Mine Signature Simulations Using Non-Overlapping Domain Decomposition with Lagrange Multipliers

Sabine SCHULZE, Ursula VAN RIENEN, Rostock University, Germany

Abstract. The metal detector is the most commonly used demining device in the world. Although it has several advantages, such as low cost and easy handling, there is one main disadvantage: the high false alarm rate. This originates in the inability of the metal detector to distinguish between a land mine and other metallic objects. A database with mine signatures can be provided allowing for an intelligent signal analysis, thus significantly reducing false alarms and increasing the operational time in demining. In order to compute synthetic signatures of the various land mines, fast and highly precise forward algorithms have to be implemented. The electromagnetic field simulation is challenging because of the small size of the metal parts in modern landmines in comparison to the coil of the metal detector and the distance between it and them. The relatively small field modification caused by these miniaturised metal parts calls for high computational accuracy. A very fine discretization is needed in and closely around the metallic object, especially in its skin depth layer and a coarser one around the coils of the metal detector and in the remaining computational domain. A non-matching, non-overlapping domain decomposition method is used in order to solve the magneto-quasistatic field problem. Continuity of the solution is enforced by Lagrange multipliers. The Finite Integration Technique (FIT) is applied as discretization method for the magnetic diffusion equation.

1. Introduction

The ability of a metal detector to discover buried metal objects is based on the eddy current principle. With the help of the acoustic signal of such a detector, a qualified mine seeker is able to determine the horizontal position of the detected metal object. However, using conventional metal detectors it is impossible to get further information, such as the vertical position, the size or the material of the object. This also makes it impossible to reach a decision as to whether the detected object is a mine or another metal object. In practice, only one signal in every hundred or thousand is actually a mine [1].

Using a comparison of measured mine signatures with calculated signatures in a database makes the identification of false alarms possible and with it the reduction of the operational time. The calculation of such signatures poses a challenge due to the difference in size between the metal detector head (diameter about 30 cm) and the metal objects within the mine (sizes only few millimetres). Furthermore, the skin effect has to be taken into account. Typical skin depths for this application range from 0.1 mm to a few millimetres. Due to these facts, different resolutions for the meshes of the metal objects and of the detector head would be desirable.

Therefore, in this paper, a domain decomposition method with Lagrange multipliers is presented to solve this problem. Here the calculation domain is decomposed into non-overlapping subdomains with different discretizations, which can be non-matching. In this application, the fine grid is chosen for around the metal object and a coarser one for the rest, including the metal detector head. A system of equations is set up for both subdomains. To obtain continuity for the values on the interface, the subdomains were coupled with interface conditions using Lagrange multipliers. Finally, a new system of equations has to be solved.

In addition, the primary field of the metal detector is considerably larger than the secondary field of the metal object. Because both fields superpose each other a high accuracy is necessary. This can be reached if the primary field is calculated initially, and the secondary field only in a second step; it is the difference in the fields between the calculations without metal object and with metal object under the coils of the detector.

2. The formulation

2.1 Analytic magneto-quasistatic equations

The transmitting coil of a standard metal detector is excited with single- or multi-frequency or with impulse signals. This paper deals with the harmonic excitation, due to the fact that the impulse excitation can be simulated by the summation of results with harmonic excitation with different frequencies. For the mine searching application, the magneto-quasistatic approach is valid for the following reason, the wavelength is much smaller than the dimension of the calculation domain and the displacement current densities are much smaller than the total current densities, so they can be neglected.

For the time harmonic case and the magneto-quasistatic approach the two following Maxwell's equations are important:

$$\begin{aligned} curl\,\underline{\mathbf{E}} &= -i\omega\underline{\mathbf{B}}, \\ curl\,\underline{\mathbf{H}} &= \underline{\mathbf{J}}. \end{aligned} \tag{1}$$

Within this system, the complex phasor of the electric field strength is denoted with $\underline{\mathbf{E}}$, the imaginary unit with i, the angular frequency with ω, the phasor of the magnetic induction with $\underline{\mathbf{B}}$ and the phasor of the magnetic field strength with $\underline{\mathbf{H}}$. The phasor of the total current density $\underline{\mathbf{J}}$ arises from the sum of conduction current density, which is the product of the conductivity σ and the phasor of the electric field strength $\underline{\mathbf{E}}$, and the phasor of the source current density $\underline{\mathbf{J}}_{\mathbf{s}}$, i.e.:

$$\underline{\mathbf{J}} = \sigma\underline{\mathbf{E}} + \underline{\mathbf{J}}_s. \tag{2}$$

Taking Maxwell's equations (1) and the formula for the total current density (2) together with the definition for the phasor of the magnetic vector potential:

$$\underline{\mathbf{B}} = curl\,\underline{\mathbf{A}}, \tag{3}$$

and the material equation for the permeability μ:

$$\underline{\mathbf{B}} = \boldsymbol{\mu}\underline{\mathbf{H}}, \tag{4}$$

the following equation for the calculation of magneto-quasistatic fields arises:

$$curl\,\frac{1}{\mu}curl\,\underline{\mathbf{A}} + i\omega\sigma\underline{\mathbf{A}} = \underline{\mathbf{J}}_{s.} \tag{5}$$

2.2 Application of the Finite Integration Technique

The Finite Integration Technique (FIT) [2], [3] is used for discretizing the calculation domain. The electromagnetic field quantities are mapped onto two grids, the primary and the dual grid, which are shifted against each other. The discrete magnetic vector potentials $\widehat{\mathbf{a}}$ are assigned to the primary grid edges and the discrete source current densities $\widehat{\widehat{\mathbf{j}}}_{\mathbf{s}}$ are allocated on the dual grid facets. In FIT the analytic curl operator is represented as the discrete primary and dual curl operators $\mathbf{C}$ and $\widetilde{\mathbf{C}}$. The material properties conductivity σ and permeability μ are averaged along the grid facets or the grid edges and are combined in the material matrices $\mathbf{M}_{\mu^{-1}}$ and $\mathbf{M}_{\sigma}$. The magneto-quasistatic equation (5) for the magnetic vector potential reads in the framework of FIT [4] as:

$$\left(\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C} + i\omega\mathbf{M}_{\sigma}\right)\widehat{\mathbf{a}} = \widehat{\widehat{\mathbf{j}}}_{\mathbf{s}}. \quad \textbf{(6)}$$

For our further considerations the system matrix $\mathbf{K}$ is defined as:

$$\mathbf{K} := \widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C} + i\omega\mathbf{M}_{\sigma}. \quad \textbf{(7)}$$

3. Domain decomposition method with Lagrange multipliers

3.1 System of equations

As mentioned above a metal detector head is much bigger than the metal parts of landmines. Therefore it is very difficult to discretize such arrangements with only one mesh and it would be favourable to have different discretizations for both parts. The domain decomposition method with Lagrange multipliers divides the calculation domain in non-overlapping subdomains with non-matching grids. Both grids can be chosen independently from each other, see the sketch in Fig.1. The same method is used in [5] for the calculation of low-frequency electric current densities in high resolution 3D human anatomy models.

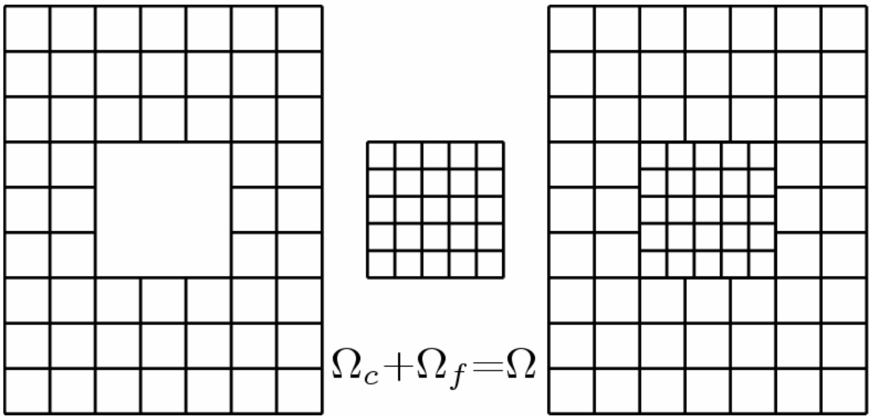


Fig. 1. Calculation domain is subdivided into coarse and fine domains.

For the mine searching application, the domain with the fine discretization is around the metal object or rather the landmine and all terms referring to this domain are denoted with the subscript f. The rest of the calculation domain including the metal detector head receives a coarser grid and the subscript c. In the first step the systems of equations for both subdomains are set up with Neumann boundary conditions on the interface. First, the coarse subdomain is set up for the whole calculation domain. In a second step, the area of the fine subdomain is deleted from the coarse one,

$$\Omega_c = \Omega \setminus \Omega_f . \tag{8}$$

Also, the dual grid of the coarse domain has to be shortened to the common interface. The magneto-quasistatic equations for both subdomains without interface conditions read as:

$$\begin{aligned} \mathbf{K}_f \widehat{\mathbf{a}}_f &= \widehat{\widehat{\mathbf{j}}}_{s,f} , \\ \mathbf{K}_c \widehat{\mathbf{a}}_c &= \widehat{\widehat{\mathbf{j}}}_{s,c} . \end{aligned} \tag{9}$$

In the following, both equations will be coupled with the help of interface conditions. The first interface condition, states that the magnetic vector potential $\widehat{\mathbf{a}}$ on the interface Γ has to be equal for both subdomains. Therefore selection operators for the fine grid $\mathbf{Q}_f$ and for the coarse grid $\mathbf{Q}_c$ choose the values on the interface from the magnetic vector potential,

$$\begin{aligned} \widehat{\mathbf{a}}_{f,\Gamma} &= \mathbf{Q}_f \widehat{\mathbf{a}}_f , \\ \widehat{\mathbf{a}}_{c,\Gamma} &= \mathbf{Q}_c \widehat{\mathbf{a}}_c . \end{aligned} \tag{10}$$

To fulfil the interface condition, the interface values of one domain have to be interpolated because the grids are non-matching. Hence we introduce a prolongation operator $\mathbf{P} : \Gamma_c \to \Gamma_f$, which calculates the coarse magnetic vector potentials on the positions of the fine grid points, illustrated in Fig.2.

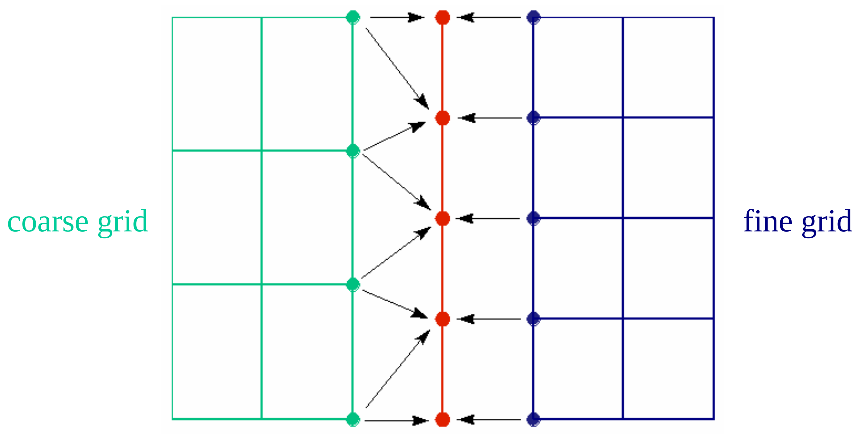


Fig.2. Interpolation and selection.

At present, a bilinear interpolation method is implemented. Therefore two linear interpolations take place on two opposite edges and another one on the intermediate values. Taking this together the first interface condition provides:

$$\mathbf{P}\mathbf{Q}_c \widehat{\mathbf{a}}_c - \mathbf{Q}_f \widehat{\mathbf{a}}_f = \mathbf{0}. \tag{11}$$

The second interface condition requires that the normal components of the currents have to be equal on the common interface. That means that the virtual current densities on the fine grid side of the interface and on the coarse grid side vanish, presented in Fig.3.

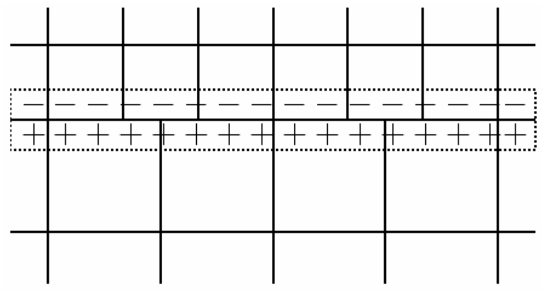


Fig.3. Virtual current densities on the interface

To chose the values on the interface we use again the selection operators $\mathbf{Q_f}$ and $\mathbf{Q_c}$. Here a restriction operator $\mathbf{P^T}:\Gamma_f \rightarrow \Gamma_c$ is necessary, which is chosen as the transpose of the prolongation operator. This has the advantage, that the resulting system of equations is symmetric. The second interface condition reads as:

$$\hat{\hat{\mathbf{j}}}_{\boldsymbol{\Gamma},\mathbf{c}} = -\mathbf{P^T}\hat{\hat{\mathbf{j}}}_{\boldsymbol{\Gamma},\mathbf{f}}. \tag{12}$$

The virtual current densities on the fine grid side of the interface serve as Lagrange multipliers λ.

Combining the magneto-quasistatic equations of both subdomains (9) and the interface conditions (11) and (12), the system of equations for the whole calculation domain reads as:

$$\begin{pmatrix} \mathbf{K_c} & \mathbf{0} & \mathbf{Q_c^T P^T} \\ \mathbf{0} & \mathbf{K_f} & -\mathbf{Q_f^T} \\ \mathbf{PQ_c} & -\mathbf{Q_f} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{a}}_\mathbf{c} \\ \hat{\mathbf{a}}_\mathbf{f} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \hat{\hat{\mathbf{j}}}_\mathbf{s,c} \\ \hat{\hat{\mathbf{j}}}_\mathbf{s,f} \\ \mathbf{0} \end{pmatrix}. \tag{13}$$

This system of equations represents a saddle-point problem and the system matrix is indefinite, complex and nearly singular. For further consideration we define the matrix $\mathbf{K_g} := \begin{pmatrix} \mathbf{K_c} & \mathbf{0} & \mathbf{Q_c^T P^T} \\ \mathbf{0} & \mathbf{K_f} & -\mathbf{Q_f^T} \\ \mathbf{PQ_c} & -\mathbf{Q_f} & \mathbf{0} \end{pmatrix}$, the vector $\mathbf{x} := \begin{pmatrix} \hat{\mathbf{a}}_\mathbf{c} \\ \hat{\mathbf{a}}_\mathbf{f} \\ \boldsymbol{\lambda} \end{pmatrix}$ and the right hand side $\mathbf{b} := \begin{pmatrix} \hat{\hat{\mathbf{j}}}_\mathbf{s,c} \\ \hat{\hat{\mathbf{j}}}_\mathbf{s,f} \\ \mathbf{0} \end{pmatrix}$.

3.2 Solving the metal detector problem

The primary field, which originates from the excitation of the transmitting coil of the metal detector, is much bigger than the secondary field, which arises from the induced eddy currents in the metal objects. Both fields superpose each other. Very high accuracies are necessary, to calculate the change of the voltage on the receiving coil of the detector if an object comes into the area of the search head. For this, we calculate at first only the primary field without any objects:

$$\mathbf{K_{g,1}x_1} = \mathbf{b}. \tag{14}$$

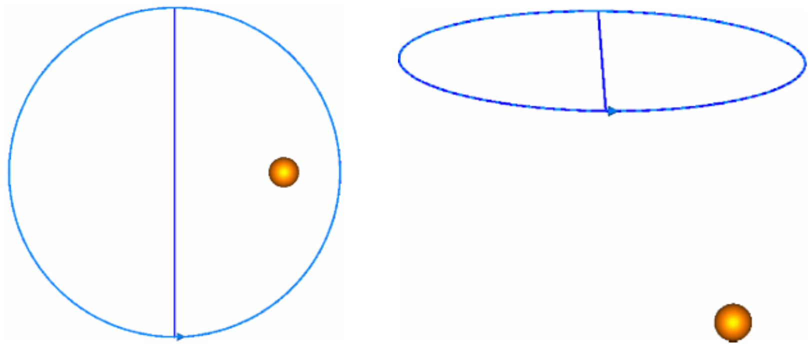
Using this result $\mathbf{x_1}$ for the primary field, in the second step the change in the fields between the detector without object and with object below is calculated. The following system of equations is obtained for the secondary field:

$$\mathbf{K_{g,2}(x_2 - x_1)} = \mathbf{b} - \mathbf{K_{g,2}x_1}. \tag{15}$$

With this approach it is possible to achieve higher accuracies than with the calculation of the superposed primary and secondary field only. Both systems of equations (14) and (15) can be solved with Krylov-subspace solvers, such as BiCG and QMR respectively.

4. Results

For validation we have chosen a model problem with a circular transmitting coil and a double-D shaped receiving coil on the same position, as shown in Fig.4. The exciting current has an amplitude of 1 A and a frequency of 2.4 kHz. Both coils have a diameter of 30 cm and the wires will be assumed as infinitely thin.



(a) from the top **(b) from the side**

Fig. 4. Model problem.

The coils are located 20 cm above a copper sphere, which has a relative permeability of 1 and a conductivity of $6 \cdot 10^7$ S/m. The horizontal position of the sphere is 10 cm away from the centre of the coils and the diameter is 2.8 cm. The imaginary part of the electric field strength $\underline{\mathbf{E}}$ of this arrangement is presented in Fig.5. The black box shows the size of the subdomain with the fine mesh.

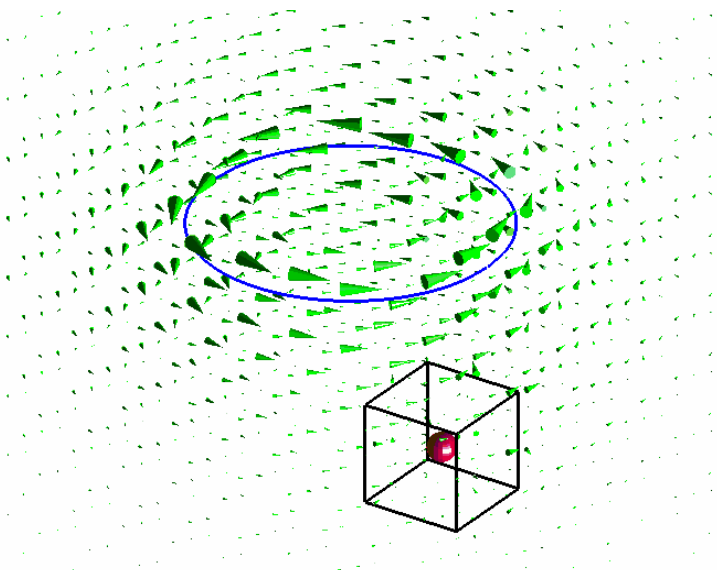


Fig. 5. Imaginary part of the electric field strength $\underline{\mathbf{E}}$ of a transmitting coil above a copper sphere.

The results with domain decomposition are compared with a reference solution and with results without domain decomposition. The reference solution was provided by Hanstein and Lange, which presented their analytic approach at [6]. Their solution is based on the assumption, that the metallic object is very small: the primary field of the metal detector can then be considered to be homogenous in the area around the object. The secondary field of the sphere is considered to be a dipole source. The calculations without domain decomposition were done with an equidistant grid in the area of the coils and the sphere. For all methods the imaginary part of the voltage of the receiving coil is calculated. In Fig.6 the absolute value of the difference between the reference solution and the results without and with domain decomposition method are shown as a function of the total number of grid points.

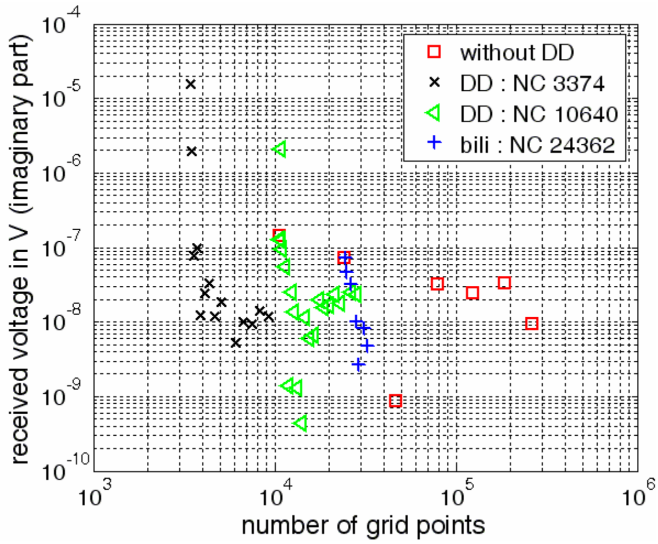


Fig. 6. Difference of the imaginary part of the voltage between reference and own calculations. (DD: Domain Decomposition, NC: Number of coarse grid points)

The absolute value for the difference between the reference solution and the results without domain decomposition, in Fig.6 presented by rectangles, slowly decreases with increasing number of grid points. The calculations with domain decomposition with Lagrange multipliers were performed with three different numbers of coarse grid points. All coarse grids in the area of the coils and the smaller subdomain are equidistant. For these three coarse grids the number of fine grid points was increased step by step. The fine grid is always equidistant. Even with a small number of coarse grid points, the absolute value of the difference is smaller than without domain decomposition. Increasing only the number of fine grid points is not sufficient, because the accuracy stagnates or decreases. An optimal value exists for the ratio between the step width of the coarse grid and the step width of the fine grid. This depends on the arrangement and has to be newly determined for every single case.

5. Summary

Due to the different sizes of the metal detector head and the metal objects in landmines, an effective discretization method is compulsory for simulations. The domain decomposition method with Lagrange multipliers, presented in this paper, enables the decomposition of the calculation domain in subdomains with different grids. These are non-overlapping and non-matching. The coupling of the domains takes place with the help of interface conditions.

Finally a saddle-point system, which is indefinite, complex and nearly singular, has to be solved.

The secondary field of the induced eddy currents in the metal object is much smaller than the primary field excited from the metal detector. Therefore it is useful to calculate the primary field first and only then the secondary field, in order to achieve a better accuracy. First results are shown for a magneto-quasistatic application of a coil pair above a copper sphere. The imaginary part of the voltage of the receiving coil with domain decomposition was compared with a reference solution and with results without domain decomposition.

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