Denoising of Computed Tomography Images using Multiresolution Based Methods

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Abstract. The present paper deals with the enhancement of X-ray computed tomography (CT) images based on multiresolution image denoising methods (wavelets, platelets). The term 'denoising' describes the attempt to remove noise and retain the signal regardless of the frequency content of the signal. The main focus of this experimental work, conducted on an industrial 3D-CT, lies in the attempt to determine and visualize small spatial structures (e.g. inclusions, blowholes) in low contrast images which are degraded by noise. A precondition for correct application of the proposed denoising filter methods is an exact noise characterization of the imaging system (poisson noise, gaussian noise). Special emphasis is put on the noise analysis of the two dimensional CT imaging detector (i.e. the spatial noise and degradations in one single image). The denoising results of various practical CT image examples are presented and compared with classical linear filter methods showing the improved image quality of multiresolution denoised CT images.

Introduction

X-ray computed tomography (CT) is a powerful method for determining the internal structure of an object. As such it finds application, e.g. in the non-destructive testing of a variety of materials. The CT image is derived from a large number of systematic observations at different viewing angles, and the final CT image is then reconstructed with the aid of a computer (Radon transform). In the last years 3D-CT systems with a matrix detector (Fig. 1+2) have become more and more popular. The main advantages of these systems are the reasonable high scanning speed and thus lower costs compared to 2D-CT systems with a line detector.

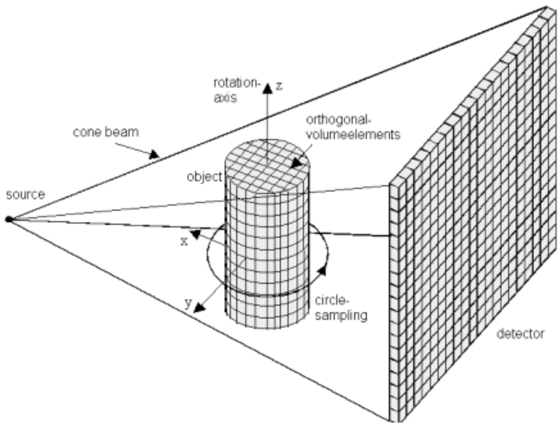

Fig. 1. Principle of 3D-computed tomography

Fig. 2. X-ray detector *PerkinElmer RID 1640 AL1 ES* of the CT *Rayscan 250 E*

For monochromatic radiation with an incident intensity I_0, the X-ray beam is attenuated after passing through a sample of thickness *d*. The magnitude of the attenuated intensity *I* is described by Lambert–Beer's law:

$$I = I_0 \exp(-\mu d) \quad (1)$$

where μ is the sample-representative linear attenuation coefficient which depends on the electron density of the material, the energy of the radiation, and the bulk density of the sample material.

In this paper we focus on the attempt to determine and visualize small spatial structures (e.g. inclusions, blowholes, faint structures) in low contrast images by applying multiresolution based digital filter methods – namely wavelets [1] and platelets [2][3]. Although noise gives an image a generally undesirable appearance, the most significant factor is that noise can cover and reduce the visibility of certain features within the image. In this work we consider image denoising as the process of removing noise without trying to additionally enhance spatial features. In this way the denoising process can be seen as a pre-processing step for further image processing (e.g. segmentation). A precondition for the correct application of the proposed denoising filter methods is an exact noise characterization of the imaging system. For this reason we applied the "Photon Transfer Technique" which will be described in the following section.
The experimental work was conducted on an industrial 3D-CT *HWM Rayscan 250 E* System that uses a *PerkinElmer RID 1640 AL1 ES* scintillation matrix detector at the Univ. of Applied Sciences Wels. The denoising experiments were programmed in MATLAB which is a commercially available mathematical software development package (www.mathworks.com).

1. Noise characterization

1.1 Detector noise

X-ray photons impinge on a surface, such as an image detector, in a random pattern. One area of the detector surface might receive more photons than another area, even when both are exposed to the same average x-ray intensity. The amount of noise is determined by the variation in photon concentration from point to point within a small image area.

In this paper the total noise of the detector is modeled by three major components: photon noise, read noise and pattern noise. The standard deviation of the detector noise $<n_{DETECTOR}>$ is modelled as a sum of these components:

$$< n_{DETECTOR} > = \sqrt{< n^2{}_{shot} > + < n^2{}_{read} > + < n^2{}_{pattern} >} \quad (2)$$

Photon shot noise is caused by the quantum nature of light exhibiting a Poisson distribution. For a Poisson random variable, the probability that *X* is some value *x* is given by the formula:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0,1,\ldots \quad (3)$$

where λ is the average number of occurrences (e.g. number of photons) in a specified interval (i.e. detector integration time or x-ray tube current). For the Poisson distribution, the expectation and the standard deviation is given by:

$$E(X) = \lambda, \qquad \sigma(X) = \sqrt{\lambda} \tag{4}$$

Read noise represents the random noise measured under totally dark conditions (i.e. independent from the signal level). Read noise is assumed to be independent of position on the detector with a simple Gaussian error distribution. Pattern noise (fixed pattern noise and photoresponse non-uniformities) manifests itself as a spatial non-uniformity most prominent in averaged dark images or flat-field images. Dark images and flat-field images are used to remove disturbing image structures [4].

1.2 Photon Transfer Technique

The photon transfer technique now makes use of the imaging characteristics described above. The photon transfer curve (PTC, Fig.5) is a response of a uniformly illuminated detector at different radiation levels. It gives the conversion constant used to convert relative digital units (ADU - analog to digital units) generated by the detector into absolute physical units (number of photo-electrons per ADU). This detector gain constant is expressed in e^-/ADU (i.e. the number of photo-electrons to cause a gray-level change of 1 ADU). Practically, the photon transfer curve is a plot of random noise σ_s as a function of mean signal (ADU). The mean signal intensity μ of two images A,B within an image region of N=mxn pixels is determined in the following way:

$$\mu_A = \frac{1}{N}\sum_{i=1}^{N} x_{i,A} \qquad \text{and} \qquad \mu_B = \frac{1}{N}\sum_{i=1}^{N} x_{i,B} \tag{5}$$

The signal noise σ_s is defined as the square root of the signal variance, with fixed pattern noise (FPN) removed. FPN is removed by subtracting two images at the same exposure level. Then the variance σ^2_{A-B} is computed over a region (e.g. 100x100 pixels) and divided by two, because the rms noise increases by $\sqrt{2}$ when two identical frames are either added or subtracted):

$$\sigma^2_{A-B} = \frac{1}{2\cdot(N-1)}\sum_{i=1}^{N}\left(\left[x_{i,A}-\mu_A\right]-\left[x_{i,B}-\mu_B\right]\right)^2 \tag{6}$$

The read noise floor represents random noise under totally dark conditions. As the radiation on the detector is increased, the noise becomes dominated by the shot noise associated with the random arrival of photons (as goverened by Poisson's statistics) characterized by a line of slope ½ in the log-log-coordinate plot (Fig.5). The extrapolated straight-line intercept of the signal-axis gives the detector gain constant g (i.e. the number of photons that are necessary to cause a gray level change of one ADU/analog-to-digital-unit in the image). This gain constant can now be identified as the necessary parameter for the platelet denoising (section 2.2), because it represents the statistical number of 'occurences' in Eq.3. The measurement results show the almost perfect increase of noise signal with intensity showing that the dominant source of noise is Photon noise (see section 4). Furthermore we found, that the gain constant varies slightly with different x-ray source energies and detector parameters [5] as indicated in Fig.5.

2. Multiresolution denoising

2.1 Wavelets

Wavelet Transform (WT) was introduced as an alternative approach to overcome problems with the frequency resolution in standard STFT (short term fourier transform). Instead of using periodic functions in the transformation kernel, it uses a waveform function, the so-called wavelet function. While STFT provides uniform time resolution for all frequencies, WT provides high time resolution and low frequency resolution for high frequencies and high frequency resolution and low time resolution for low frequencies. This is obtained by scaling and translating a basis function, which is called the mother wavelet. In general, the mother wavelet can be used to obtain wavelet basis functions. These functions can be expressed as:

$$\psi_{s,u} = \frac{1}{\sqrt{|s|}}\psi(\frac{t-u}{s}), \qquad s,u \in \Re \quad s \neq 0 \tag{7}$$

where ψ (t), referred to as the mother wavelet, is a time/space function with finite energy and fast decay, and s and u represent the dilation and translation parameters respectively. The continuous wavelet transform (CWT) is defined as:

$$CWT(s,u) = \int_{-\infty}^{\infty} \psi_{s,u} z(t) dt \tag{8}$$

Hence, with the CWT a signal is decomposed into its frequency components by scaled wavelet functions. The WT provides a tool for analyzing a signal at different scales (or resolutions) for an entire space range. The WT results in wavelet coefficients at every possible scale. The Discrete Wavelet Transform (DWT) is a special case of the WT and is based on dyadic scaling and translating which makes the WT computationally efficient. It is basically a filtering procedure that separates high and low frequency components with high-pass and low-pass filters by a multiresolution decomposition algorithm [1]. For most practical applications, the wavelet dilation and translation parameters are discretized dyadically. Hence, the DWT is represented by the following equation:

$$W(j,k) = \sum_j \sum_k x(k) 2^{-j/2} \psi(2^{-j} n - k) \tag{9}$$

Accordingly, the fundamental principle of wavelet-analysis is to represent a function f as a linear combination (superposition) of ‘simple’ basic functions ψ_k.

$$f = \sum_{k \in J} c_k \psi_k \tag{10}$$

The idea behind denoising is a ‚thresholding’ of wavelet coefficients c_k. Thresholding means, that certain coefficients with an absolute value below a threshold τ are set to zero (‘hard-thresholding’):

$$\tilde{c}_k := \begin{cases} 0 & if \quad |c_k| \leq \tau \\ c_k & if \quad |c_k| > \tau \end{cases} \tag{11}$$

Fig.3: Estimation $\tilde{f}$ of the signal through a thresholding of wavelet coefficients.

The aim is to define a threshold that mainly eliminates coefficients which are responsible for the noise signal. As a statistical model for the noise distribution, usually white Gaussian noise with standard deviation σ is assumed. It can be shown that in the case of an orthonormal basis the coefficients c_k also undergo a Gaussian noise distribution with the same standard deviation σ [6]. It is proposed to set the threshold in proportion to the number N of coefficients :

$$\tau = K \cdot \sqrt{2 \cdot \ln(N) \cdot \sigma} \quad \text{with } K \approx 1 \tag{12}$$

A practical problem is the fact that relatively small contributions of the true signal are often enough to lift a certain coefficient (that is mainly caused by noise) above the threshold level. As a result, impulses in the form of the basis function become visible in the reconstructed signal (artefacts). A possible solution is the introduction of a ‘shrinkage’ of the coefficients (‘Soft-thresholding’):

$$\tilde{c}_k := \begin{cases} 0 & if \quad |c_k| \leq \tau \\ sign(c_k)(|c_k| - \tau) & if \quad |c_k| > \tau \end{cases} \tag{13}$$

In most cases signal noise is modelled as Gaussian white noise with a constant noise level σ. When non-white noise is suspected, thresholds must be rescaled by a level-dependent estimation of the level noise.

2.2 Platelets

Despite the excellent results in a variety of applications, wavelet-based methods have shortcomings in the treatment of edge structures in images resulting in artefacts (like ‘ringing’) around edges in the final image. Additionally, most wavelet-based approaches are based on Gaussian approximations to the Poisson likelihood which can be especially inaccurate when low count levels are expected. Recently, a new computationally fast and spatially adaptive method was presented by R. M. Willett and R. D. Nowak [2] which is based on the analysis of the tree structure of a dyadic image decomposition (recursive partitioning). They propose a new multiscale image representation based on ‘platelets’. The platelet is based on the wedgelet-approach of D. Donoho which was originally developed for better edge representation in images [7]. Each wedgelet is defined on a dyadic square with certain edge-points. Instead of approximating the partitions with a constant, the platelet decomposition introduces planar surfaces and in this way additionally gradients. This representation can be coded in a tree structure (Fig.4) and an adaptive ‘pruning’ process eliminates the noisy signal portions. Additionally, this approach is very well suited to applications with Poisson data, unlike most other multiscale signal and image representations [2][8]. In this way, platelets can produce highly accurate, piecewise linear approximations to images consisting of smooth regions separated by smooth boundaries. These are the required characteristics when imaging ‘geometric’ image features. The only necessary parameter which has to be determined for a statistically correct implementation of this denoising procedure is the Poisson parameter λ (see Eq.3). We propose to derive this parameter from the Photon Transfer Technique described in section 1.2.

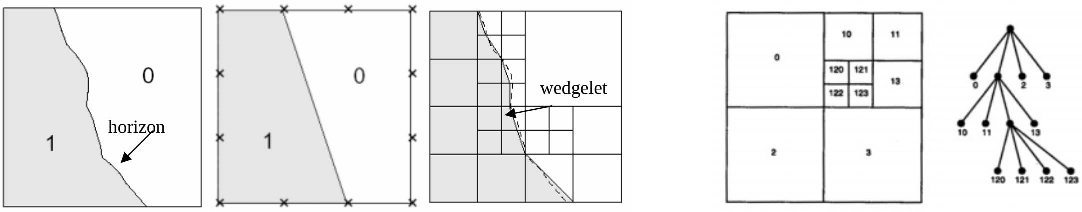

Fig.4: left: wedgelet decomposition; right: tree coding

4. Experimental results

4.1 Photon Transfer Curves (PTC)

The Photon Transfer Curve (see section 1.2) was determined by imaging a copper-structure (Fig.5a,b) which produces intensity steps in the radiogram. Within these regions the associate noise standard deviation was determined. The results clearly show the almost perfect Poisson noise characteristic (dashed line) of the measurements conducted at various x-ray source energies / detector parameters. The corresponding gain constants can also be found in Fig.5.

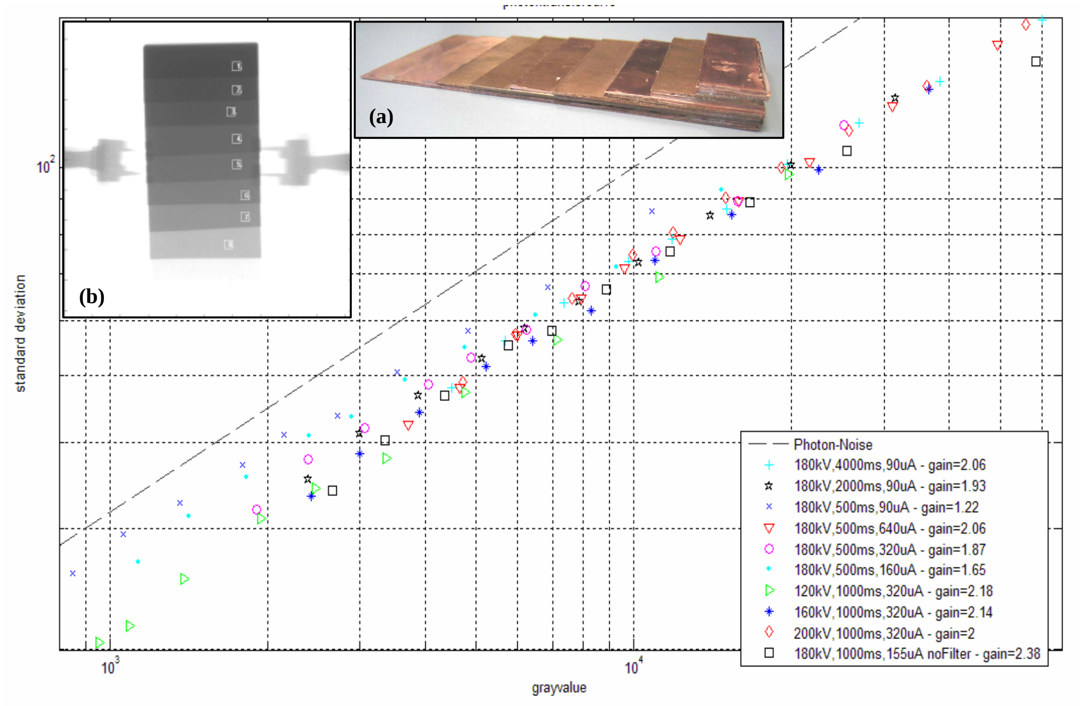

Fig. 5. Photon Transfer Curves at various x-ray source energies / detector parameters and corresponding gain constants (section 1.2), (b) shows a radiogram of the cupper step (a) used for the Photon Transfer Curve generation (used pre-filter Pb 0,1 mm; Cu 0,2 mm; post filter Al 1 mm except the last parameter combination)

4.2 Denoising results

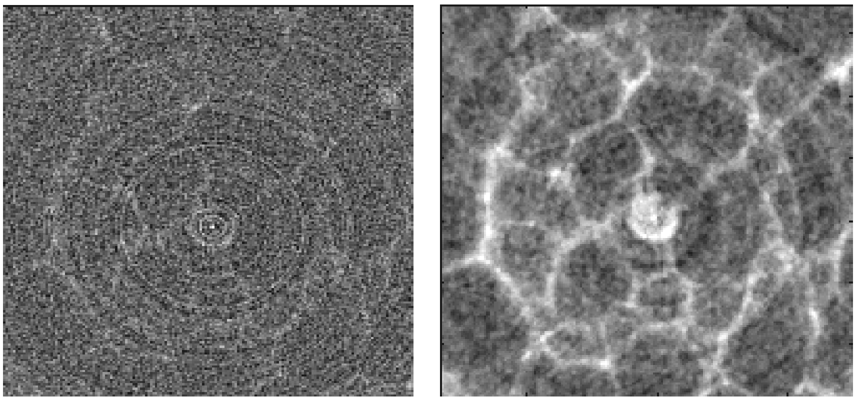

Fig. 6. Result of denoising process (polymeric foam structure), left: original CT-picture; right: platelet-denoised

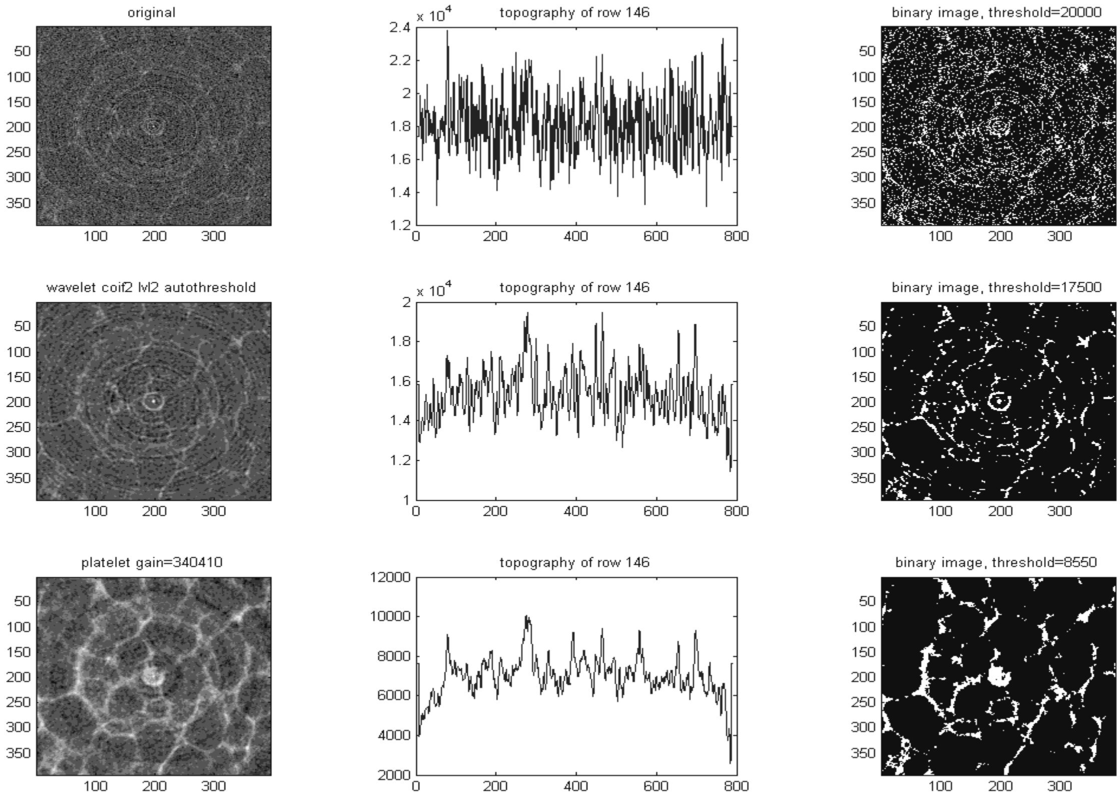

Fig. 7: Result of denoising process (polymeric foam structure) with binary threshold (right column).

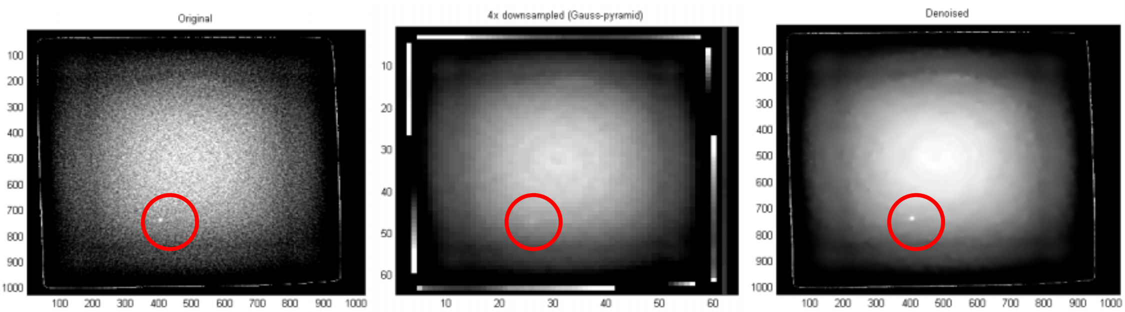

Fig. 8: Comparison: detection of small inclusion within steel (left: original CT cross-sectional picture; middle: linear filter - Gaussian pyramid level4; right: platelet-denoised)

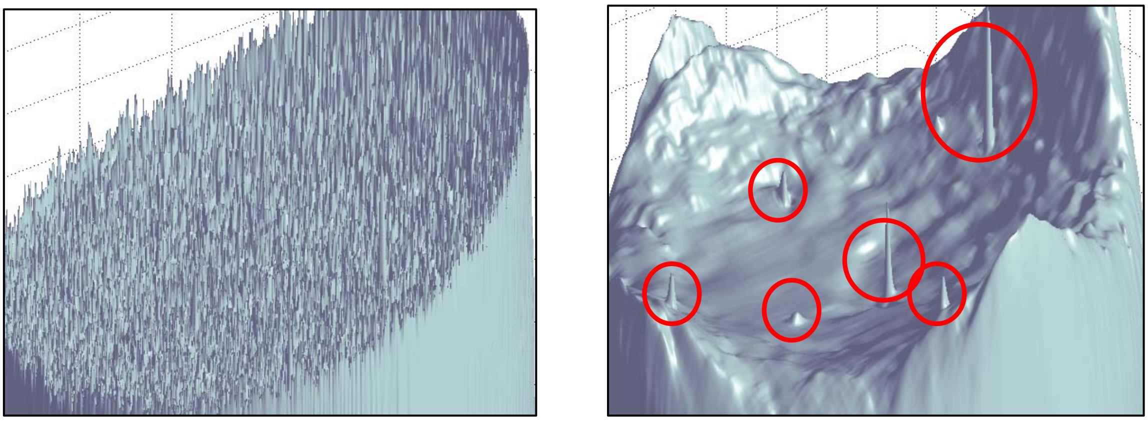

Fig. 9: Detection of small inclusions (left: original CT-results; right: platelet-denoised). The inclusions are marked with circles.

Fig.6 – Fig.8 show some of the denoising results obtained by the methods described. Fig.6 illustrates the difference between the original image and the denoised version of a faint plastic foam-like structure. As mentioned above, the result is not enhanced for visual appearance but for further image processing (like segmentation). This can be seen from Fig.7. which shows significant differences in the resulting binary images (after introduction of a simple grey level threshold). Fig.8 and Fig.9 show the advantage of the statistically based denoising process when trying to detect small spatial structures (like inclusions in a metallic object).

5. Conclusions

It is shown that multiresolution denoising methods (in particular wavelet filters and the platelet filter) can improve the quality of CT images considerably. Especially, if small spatial structures in low contrast images, which are almost hidden in signal noise, are expected. The platelet denoising filter is based on the statistical model of Poisson noise. The only necessary parameter for correct implementation of the platelet filter is the Poisson parameter λ. It was proposed that the Poisson parameter λ can be determined by an exact noise characterization of the detector system (Photon Transfer Technique).

Further work focuses on the extension of multiresolution based denoising schemes to 3D-voxel data. Additionally, a detailed comparison of the presented denoising schemes before and after a CT-reconstruction is planned [5].

6. Acknowledgements

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