

# Semi-Analytical-Fem Hybrid Modeling of Ultrasonic Defect Responses

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**Abstract.** A hybrid model based on the FEM code ATHENA developed by EDF and the pencil model implemented in CIVA (CEA software platform) has been developed. The goal of this model is to certify the validity of simplified beam-defect interactions models and to provide an alternative to these models in situations involving complicated scattering phenomena (complex flaw geometries, generation of surface wave, etc...). The hybrid approach tends to combine the advantages of both methods: a semi-analytical computation (CIVA pencil model) of the propagation, and a wave/defect interaction numerical computation (ATHENA) which is applied in a small region surrounding the defect. In this paper we describe the model itself and its implementation. A few examples of applications are studied and show the benefits of such a hybrid approach for NDT modeling.

## Introduction

Semi-analytic models for UT simulation provide quantitative predictions in a wide range of situations. They allow fast computations, a crucial requirement in the industrial context. However, semi-analytical approximations may fail at predicting responses from defects in some complex configurations. Particularly, when defects of complex geometry or neighbouring defects are considered, all physical processes observed on scattering are not taken into account.

Numerical schemes such as FEM do not rely on physical approximations for computations of elastic waves phenomena. The numerical resolution of the wave propagation equation could be quantitatively well defined by theoretical considerations ensuring the accuracy of the model. However, numerical schemes are computer intensive (computation time, memory) considering typical wave paths of hundreds of wavelengths in NDT configurations, especially in three dimensions.

In order to combine the advantages of both methods while minimizing their inconveniences, a hybrid model has been developed. The pencil method used for beam computations in CIVA is applied to deal with most of the propagation, while intricate phenomena located in a small region surrounding the defects are computed numerically by the FEM code ATHENA developed by EDF/INRIA. Finally, the echo-response from the defect is synthesized by deriving an extended formulation of Auld's reciprocity principle to the transient case, coupling computed results from both codes.

In the first part of this paper, the two theoretical models and the coupling principle are reminded. Then, two examples are provided to illustrate the capabilities of the hybrid method for UT simulation. The first case deals with the inspection of a surface breaking notch in a sample with various densities of thermal fatigue cracking. The second case deals with the comparison between simulation and experiment on a branched crack.

## 1. Theory

### 1.1 Coupling principle of the ATHENA FEM and CIVA semi-analytical pencil model

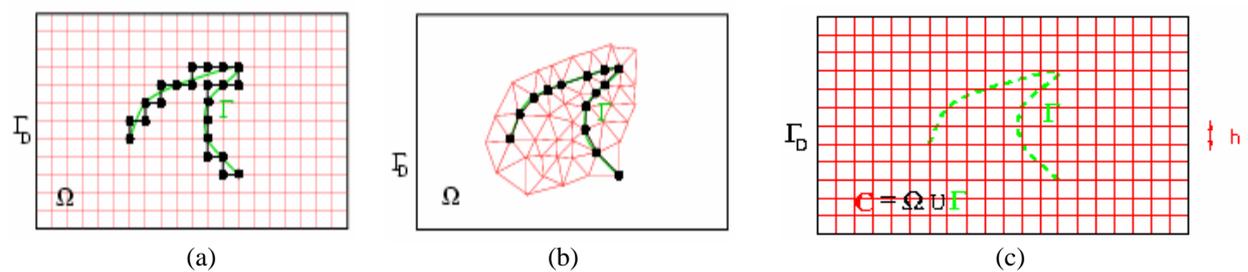
#### 1.1.1 ATHENA, finite element model for wave interaction with complex defects

The modeling code ATHENA comes from a collaboration between INRIA and EDF. It allows to simulate wave propagation in heterogeneous and anisotropic media, in the presence of complex-shaped defects using the fictitious domain method to represent it into the principal mesh [1-2]. The mathematical formulation combines both the accuracy of finite element and the performance of the finite differences.

While the spatial discretization is done with mixed finite element on a regular mesh, which allows to use the condensation of the mass matrix, the time discretization is performed with an explicit scheme obtaining a virtually explicit and therefore very rapid resolution method compared to usual FEM methods.

The second improvement lies in the way to consider defects. Taking the defect into account requires imposing zero stress on the defect boundaries, which can be done by applying two methods. The first one approximates the defect geometry within the regular mesh leading to spurious diffraction, see Fig. 1 (a). The other one uses an irregular mesh adapted to the geometry of the defect, see Fig. 1 (b), which is not convenient for parametric studies, where each configuration requires a new mesh. Furthermore, the irregularity of the mesh with respect to the stability condition requires a very fine temporal discretization, and thus more computation time for a given precision. The ATHENA code proposed a third method called the fictitious domain method. In this method, a regular mesh is maintained and a discontinuous displacement over the defect is introduced as an additional unknown. This displacement is defined over an additional mesh well adapted to the defect geometry, see Fig. 1 (c). This approach requires the resolution of a new system of linear equations; the size of this system depends on the resolution of the defect discretization. The method ability to handle a complex shaped defect within a regular mesh far outweighs this drawback.

**Figure 1.** Different meshes for the defect in FEM: (a) regular mesh, (b) irregular mesh, (c) fictitious domains.



#### 1.1.2 CIVA, semi-analytical model for wave propagation

The semi-analytical model for ultrasonic beam calculation has been developed at CEA for about a decade, into the CIVA software [3]. CIVA can predict the transient field radiated by a transducer in a component. The transducer can be monolithic or a phased array, either wedge-coupled or immersed. The samples can be of complex geometry either homogeneous or heterogeneous with isotropic or anisotropic properties.

The model is based on the Rayleigh integral formulation of the radiated wave. The transducer is therefore considered as a distribution of particle velocity sources over the

emitting surface. The transmitted field results from the summation of the various source contributions, each of them being evaluated by the so called pencil method [4].

This method consists in describing the evolution of a pencil of rays emitted by the source and centered on the geometrical path. The amplitude is evaluated using the divergence of a pencil linked to the elementary surface surrounding the geometrical path. The evolution of this elementary surface depends on the properties of the media and interfaces crossed by the pencil, and is mathematically described by a product of matrices. This model has been experimentally validated for several configurations and compared to other approximate or exact models.

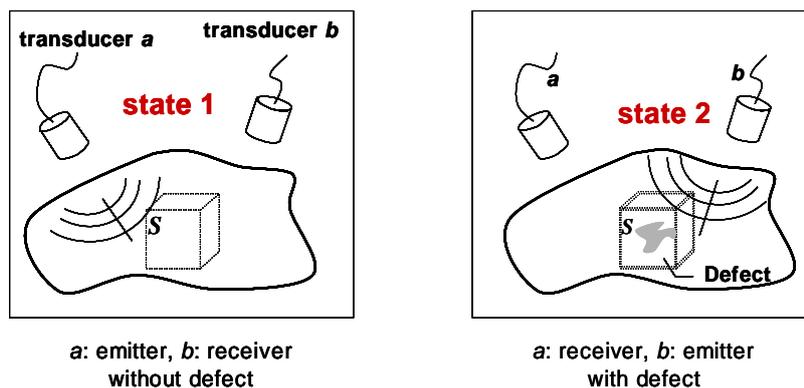
### 1.1.3 Derivation of the coupling integral formula

The goal of the coupling of the two previous models is to separate the computation into two parts: the wave propagation done by the semi-analytical model and the interaction with a defect performed with the numerical FEM model. With this aim, a coupling equation has been proposed based on an integral formulation extending Auld's reciprocity principle to the transient case [5]. Indeed, Auld establishes a relationship between two typical NDT configurations. The first one, called state 1, involves a component without defect, whereas the second one, called state 2, takes the flaws into account (see Fig. 2). The extracted signal is computed from an integral over the surface  $S$  of a volume containing all flaws, using particle velocity and stress tensor quantities [6]. The idea is that CIVA and ATHENA will be devoted to the computation of state 1 (without defects) and state 2 (with defects), respectively.

For practical reasons, the region in which the FEM computation is done must be as small as possible to avoid intensive computation. Then, state 2 is only considered inside the surface  $S$  surrounding the defect, called FEM box. This box is a rectangle for 2D computation and a parallelepiped for 3D computation. The coupling computation is composed of 3 steps:

- First, on the face of the box first crossed by the incident beam, the incident field is computed using the pencil method.
- This field is then used as an input into ATHENA computation. The boundary conditions applied on the faces of the box are absorbing conditions (Perfectly Matched Layer), so no artificial reflection comes back from these faces. During a limited number of time steps, the calculation is done by modeling both the complex propagation and the interaction with defects and for each of these time steps the elastodynamic quantities on the boundaries of the box are saved.
- Finally, the field corresponding to state 1 is computed using the pencil method in terms of impulse responses, for particle velocities and stresses. The coupling integral can then be calculated giving the signal due to the presence of defects.

**Figure 2.** Definition of state 1 and state 2 used for Auld's reciprocity principle.



## 2. Simulated Inspections Based on the Hybrid Model

In this part, two different simulations have been performed using a 2D box for the FEM calculation. For 3D FEM calculation the computation time and the limitations observed in terms of box size or defect size do not allow us to obtain BScan images easily. However, developments such as parallelization or optimization of the hybrid coupling are considered to overcome these problems.

At first, a sample containing thermal fatigue cracking is studied demonstrating the capability of the hybrid model to simulate multiple interactions between cracking and flaws.

Next, a comparison between experimental and simulated BScan on a branched crack is shown and emphasizes the ability of the hybrid model to simulate most of the complex phenomena that may occur.

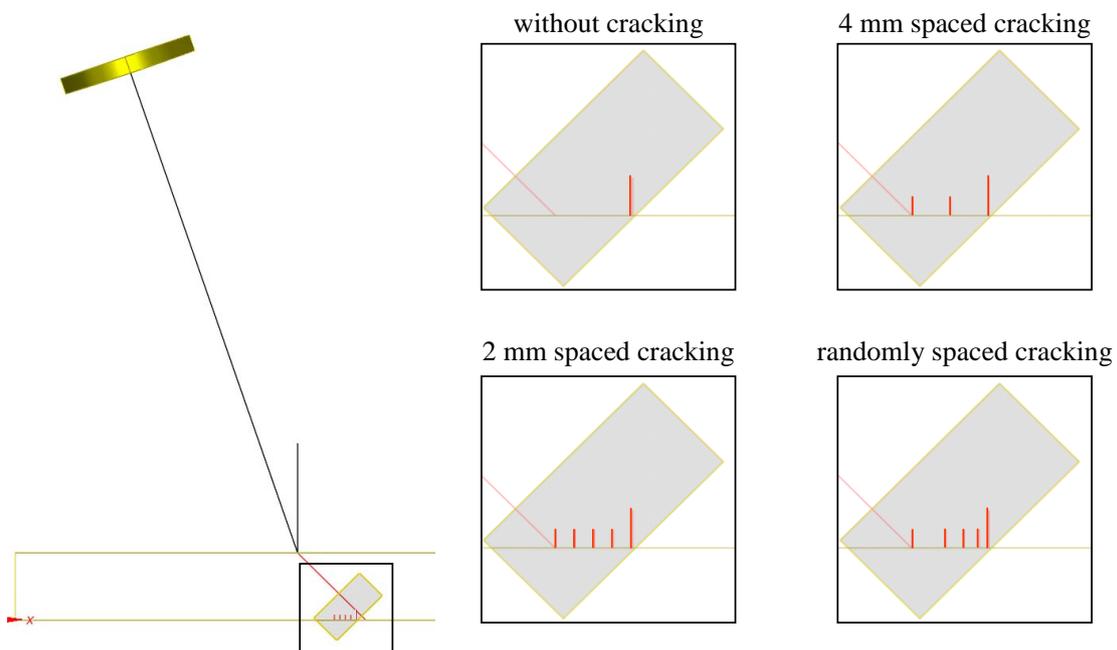
### 2.1 Defect response simulation in a sample with thermal fatigue cracking

The transducer used for this simulation is a bifocal transducer with a 50 mm diameter working at the central frequency of 4 MHz. For this frequency, the transducer is considered to have a wide aperture. For this configuration, 45°-shear waves are generated into the sample and the energy is focused at a depth of 25 mm with a high spatial resolution. (1.7-mm beam diameter at the focal depth).

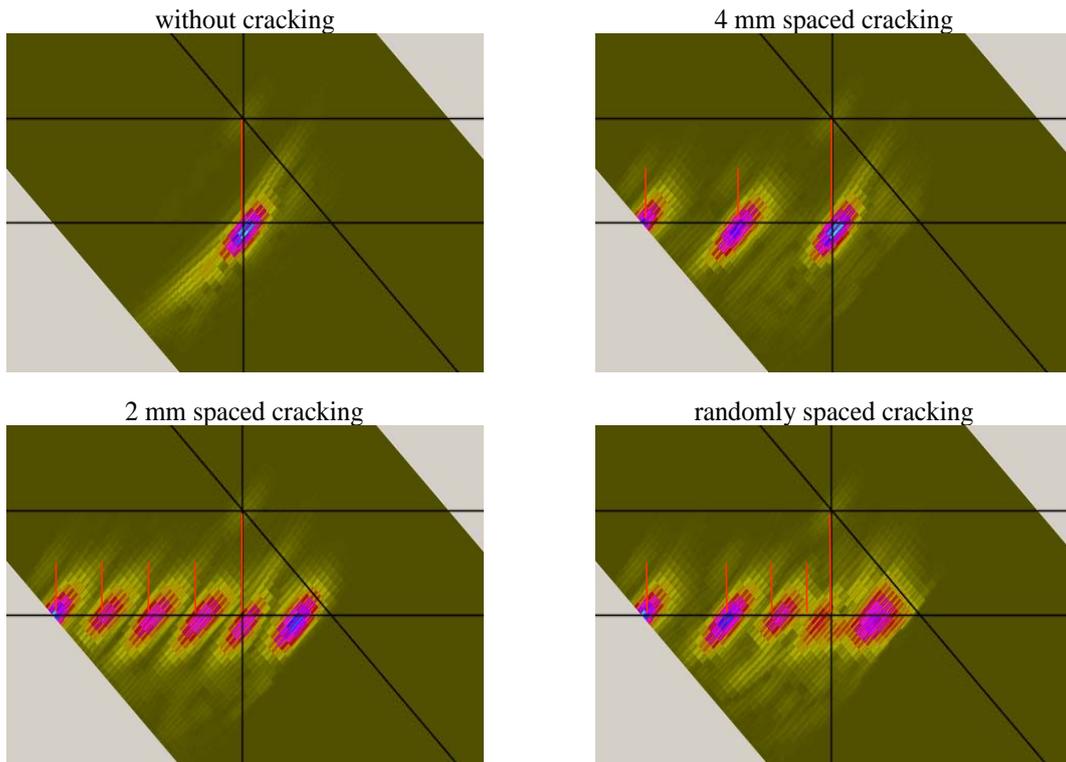
We propose to simulate the effect of thermal fatigue cracking on the detection of a 4 mm surface breaking notch. We describe thermal fatigue cracking as a set of 2-mm high planar surface cracks. Calculations were made considering various cracking density: at first without cracking, which is used as a reference, the 3 others configurations are for 4-mm, 2-mm and randomly spaced cracking (see. Fig.3). The sample is 28 mm thick and the configuration is optimized to detect the diffraction echo to determine the height of the surface breaking crack.

For each position the coupling formula previously described is applied to provide an Ascan signal. The set of all Ascan is processed in order to reconstruct a true Bscan positioned into the sample as shown in Fig. 4.

**Figure 3.** Control configurations performed for cracking effect on flaw detection.



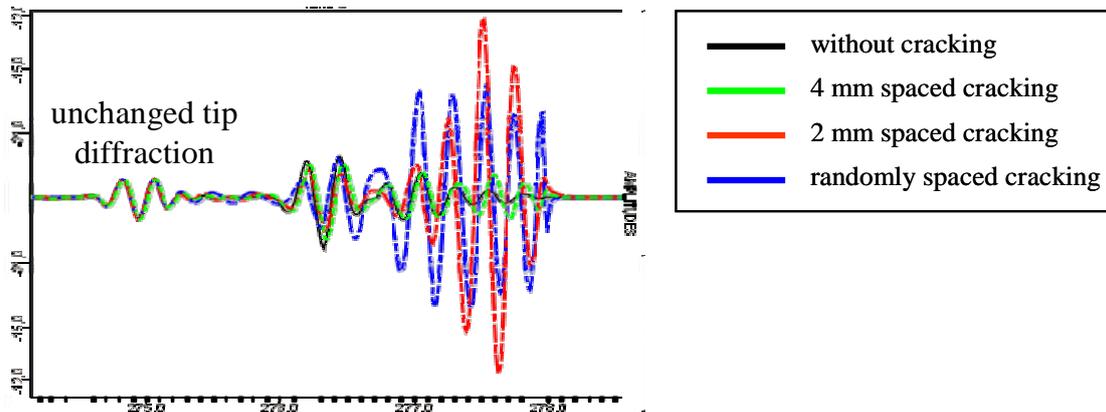
**Figure 4.** Bscan images provided for the different cracking densities with a 45°-SW control.



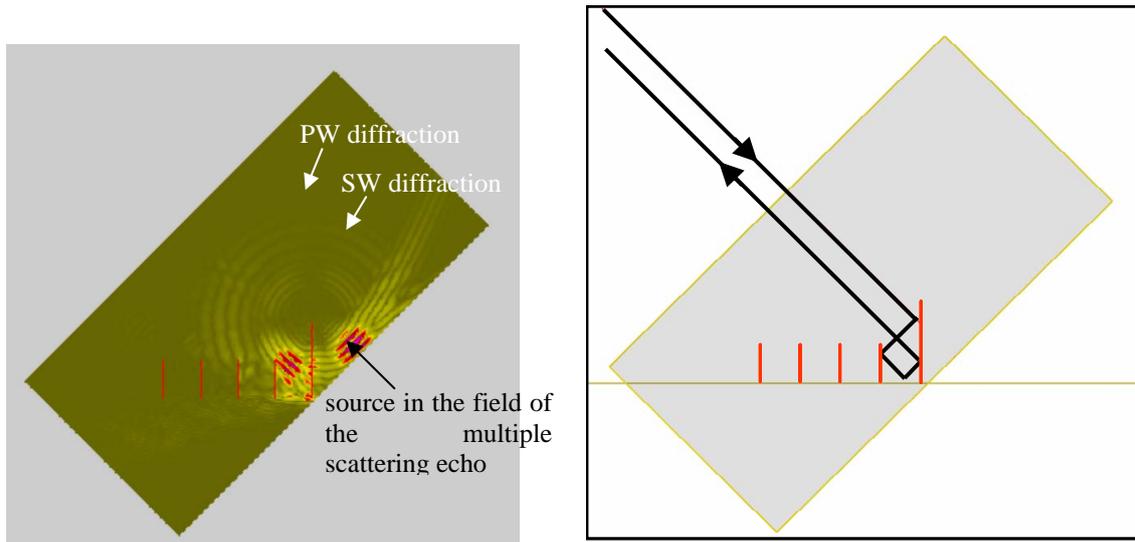
It is easy to distinguish the diffraction echo at the tip of the notch whatever the cracking density is. This is due mainly to the high-spatial resolution of the beam in the crack tip surrounding. If the corner echo of the surface breaking crack is always identified for the 4-mm spaced cracking, it becomes however difficult to localize it for the two others densities of cracking. For these cases, the corner echo seems to be split and shifted along the bottom part of the component (large echo on the right of the original corner echo position).

The processing of the Ascans shows that the amplitude of the diffraction echo is unchanged (see Fig. 5) for all the configurations. Moreover, another echo is detected when cracking is considered. This contribution doesn't seem to be a numerical artefact. Indeed, by looking at ATHENA snapshots for this position, it appears that this contribution can be geometrically interpreted and comes from multiple reflections between cracking and the planar crack (see Fig. 6).

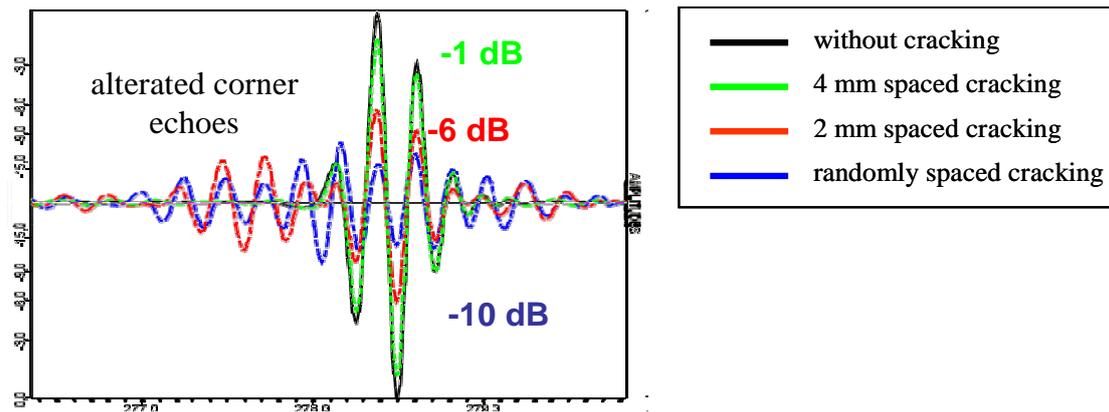
**Figure 5.** Ascans signals at the position corresponding to the maximum of the diffraction echo of surface breaking notch.



**Figure 6.** On left, a snapshot of the field at the position corresponding to the maximum of the diffraction echo. On right, the geometrical interpretation of one of the path which contributes to the detection of the multiple scattering echoes between cracking and the notch.



**Figure 7.** Ascans signals at the position corresponding to the maximum of the corner echo of surface breaking notch.



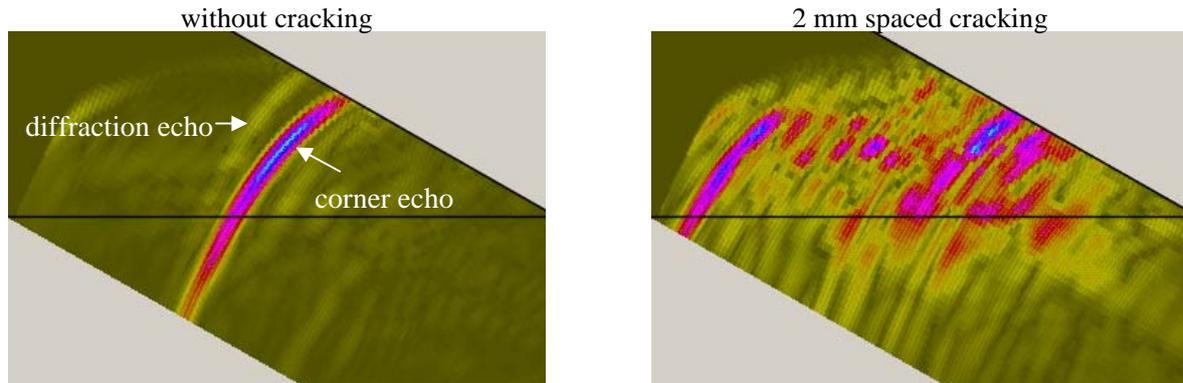
Another interesting effect simulated by the coupling model is the shadowing effect of cracking on the notch corner echo. The Ascans obtained at the position of the maximum of this echo illustrate the influence of cracking density on the amplitude of the echo (see Fig. 7). The amplitude for the 4-mm , 2-mm and randomly spaced cracking is respectively 1dB, 6dB and 10dB lower than the corner echo obtained without cracking.

The configurations previously presented were obtained for a high resolution beam, which allows easy interpretation of the results. A planar contact transducer with a 3.2 mm diameter working at the central frequency of 5 MHz is used to generate a 60°-P wave in steel.

Contrary to the cases shown in Fig. 4 the beam diameter emitted by the transducer is very large with respect to the size of the notch. As shown in Fig. 8, the response with 2-mm spaced cracking presents a very high back-scattering noise compare to the result without cracking. For this configuration control, the detection of the diffraction echo is not possible. Note that this simulation does not take into account PW mode conversion into SW.

The hybrid model provides additional help for the conception and the qualification of control techniques. Cases that require taking into account the interaction between flaws can be simulated.

**Figure 8.** Bscan images obtained by the hybrid model on samples containing the same 4 mm surface breaking notch without and with 2 mm spaced cracking for a 60°-PW control.



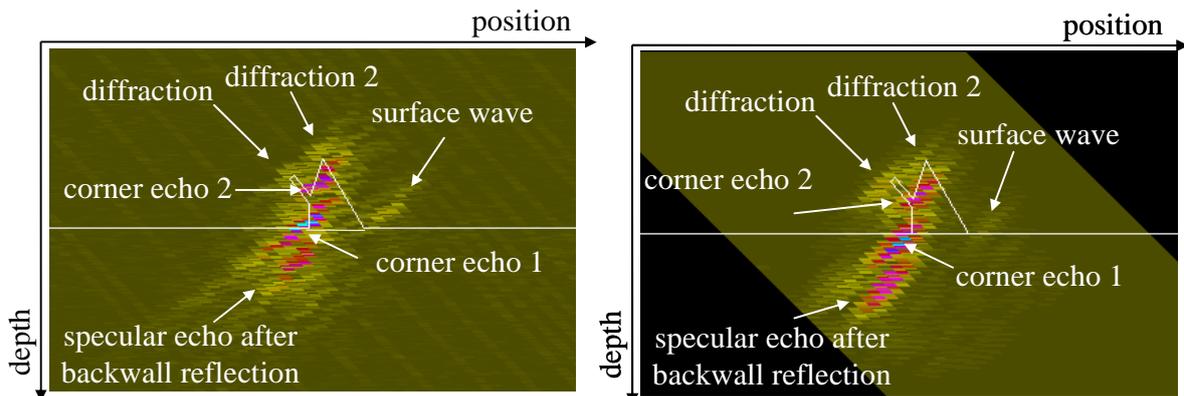
### 2.2 Inspection simulation on a branched crack

In this section, the simulated results have been compared with experimental ones. The controlled flaw is a surface-breaking crack with a branched geometry. The transducer is the same bifocal transducer used in the previous section which generates 45°-shear waves into the sample.

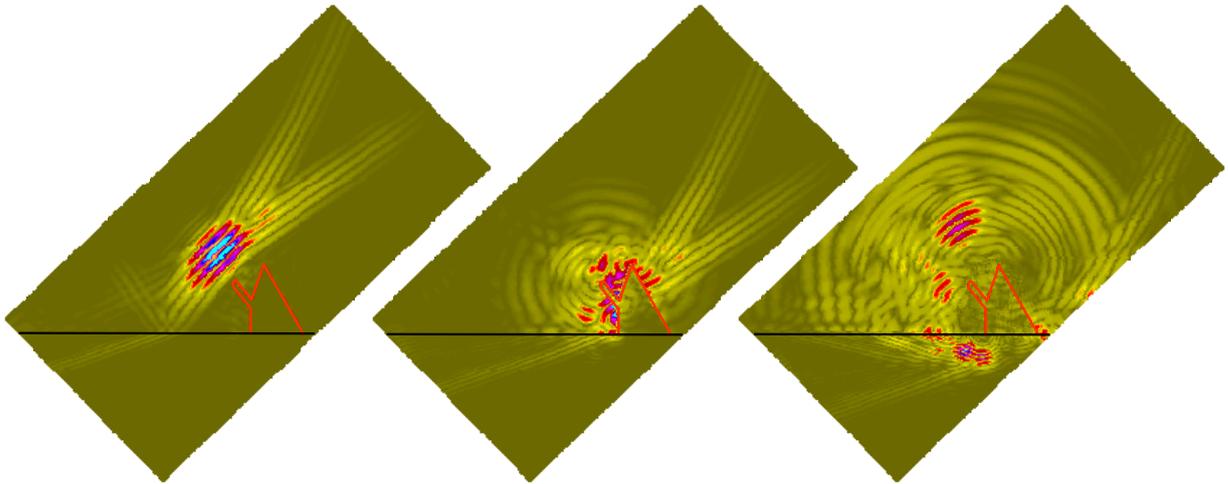
Both true BScan images are illustrated in Fig. 9. The experimental image presents multiple echoes due to the complex shape of the defect. At the base of the crack, the classical corner echo is generated but another one is observed on the top of the crack coming from reflections between 2 contiguous faces. A specular echo after a backwall reflection is observed arising from one branching of the crack. At last, 3 echoes with lower amplitudes are distinguished. Two of them are located on the tips of the crack and correspond to diffraction echoes. The other should be due to a surface wave propagating on the crack boundary. By comparing simulated result with the experimental one, all of these phenomena are recovered. The locations of the echoes are well defined although computed image outlines sharp contours of the echoes as a consequence of the idealized crack representation. The relative amplitudes between echoes are in general well evaluated except for the surface wave. The analysis of the field snapshots at different transit times of the FEM computation confirms the cause of the corner echo 2 as a multiple reflection between faces (see Fig. 10). Examining such an animation makes more comprehensible these complex phenomena.

Once more, this example demonstrates the capability of the coupling model to simulate complex beam/defect interactions, notably the wave interaction between faces of a same crack.

**Figure 9.** Bscan images provided experimentally (at left) and from computing (at right) for a 45°-SW control of a surface-breaking crack with a branched geometry.



**Figure 10.** Successive snapshots of the field computed by ATHENA describing the propagation and interaction of the incident beam with the branched crack.



## Conclusion

A hybrid approach for coupling the finite element code ATHENA and the semi-analytical model for CIVA US-field computation has been developed to determine defects in complex configurations. It is based on a transient form of Auld's reciprocity principle, in which the results of both models are linked together combining advantages of both computations.

The two illustrated examples have demonstrated the capability of this hybrid model to simulate complex phenomena, such as shadowing or multiple scattering, occurring when complex configurations are considered.

The model allows to evaluate amplitude decrease between a standard idealized defect and a realistic one. For NDT methods in complex configurations, this new simulation tool is then a complementary help for conception and qualification of technique in such configurations. However, if computation time for 2D calculation is adequate with parametric studies, performances obtained in 3D FEM computations are prohibitive for the present time.

Further developments of the model will now focus on the improvement of time performance for 3D computation to simulate specific 3D phenomena due to the shape of a defect or spatial distribution of the defects.

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