrial Technology Division, Malaysian Nuclear Agency, Bangi, 43000 Kajang, Selangor, Malaysia; Phone:+603 89250510, Fax:+60389250907; e-mail: Razak_Hamzah@nuclearmalaysia.gov.my, e-mail: madiha_amir@nuclearmalaysia.gov.my

Abstract
Wall thickness measurement of pipe is very important of the structural integrity of the industrial plant. However, the radiography method has an advantage because the ability of penetrating the insulated pipe. This will have economic benefit for industry. Moreover, the era of digital radiography has more advantages because the speed of radiographic work, less exposure time and no chemical used for film development. Either the conventional radiography or digital radiology, the wall thickness measurement is using the tangential radiography technique (TRT). In case, of a large diameter, pipe (more than inches) the determination maximum penetration wall thickness must be taken into the consideration. This paper is revisited the mathematical derivation of the determination of wall thickness measurement based on tangential radiography technique (TRT). The mathematical approach used in this derivation is the Pythagoras theorem and geometrical principles. In order to derive the maximum penetration wall thickness a similar approach is used.
Keyword : wall thickness measurement, tangential radiography, maximum penetration wall thickness

1. Introduction

The most prominent advantage in radiography as Non-Destructive Testing (NDT) techniques regardless the conventional or digital radiography is the ability to penetrate the insulated pipes. As we know industrial radiography is more popular to detect defects in the welding and casting. However, a few years back, wall thickness measurement has becoming more popular and it is very crucial in the assessment of structural integrity of industrial plants.

There are two methods in assessing wall thickness; the double wall method and Tangential Radiography Technique (TRT). The double wall method is an established method used by the radiographers. For tangential radiography, a certain geometrical set-up for pipe inspection is needed. In the paper, we are revisited the mathematical derivation the wall thickness measurement of pipe using radiography method.

2. Tangential Radiography

Tangential radiography is a method employed for pipe inspection to monitor corrosion and evaluate wall thickness without removing the insulator. In this method, a certain geometrical set-up for pipe inspection is needed. In this method, only parts of the radiograph, which lies below the tangential location, are interpreted. The middle part of the image is ignored [1].

In obtaining the good image at the tangential location that is the chord, the selection of optimum energy is very important. This is due to the chord segment, which has difference in thickness that varies from the thinnest to the maximum penetrated wall thickness. The selection of optimum energy is crucial to prevent burn-off effect at the thinnest portion of the sample and scattering or too bright image at the thickest segment. Therefore, the selection of the irradiation source used must be according to the maximum of the penetrated thickness.

2.1 Wall Thickness Derivation – revisited [2]

The main purpose of the Tangential Radiography technique is to evaluate the wall thickness. In deriving the wall thickness equation, the mathematical approach on Pythagoras theorem and geometrical principles are used. The determination of the wall thickness using tangential method is represented in Figure 1.

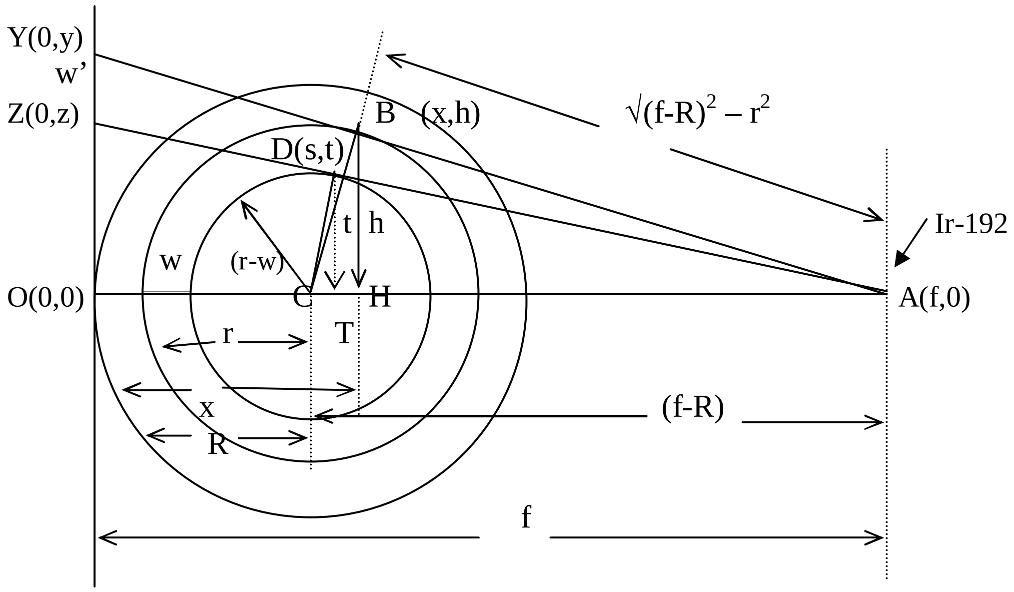

Figure1. The schematic diagram for determination of wall thickness

From Fig.1, *AY* is the tangent to the circle of radius *r* at point *B (x,h)*. *h* is the length of the perpendicular dropped from *B* to *H* on *OA*. Using Pythagoras theorem,

$$AB = \sqrt{AC^2 - CB^2} = \sqrt{(f-R)^2 - r^2}$$

also

$$h^2 = AB^2 - AH^2 = (f-R)^2 - r^2 - (f-x)^2 \quad \text{.......................(1)}$$

Area of $\Delta ABC = \frac{1}{2} h \cdot AC = \frac{1}{2}(f-R)h$ but also area of $\Delta ABC = \frac{1}{2} BC \cdot AB$ $= \frac{1}{2} r\sqrt{(f-R)^2 - r^2}$. Hence, $\frac{1}{2} r\sqrt{(f-R)^2 - r^2} = \frac{1}{2}(f-R)h$ and solving for *h*,

$$h = \frac{r\sqrt{(f-R)^2 - r^2}}{(f-R)}$$

or

$$h^2 = \frac{r^2\left[(f-R)^2 - r^2\right]}{(f-R)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Therefore (1) and (2) imply

$$(f-x) = \frac{(f-R)^2 - r^2}{(f-R)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Next, since the gradient of *AB* is the same as that of *BY*, $= \frac{0-h}{f-x} = \frac{h-y}{x-0}$. This implies

$$\begin{aligned} y &= \frac{hf}{(f-x)} \\ &= \frac{r\sqrt{(f-R)^2 - r^2}}{f-R} \cdot f \cdot \frac{(f-R)}{(f-R)^2 - r^2} \\ &= \frac{fr}{\sqrt{(f-R)^2 - r^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4) \end{aligned}$$

From Fig.1, *AZ* is the tangent to the circle of radius *r*-*w* at point *D(s,t)*. *t* is the length of the perpendicular dropped from *D* to *T* on *OA*. By Pythagoras theorem,

$$AD = \sqrt{AC^2 - CD^2} = \sqrt{(f-R)^2 - (r-w)^2}$$

and by Pythagoras theorem,

$$\begin{aligned} t^2 &= AD^2 - AT^2 \\ &= (f-R)^2 - (r-w)^2 - (f-s)^2 \quad \ldots\ldots\ldots\ldots\ldots (5) \end{aligned}$$

The area of $\Delta ACD = \frac{1}{2} t \cdot AC = \frac{1}{2} t(f-R)$ but also area of $\Delta ACD = \frac{1}{2} CD \cdot AD$ $= \frac{1}{2}(r-w)\sqrt{(f-R)^2 - (r-w)^2}$. This implies $\frac{1}{2} t(f-R) = \frac{1}{2}(r-w)\sqrt{(f-R)^2 - (r-w)^2}$ and solving for *t*, we obtain

$$t = \frac{(r-w)\sqrt{(f-R)^2 - \left(r-w^2\right)}}{(f-R)}$$

or

$$t^2 = \frac{(r-w)^2\left[(f-R)^2 - (r-w)^2\right]}{(f-R)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Therefore (5) and (6) implies

$$(f-R)^2-(r-w)^2-(f-s)^2=\frac{(r-w)^2\left[(f-R)^2-(r-w)^2\right]}{(f-R)^2}$$

or

$$f-s=\frac{(f-R)^2-(r-w)^2}{f-R} \qquad \text{……………………..………….…………….(7)}$$

The gradient of *AD* is the same as that of *DZ*, $\frac{0-t}{f-s}=\frac{t-z}{s-0}$. This implies,

$$\begin{aligned} z &= \frac{tf}{f-s} \\ &= \frac{(r-w)\sqrt{(f-R)^2-(r-w)^2}}{f-R}\cdot f\cdot\frac{f-R}{(f-R)^2-(r-w)^2} \\ &= \frac{(r-w)f}{\sqrt{(f-R)^2-(r-w)^2}} \qquad \text{…………..………………………………….(8)} \end{aligned}$$

Now $w'=y-z=\frac{rf}{\sqrt{(f-R)^2-r^2}}-\frac{(r-w)f}{\sqrt{(f-R)^2-(r-w)^2}}$ so that

$\frac{(r-w)}{\sqrt{(f-R)^2-(r-w)^2}}=\frac{r}{\sqrt{(f-R)^2-r^2}}-\frac{w'}{f}$ and squaring both sides of the equation,

$\frac{(r-w)^2}{(f-R)^2-(r-w)^2}=\left[\frac{r}{\sqrt{(f-R)^2-r^2}}-\frac{w'}{f}\right]^2$.

Let $\theta=\frac{r}{\sqrt{(f-R)^2-r^2}}-\frac{w'}{f}$. Now $\frac{(r-w)^2}{(f-R)^2-(r-w)^2}=\theta^2$ implies $w=r-\frac{(f-R)\theta}{\sqrt{1+\theta^2}}$ and substituting for θ,

$$w=r-\frac{(f-R)\left[\frac{r}{\sqrt{(f-R)^2-r^2}}-\frac{w'}{f}\right]}{\sqrt{1+\left[\frac{r}{\sqrt{(f-R)^2-r^2}}-\frac{w'}{f}\right]^2}} \qquad \text{…………………….(9)}$$

2.2 Maximum Penetration Wall Thickness [3]

Tangential radiography is a method used for assessing the residual wall thickness of pipelines where corrosion or erosion is likely to occur. However, there exist limit in adopting this method especially in determining the source used. In obtaining the correct value of the residual wall thickness the calculation on maximum penetration wall thickness, L_{max} must take into consideration, as this is to ensure the appropriate radiation energy being used.

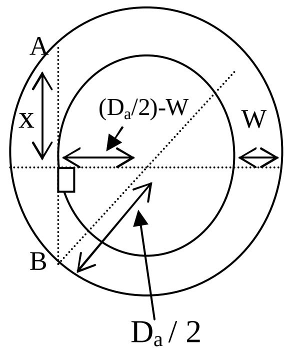

Figure 2. The schematic diagram to determine L_{max}.

Let AB be the maximum penetrated wall thickness L_{max}., W is the true wall thickness and D_a is the outer diameter. Using Pythagoras theorem,

$$\left[\frac{D_a}{2}\right]^2 = x^2 + \left[\frac{D_a}{2} - W\right]^2$$

$$\left[\frac{D_a^{\ 2}}{4}\right] - \left[\frac{D_a^{\ 2}}{4}\right] + D_a W - W^2 - x^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Solve (13) and yield

$$\frac{x^2}{W^2} = \frac{D_a}{W} - 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Square root (14) on both sides of the equation. This yields,

$$x = W\sqrt{\frac{D_a}{W} - 1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Since AB=L_{max} and AB=2x, hence,

$$x = \frac{L_{max}}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Substitute (16) in (15) to obtain

$$L_{\max} = 2W\sqrt{\frac{D_a}{W} - 1} \quad \text{...(17)}$$

3. Conclusion

It is important for industrial radiographer to understand the derivation of the wall thickness measurement at least they know the basic parameters and the limitation of the technique when performing the experiment.

Acknowledgements

The authors would like to thank the IAEA for coordinating and the financial supporting on this research project.

References

1. Validation of Protocols for Corrosion and Deposit Evaluation in Large Diameter Pipes by Radiography, Report of the 2nd RCM of the CRP, Istanbul Turkey, pp 13, March 2004, IAEA.

2. S.M.M.Amir, A.R.Hamzah, H.A.Kassim and M.B. Zubir, 'Analytical Studies Of Tangential Radiography Method – Revisited', presented at the Conference on Advance in Theoretical Sciences (CATS 2003) /Advanced Technology Congress (ATC 2003) 2003.

3. S.M.M.Amir, A.R.Hamzah, S.Sayuti, A.Amat and S.Rejab, 'Tangential Radiography Technique(TRT) For Pipe Wall Thickness Measurement Using Radiation – Mathematical Approach', proceeding MINT R&D Seminar Research and Development, Seminar B, pp270-276, MINT, July 2004.