Finite-Difference Simulation of Ultrasonic Waves in a Medium Which Includes a Fluid-Solid Boundary

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Abstract. A finite-difference program, where a fluid-solid boundary was taken into consideration, was developed for computer simulation of the immersion method for ultrasonic testing. Since variables of wave equations for both fluid and solid regions are different, some consideration was given to the boundary conditions. For the visualization of simulation results, the behavior of ultrasonic waves was reproduced precisely in animated form on a computer display. With regard to the solid region, the longitudinal and shear components of ultrasonic waves were identified by different colors. Transmission and reflection characteristics of ultrasonic waves at the water-steel boundary were studied as well. It was shown that essential knowledge about the behavior of ultrasonic waves in immersion testing can be obtained through computer simulation, which includes knowledge about the fluid-solid boundary.

1 Introduction

For research and development of ultrasonic testing, it is essential to have an accurate understanding of the behavior of ultrasonic waves in the objects. In solids however, the behavior of ultrasonic waves is complicated due to the existence of various wave modes, mode conversions, reflections, diffractions, and other phenomena related to wave motions. In addition, a boundary between fluid and solid regions should be considered for the analysis of such methods as immersion testing. Consequently, the numerical calculation of ultrasonic waves by analytical methods is very difficult. Under these circumstances, computer simulation can be an effective tool for tracing ultrasonic waves.

A finite-difference program, where a fluid-solid boundary was taken into consideration, was developed for computer simulation of the immersion method for ultrasonic testing. Since variables of wave equations for both fluid and solid regions are different, some consideration was given to the boundary conditions. For the visualization of simulation results, the behavior of ultrasonic waves was reproduced precisely in animated form on a computer display. Transmission and reflection characteristics of ultrasonic waves at the water-steel boundary were presented as well.

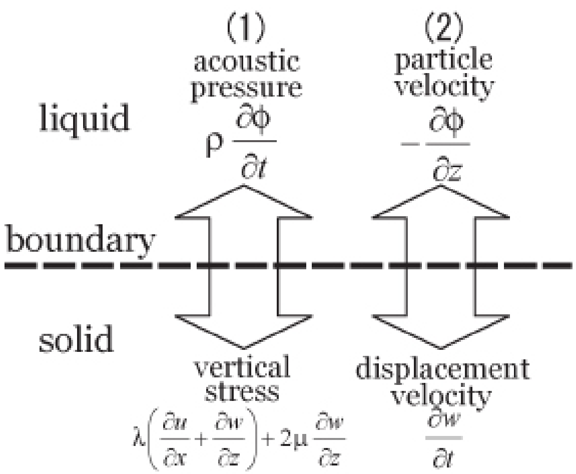


Fig.1. Model of Liquid-Solid Boundary

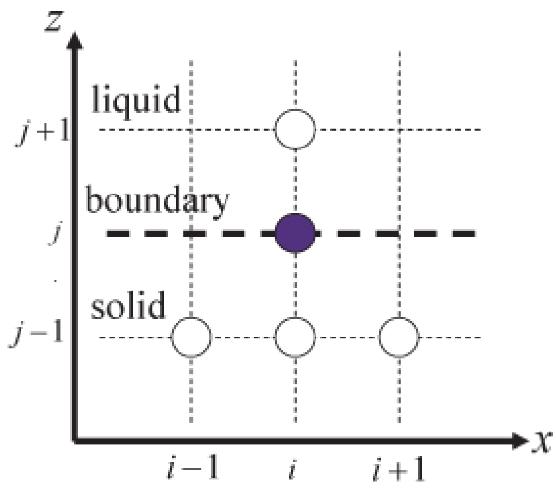


Fig.2. Lattice Arrangements around Liquid-Solid Boundary

2 Theory

2.1 Theoretical Consideration

Figure 1 shows a model of the liquid-solid boundary. Since variables of wave equations for both the fluid and solid regions are different, the boundary conditions between them are supposed as follows [1];

1. The sound pressure of the fluid is equal to the vertical stress of the solid.
2. Vertical component of the particle velocity of the fluid is equal to that of the displacement velocity of the solid.
3. The shearing stress of the solid at the boundary becomes zero.

2.2 Finite-Difference Program

A finite difference method was employed for the simulation. Finite-difference calculations for the liquid-solid boundary are carried out by using the following equations;

$$\phi_{t+\Delta t}(i,j) = -\frac{\Delta z}{\Delta t}\left(w_{t+\Delta t}(i,j-1) - w_t(i,j-1)\right) + \phi_{t+\Delta t}(i,j+1),$$

$$w_{t+\Delta t}(i,j) = \frac{\rho\Delta z}{\Delta t(\lambda+2\mu)}\left(\phi_{t+\Delta t}(i,j+1) - \phi_t(i,j+1)\right) - \frac{\lambda\Delta z}{2\Delta x(\lambda+2\mu)}\left(u_{t+\Delta t}(i+1,j-1) - u_{t+\Delta t}(i-1,j-1)\right) + w_{t+\Delta t}(i,j-1),$$

$$u_{t+\Delta t}(i,j) = -\frac{\Delta z}{2\Delta x}\left(w_{t+\Delta t}(i+1,j-1) - w_{t+\Delta t}(i-1,j-1)\right) + u_{t+\Delta t}(i,j-1) \qquad (1.2.1)$$

Here φ is the velocity potential of the liquid, *w* and *u* are the displacements of the solid in the vertical and horizontal directions, respectively. λ and μ are Lame's constants of the solid, and ρ is the density of the liquid. (*i, j*) represents coordinates of the lattice point, and Δx and Δz are lattice spacing, and Δt is a time step of the finite difference calculation.

Figure 2 shows the lattice arrangements around the liquid-solid boundary. As to the lattice points on the boundary, three variables, φ , w, u, should be simultaneously considered in the finite difference calculation [2][3].

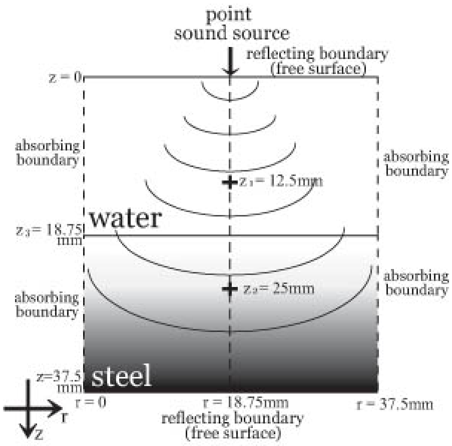


Fig.3. Simulation Model

TableⅠ . Material constants used for the simulation.

Water	Acoustic Velocity	1.50×10^{3} (m/s)
	Density	1.00×10^{3} (kg/m^{3})
Steel	Young's Modulus	2.06×10^{11} (N/m^{2})
	Poisson's Ratio	0.28
	Density	7.70×10^{3} (kg/m^{3})
	Longitudinal Wave's Velocity	5.85×10^{3} (m/s)
	Shear Wave's Velocity	3.23×10^{3} (m/s)
	Lame's Constant λ	1.02×10^{11} (N/m^{2})
	Lame's Constant μ	8.05×10^{10} (N/m^{2})

3 Method

Figure 3 shows the two-dimensional model for the simulation which includes the liquid-solid boundary. The height and width are 37.5mm, and the liquid-solid boundary was set horizontally in the middle. Waveforms of the particle velocity and displacement velocity ware observed at the points z_1 in the liquid and z_2 in the solid, respectively.

For the immersion method for ultrasonic testing, the liquid and solid ware supposed to be water and steel. TableⅠ shows the constants used for the simulation. The top of the liquid and the bottom of the solid were set to be free boundaries. Both sides of the medium were set to be absorbing boundaries.

4 Result

Figure 4 shows snapshots of velocity potential in the water and displacement in the steel. A point sound source of acoustic pressure was applied to the center of the top of the liquid. A three-wave tone burst of 1.25MHz with a squared-cosine envelope was employed as the waveform of the applied pressure[4].

The generated waves propagate in the water and attain the liquid-solid boundary at t=12.7μs. Reflected waves into the water and transmitted waves into the steel are observed.

Figure 5 shows the waveforms of particle velocity at z_1 in the water and displacement velocity at z_2 in the steel.

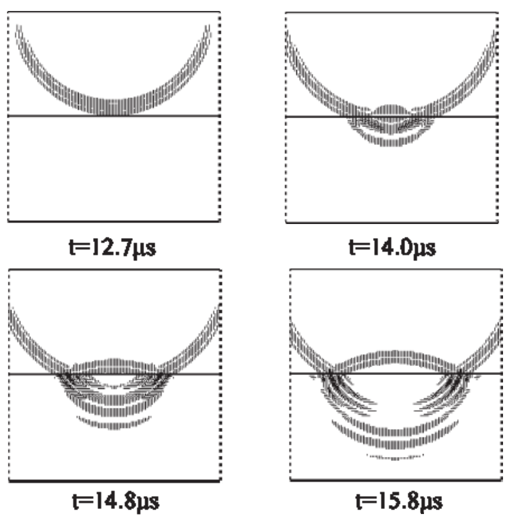
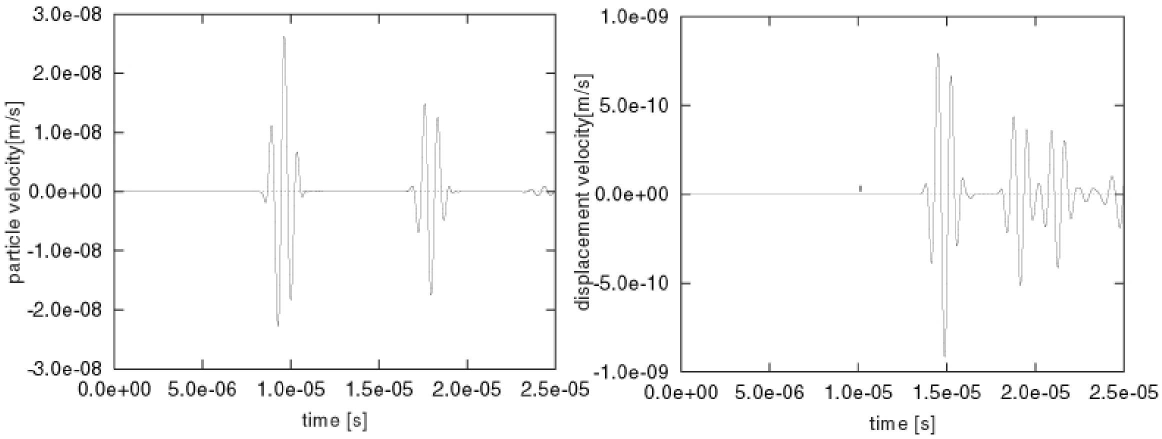


Fig.4. Snapshots of Velocity Potential in Water and Displacement in Steel



(a) $z=z_1$(Water) (b)$z=z_2$(Steel)

Fig.5. Waveforms of Particle Velocity in Water and Displacement Velocity in Steel

5 Consideration

5.1 Reflection and Transmission of Plane Waves

With respect to the particle velocity in liquid and displacement velocity in solid, the reflectance *r* and transmittance *t* of the plane wave at the boundary, are generally described by the following equations [5];

$$r = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \qquad t = \frac{2Z_1}{Z_1 + Z_2} \tag{5.1.1}$$

Here, Z_1 and Z_2 are acoustic impedance densities of liquid and solid, respectively.

5.2 Geometrical Attenuation

For the estimation of actual reflectance and transmittance, it is necessary to consider geometrical attenuation due to wave propagation. Since a two-dimensional model was employed, the waves propagate cylindrically. Wave amplitudes are inversely proportional to the square root of distance. Figure 6 shows wave amplitudes as a function of distance from the source. In the figure, the solid line indicates the simulation result and broken line indicates the theoretical curve of geometrical attenuation for cylindrically waves [6].

5.3 Reflection and Transmission Considering Geometrical Attenuation

The actual reflectance and transmittance at the liquid-solid boundary z_3 are calculated by the following equations [7];

$$r = \frac{A_2}{A_1}\sqrt{\frac{2z_3 - z_1}{z_1}}, \qquad t = \frac{A_3}{A_1}\sqrt{\frac{z_2 + (\alpha - 1)z_3}{\alpha z_1}}, \qquad \alpha = \frac{\text{sound velocity in water}}{\text{sound velocity in steel}} \tag{5.3.1}$$

Here, wave amplitudes of the incident wave and the reflected wave at z_1 in the water are A1 and A2, respectively, and wave amplitude of the transmitted wave at z_2 in the steel is A3.

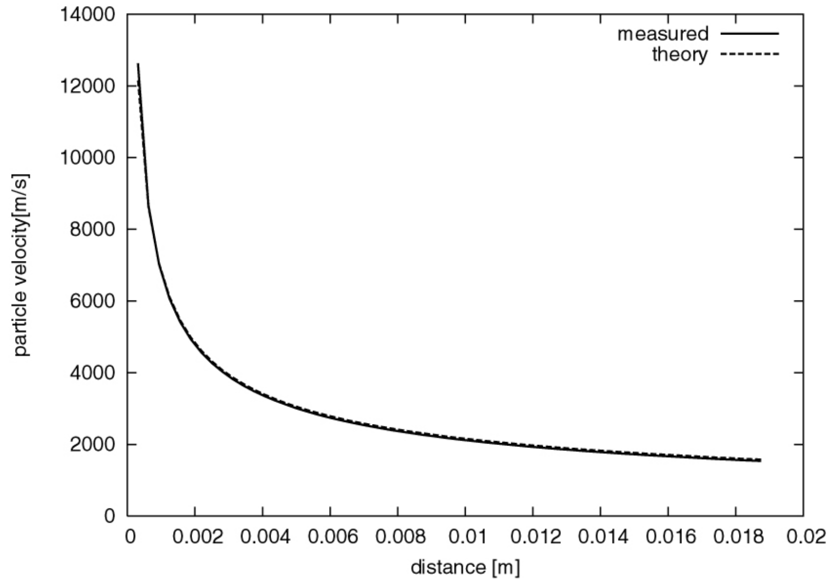


Fig.6. Geometrical Attenuation in Liquid

Table Ⅱ compares the calculated and theoretical values of the reflectance and transmittance. Theoretical values are derived from acoustic impedance densities by using equation (5.1.1). Calculated values are based on a result of simulation which employed a two-dimensional model.

Table Ⅱ . Comparison of calculated and theoretical values.

Reflectance	Theoretical	0.9355
	Calculated	0.9293
Transmittance	Theoretical	0.0645
	Calculated	0.0647

References

[1] N.Kawai, H.Hatano, S.Suzuki; "Computer Simulation of Elastic Waves in Multilayered Media", Proceedings of the Fifth Symposium on Ultrasonic Testing, January 27-28,1998, Tokyo, pp.35-36.

[2] A.Taflove, S.C.Hangness; "Computational Electrodynamics: The Finite-difference Time-Domain Method", Artech House, 2000, pp.1-34.

[3] Almoga Ilan, Abraham Unger and Zipora Alterman; "An Improved Representation of Boundary Conditions in Finite Difrference Schemes for Seismological Problems", Geophys.J.R.astr.Soc, 1975, pp.727-745.

[4] Alexander Sedov, Lester W. Schmerr, Jr.and Sung Jin Song; "Ultrasonic scattering by a flat-bottom hole in immersion testing: Analytical model", Acoustical Society of America Volume 92 No. 1, 1992, pp.478-486.

[5] K.Kido; "Acoustical Engineering", Corona Publishing Co., Ltd., 1982, pp.8-14.

[6] S.Nishiyama; "Acoustic and Vibration Engineering", Corona Publishing Co., Ltd., 1979, pp.40-56.

[7] M.Kishigami, K.Kimura, H.Uragaki, H.Matsuyama; "Ultrasonic Testing Ⅲ ", Japanese Society of Non-Destructive Inspection, 1989,pp.25-29.