A Ray Technique to Calculate the Multiple Reflections and Leaky Wave Propagation from a Single or Multilayer Plate for the Detection of Critical Disbonds in Layered Media

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Abstract: As an acoustic wave travels in a single or multilayer plate it reflects multiple times with the plate's outermost boundaries and any internal interfaces. Due to the coupling between the longitudinal and transverse (shear vertical) waves the number of waves to track, and in turn the number of reflection and transmission calculations, increases exponentially with each internal reflection (or the distance traveled through the plate). By utilizing a ray technique and considering some basic properties of physics, mathematics, acoustics, and computing it is possible to simplify the process of tracking the acoustic waves, and reduce the number of calculations which determine the multiple reflections in the plate, and the transmissions of leaky waves outside of the plate. This allows the leaky waves to be calculated in a more efficient manner, especially as the distance from the acoustic source increases (or the number of internal reflections increase). By extending the ray to a plane wave (or wave of finite size) and considering the properties of acoustic waves (such as frequency, pulse shapes, and superposition effects) more realistic leaky waves can be calculated. In addition the same approach used in the single plate can be can be expanded upon and used in multilayer plates enabling a more efficient examination of the changes due to bad bonding

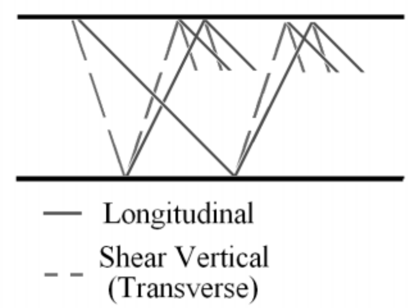
1. Description of Problem

Consider a single layer plate (a multiple layer plate may also be considered), surrounded by two half-spaces. An acoustic wave, which enters this plate (with some angle of incidence) from either of the two half-spaces, travels through the plate, and is repeatedly reflected back and forth from one interface to the other (See Figure 1 and Figure 2). Along with these multiple internal reflections, the acoustic wave may also be partially transmitted to the surrounding half-spaces producing 'leaky' acoustic waves. In the case of a solid isotropic plate and longitudinal (or shear vertical) polarized waves (see Section 2.1, for the specific theoretical considerations), it should also be considered that there exists a coupling between the longitudinal and shear vertical (transverse) polarizations. This coupling will cause each internal reflection within the plate to produce two reflected waves (one longitudinal and one shear vertical polarized wave) for each incident wave. These mode converted waves also reflect multiple times in the plate, and each reflection will produce two additional waves.

With only these considerations, it would appear that the number of acoustic waves to track increases by a factor of 2 with each internal reflection (this incorrect ray model can be seen in Figure 1). Due to this apparent 2^N (N is the number of internal reflection) increase in the number of waves to consider (and the 2^N calculations for the reflected and transmitted waves), many studies of waves in layered media, and multiple scattering problems, consider cases with liquid layers, shear horizontal waves [1], or waves with normal incidence [2,3]. (These papers do have their advantages in that they obtain analytical solutions, analyze specific experiments or experimental features, or are usable in situations not suitable for this theory such as anisotropic media, or cylindrical samples). With these changes the coupling of polarizations is no longer an issue, and only a single wave is now tracked and a single reflection transmission calculation is needed at the plate's boundaries. Those problems dealing with solid layers and allow coupling typically consider infinite plane waves [4,5] to help analyze the problem.

However, with careful consideration of the paths that the waves travel, the number of waves to consider can be reduced to a more manageable scale. This will allows for the multiple internal reflections, and the external transmissions (the leaky waves) to be calculated in a more time efficient manner. In addition by modeling the acoustic waves as rays this theory will obtain calculations independent upon the frequency, physical size, and time domain properties of the incident acoustic wave. The transmitted rays can then be expanded into waves of finite size, and by considering the properties of acoustic waves (such as frequency, pulse shapes, and superposition effects) the results of more realistic leaky waves can be determined via a separate set of calculations. This allows the theory to calculate results, which can be used to test a variety of acoustic sources (under the condition of constant sample structure, and constant angle of incidence of the acoustic source).

While the focus of this paper will be on the case of a single layer plate, the theory can be expanded upon for the multilayer plate situation as will briefly be discussed in Section 4.2



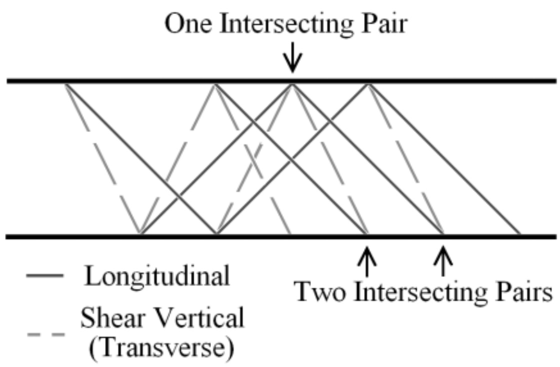


Figure 1 (Left): Incorrect Ray Model: The number of waves appears to double at each reflection
Figure 2 (Right): Correct Ray Model: Intersecting and coincident rays lower the number of waves to consider

2. Overview of Theory

The quick analysis of Section 1 yielded a model requiring calculations which increase on the order of 2^N. Such an exponential increase in the number of calculations causes the problem to quickly become a very time consuming process as the number of internal reflection increases. As well the problem would also require storage space for the calculations and results, which also increases on the scale of 2^N. What has not been considered is that many of the multiple internal reflections will travel equivalent paths (in total) and intersect in space and time at the plate's outermost boundaries in a predictable manner. After a more careful examination of the problem, the correct ray diagram for an isotropic plate is as shown in Figure 2. (This has also been shown correctly by other authors as well [6]). Examining Figure 2 it is noted that the waves which have under gone at least one mode conversion intersect in pairs of opposite polarizations at the same position on the plate's outer interface (the waves also have taken the same time of flight to reach this position). Although specific path may be different, these mode converted rays intersect as they have traveled equivalent paths in total, as the number of sections traversed as longitudinal and shear vertical polarized waves are the same. (For example, Figure 2 shows that on the second reflection the LV ray path intersects with the VL ray.) In addition, due to Snell's Law, it is noted that the angle of incidence of the intersecting pair is such that they produce reflected and transmitted waves, which are coincident for each polarizations. This coincidence allows for superposition to be used and reduces the number of waves to consider at each internal reflection from 2^N to N+1 (N is the number of internal reflections), and in addition the number of calculations to obtain the amplitude of the reflected waves (and transmitted waves) from each interface is in turn reduced. The following sections will examine each of these observations in more details in order to show its validity, beginning with the examination of equivalent paths.

2.1 Theoretical Considerations

This theory assumes that the plate and half-spaces are made of "ideal" acoustic materials (i.e. the materials are linear, non-dispersive, isotropic, homogeneous materials of known acoustic properties, and the layers of the plate are of constant known thickness and have no attenuation). The interfaces between the plate and the half-spaces (and multiple layers of the plate) are bonded and satisfy the perfectly bonded boundary conditions, or alternatively the imperfectly bonded boundary conditions. If any of the layers or half-spaces is defined to be a liquid the boundary conditions are altered appropriately and in addition the shear vertical (transverse) wave in this layer will be defined to have the necessary amplitude of zero.

As the boundary condition models have been developed from plane wave models, the rays that travel in the medium are assumed to have planar wave fronts. Thus the expansion of the rays to more realistic waves is limited to the accuracy in which one can represent the acoustic source as one or more planar rays. Finally the theory limits itself to currently considering only waves which travel at real angles, that is there are no evanescent waves in any layer, and no interface waves (Rayleigh, Stonely, or Stonely-Sholtze waves).

2.2 Equivalent Paths

With two polarizations, and the possibility of each internal reflection creating both reflected longitudinal and shear vertical waves, the total path the ray has traversed can be denoted by a series of individual longitudinal (L) or shear vertical (V) sections. This makes the listing of all possible paths for a particular reflection the familiar two state problem, where the indistinguishable factor is the 'order' in which the polarized rays have traversed a path. In order to show this is indeed true it is first noted that by utilizing Snell's law, and some principles of geometry, it can be shown that all longitudinal polarized waves in the same layer of the plate will posses the same incident and reflected angle (α), and as well, all shear vertical polarized waves posses the same incident and reflected angle (β). These angles are constant, regardless of the path taken or the direction through the plate (these angles can be determined by the angle of the incident wave (θ) via Snell's Law). From these angles the horizontal distance (x) traveled by each polarization as it travels through the plate to the opposite interface can determined (where the thickness of the layer d is known)

$$x_L = d\tan(\alpha), \qquad x_V = d\tan(\beta). \tag{1}$$

In addition, the time of flight (tof) taken can be determined by these angles (and the velocity of the polarization v_L and v_V.)

$$tof_L = d\cos(\alpha)/v_L, \qquad tof_V = d\cos(b)/v_V. \qquad (2)$$

Because the waves in the plate travel at constant angles, the horizontal distance and time of flight for any section of the path is constant. The total distance and time of flight of the path is sum of its sections. Thus, for these quantities, the order in which the path is traversed does not matter, only the total number of longitudinal and shear vertical sections. It should be noted that we are making no assertion that the amplitude of the wave (reflection coefficient to this point) is equal for the equivalent paths. In fact, some paths will have different overall coefficients due to the initial shear vertical and longitudinal transmission coefficients, and in addition the reflection coefficients in the upper and lower interfaces may differ as well. (This is discussed in more detail in Section 3.1.)

2.3 Visualizing the Paths & Coincident Waves

To visualize the unique wave paths, and the intersection of indistinguishable paths it is possible to draw the paths as a tree structure (Figure 3). Here the right branches of the tree denote longitudinal paths, and left branches shear vertical paths, with each new level corresponding to an additional internal reflection. The branches that connect represent waves, which have traveled equivalent paths (they have the same total number of longitudinal and shear vertical sections). These waves have traveled the same horizontal distance, and have the same time of flight.

From Figure 3 we see that the equivalent paths intersect in pairs of the opposite polarizations (like polarizations will not intersect at the interfaces as they travel parallel). The waves reflected (and transmitted) from these intersecting pairs posses the same origin in both space and time, and as well the reflected waves (of like polarizations) travel from this point at the same angle. Thus the multiple longitudinal waves reflected from the intersecting pairs are traveling along the same path in space and time, or in other words, they are coincident. (The same can be said for the shear vertical waves, as well as any transmitted waves of like polarizations). Superposition will allow these coincident waves to be combined in a single wave. Because these intersecting pairs of rays have the same time of flight the two rays will be in phase (a sign convention in the reflection and transmission coefficients is used to keep track of any 180 degree phase shifts, or amplitude sign change). Thus, this superposition need only concern itself with the wave’s amplitude, even for waves with complicated time domain behaviors.

```
      Incident
        /  \
       V    L        - waves incident on 1st Reflection
      / \  / \
    VV   VL   LL     -- waves incident on 2nd Reflection
   /  \ /  \ /  \
 VVV  VVL  VLL LLL --- waves incident on 3rd Reflection
 /  \/   \/   \/  \
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Figure 3: Tree diagram of wave paths

3. Reflection and Transmission Coefficients Calculations

For a particular reflection it is possible to enumerate the number of each paths permutations, and find possible each permutation, and from this the position and time of flight from the wave can also be calculated. However, to find the reflection and transmission coefficients for each of the multiple reflections there is a problem, as the multiple paths may not have the same incident amplitude (displacement, pressure, or energy amplitude).

3.1 Non Equivalence of Reflected Amplitudes

The non equivalence of the reflected amplitudes comes about due to multiple reasons. First there exists a difference initial transmission coefficients of the transmitted longitudinal and shear vertical waves. Secondly the bottom and top half spaces may possibly be made of different materials, and thus have different reflection coefficients. Both of these factors make the paths that are indistinguishable in terms of horizontal distance and time of flight, distinguishable in terms of their amplitudes.

As an example consider the waves incident on the second reflection, examining the tree diagram (Figure 3), shows LLV is an equivalent path where a ray could have taken the path LLV, LVL, or VLL to reach this point. Noting that the first waves are created from a transmission the longitudinal and shear vertical waves amplitude (displacement, or pressure amplitude) are given by the transmission coefficients T_{LL} and T_{LV} respectively (assume the incident wave is a unit in size, with longitudinal polarization). In this first section of the path, one of the waves already has a different amplitude. After two reflections the coefficient to this point will be

$$\begin{aligned} &\text{for path LLV: } T_{LL}(1,2)\ R_{LL}(2,3)\ R_{LV}(2,1),\\ &\text{for path LVL: } T_{LL}(1,2)\ R_{LV}(2,3)\ R_{VL}(2,1),\\ &\text{for path VLL: } T_{LV}(1,2)\ R_{VL}(2,3)\ R_{LV}(2,1), \end{aligned} \tag{3}$$

Here T denotes a transmission coefficient, R denotes a reflection coefficient, the subscripts denote the polarization of the incident wave and reflected wave (or transmitted wave) respectively, and the numbers in brackets denote the incident and transmitted media (1 is the upper half-space, 2 is the plate, and 3 is the lower half-space). Even after two reflections it is possible that none of these paths have the same amplitude in the case of the upper and lower half-spaces being made of different materials (i.e. $R(2,1) \neq R(2,3)$ for any polarization).

3.2 Calculation of Waves

The tree structure (Figure 3) has already been found to be a useful method to show the unique ray paths, and intersection of ray paths. The tree is also a useful tool to show the computations needed for calculating the reflections and transmissions. In addition the tree translates nicely into a from for useful in computer programming as any point in the tree can be defined and accessed via any two of the three properties of the tree (tree level, number of longitudinal paths, and number of shear vertical paths). To calculate the multiple reflections and transmissions, one needs to follow the tree, one level at a time, starting with the incident wave and applying the results from the appropriate boundary conditions to find the amplitude of the ray represented by each branch (i.e. multiply the incident wave's amplitude by the reflection and transmission coefficients for the interface). These results are then used as the waves incident on the next interface, and the boundary equations can be applied again. For those sections of the tree with two incident waves (two intersecting branches) superposition is used to find a single reflected wave of each polarization.

$$\begin{aligned}&\text{Reflected L Wave: } In_L\, R_{LL} + In_V\, R_{VL},\\ &\text{Reflected V Wave: } In_L\, R_{LV} + In_V\, R_{VV},\end{aligned} \qquad (4)$$

where In is the current incident wave's amplitude (the subscripts denote the wave's polarization), and R is the reflection coefficient (the subscripts denote the incident and reflected wave polarization respectively). Note this case of superposition is more of a rule than an exception, as only two paths in the tree have only one incident wave (the pure longitudinal and pure shear vertical). Also recall that the intersecting pairs of waves posses the same time of flight, so that the two incident waves are in phase, thus the superposition need only concern itself with the wave's amplitude (Recall any 180 degree phase shift, or amplitude sign change, is covered in the reflection and transmission coefficients via a sign convention). This process then repeats for the remaining levels of the tree. Note that each interface may have different reflection and transmission coefficients, thus the reason for stating that the results from the "appropriate boundary conditions" must be used.

While it is possible to use the use the tree structure to find the symbolic equations for each wave transmitted to the half-spaces, the equations grow more complicated with every internal reflection. While the current computerized method of the model that has been developed is numerical, it may be possible to adapt the programming to yield a symbolic result. For example the solution for the LLV path of waves incident on the third reflection is the sum of the three equations of 3. Even this solution is somewhat too complicated to display properly, and further results could become hopelessly complicated, especially for multilayered cases. In addition the solutions would only include the results for rays and would not consider the expansion of rays into more realistic waveforms.

By following the tree structure, and using the superposition principle results in tracking N+1 waves at the N^{th} internal reflection, and 2N-1 calculations to find the reflected waves (these are simply multiplications and addition calculations, as the boundary conditions are assumed to have been solved). During this process of calculating the multiple reflections the amplitude of the transmitted rays (and their time of flight and horizontal position on each of the plate's boundaries) are also calculated, thus the leaky rays are determined. These results for rays can now be extended to waves of finite size to provide more realistic results.

4. Other Considerations

4.1 Extending the Results from Rays to Waves

Recall that to this point the acoustic waves have been modeled as rays, we have calculated factors such as the time of flight, the horizontal position, and the amplitude of the ray at the interfaces for a particular angle of incidence and material composition. To extend the rays to more realistic acoustic waves factors such as the frequency, the finite physical size of the wave, and any time domain (and space domain) properties which one wishes to account. These factors can be accounted for via a separate set of calculations, thus allowing the theory to calculate results for a variety of acoustic sources (under the condition of constant samples, and constant angle of incidence of the acoustic source).

While the specifics of this process differ for the exact incident wave one wishes to consider, in general the extension of rays to waves is a matter of considering the superposition of any acoustic waves that will overlap in the time and space domain due to the acoustic wave's physical size, and time domain variations.

4.2 Multilayer Plate Considerations

The same ideas used to examine the single layer plate can be expanded upon and used in a multilayer plate system. This causes many of the observations for the single layer plate to still be true in a multilayer plate situation. The important change is the possibility of the wave traveling in multiple layers of different materials, where the various attributes of the rays (angles, horizontal distance traveled, time of flight) are separate constants for each layer of the plate. This difference in attributes by media causes the medium in which the rays travel to be an additional distinguishing factor for the various wave paths. As well there are more paths to be considered due to accounting for multiple transmissions and multiple reflections in the plate structure. However, many of the paths will again be indistinguishable (in terms of horizontal distance and time of flight), and superposition can be used to combine the coincident rays. This in turn reduces the number of rays to track, and number of reflections and transmission calculations to be made allowing the necessary calculations to be done in a time efficient manner. In the multilayred cases the exact unique paths become difficult to visualize as the tree structure becomes multi-dimensional. A computerized method, however, can easily combine the necessary rays as each path can be defined and accessed via the distinguishable properties of the ray's path (such as total number of longitudinal or shear vertical polarizations in each media).

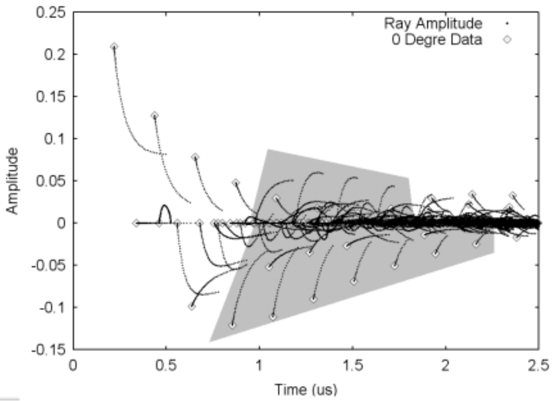
5. Theoretical and Experimental Results

5.1 Detection of Bonding and Disbonding Aluminum plates bonded with Epoxy

Consider a theoretical sample composed of two aluminum plates bonded with epoxy (properties in Table 1). We wish to use the theory to examine changes in the results between cases where the second interface (epoxy-aluminum) is fully bonded, and where it is fully disbonded (represented by an epoxy-vacuum interface). In order to examine the reflections from the epoxy-aluminum (or epoxy-vacuum) interface the lower aluminum plate will be considered a half-space, this way there are no reflections from the aluminum half-space interface in the results. Finding the results of the theory via computerized calculations gives Figure 4 for the bonded case, and Figure 5 for the unbonded case. Both figures show the amplitude of the acoustic waves (in an upper half-space of water) produced from incident rays with angles from 0 to 89 degrees (measured in aluminum, in 1 degree increments) whose rays fall in any horizontal position from 0 to 3mm on the upper aluminum-water interface. (Note, the data is represented only as raw data points, no extra superposition on the leaky waves has been done in the time or space domains. In addition the initial Water-Aluminum reflection is not plotted, as it would overshadow many of the data points). A comparison of the two figures finds that there exist two regions of data points, which have a sign change between two cases (denoted on the figures, and occur from approximately 1 μs onward). Examination of the reflection and transmission coefficients for each layer discovers that this sign change is due to the phase change of waves reflected from the epoxy-aluminum boundary relative to the epoxy-vacuum case, and is present at all real angles, and for all waves reflected from this interface. It is noted that one of the sign changing regions (the positive region in the unbonded case of figure 4) is a maximum at normal incidence, and the second region is approximately zero at normal incidence, and increases when there is a non normal incident angles. This second region offers an alternative option for testing of disbonding other than the typically used normal incidence method.

Table 1: Material Properties

Material	Longitudinal Velocity (m/s)	Shear Velocity (m/s)	Density (kg/m^3)	Thickness (mm)
Aluminum	6420	3040	2700	0.7
Epoxy	2400	1201	1154	0.5
Steel	5909	2954.5	7500	1.95
Delay	1111	555.5	2000	Half-Space



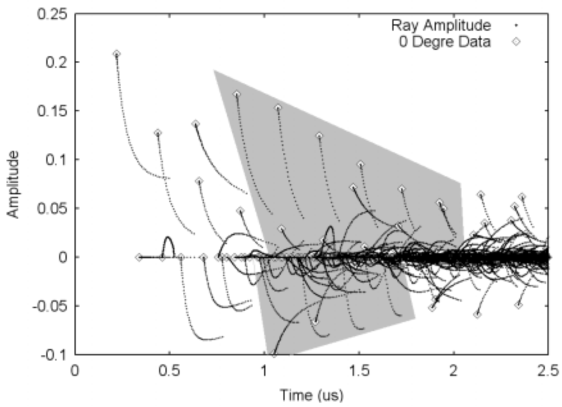


Figure 4 (Left): Bonded Situation: Transmission to water from Aluminum-Epoxy-Aluminum sample
Figure 5 (Right): Unbonded Situation: Transmission to water from Aluminum-Epoxy-Vacuum sample

5.2 Explanation of Additional Reflected Waves in Experimental Data

The theory has also proven useful to fully understand experimental results. Figure 6 shows experimental data of an A-Scan (data courtesy of Gilbert Chapman, Centre for Imaging Research and Advanced Materials Characterization, University of Windsor) of a steel plate (properties in Table 1), the large peaks in Figure 6 correspond to the multiple reflections from the steel's surfaces. In addition, however, between these are smaller peaks, which have an amplitude that slightly increases with time. These were experimentally determined not be effects of the transducer, or coupling material. Modeling the experimental setup as a half-space for the transducer delay-line, layer of steel, and air half-space (all fully bonded) and assuming there is a 1 degree angle of the acoustic wave in the steel, produces the results seen in Figure 6 (a 20MHz sine wave of 1.5 periods is used as the theoretical incident wave). The 1 degree off set from normal incidence allows for mode converted waves to be generated, and produces the additional peaks located between the main steel reflections. In particular these peaks are the mode converted longitudinal to shear vertical waves created from the reflection off the steel-air interface. The multiple reflections, and additional mode conversions that are possible are found to overlap in the time domain and superposition effects cause the peaks to gradually grow in size due to constructive interference. (The shear velocity of the steel has been set to exactly half the longitudinal velocity in order to maximize the number of overlapping multiple reflections). In addition an examination of the position domain data (not shown) shows that in the time frame examined the acoustic wave has traveled less than 0.5 mm horizontally, allowing the acoustic wave to be able to enter the delay line and be detected by the transducer.

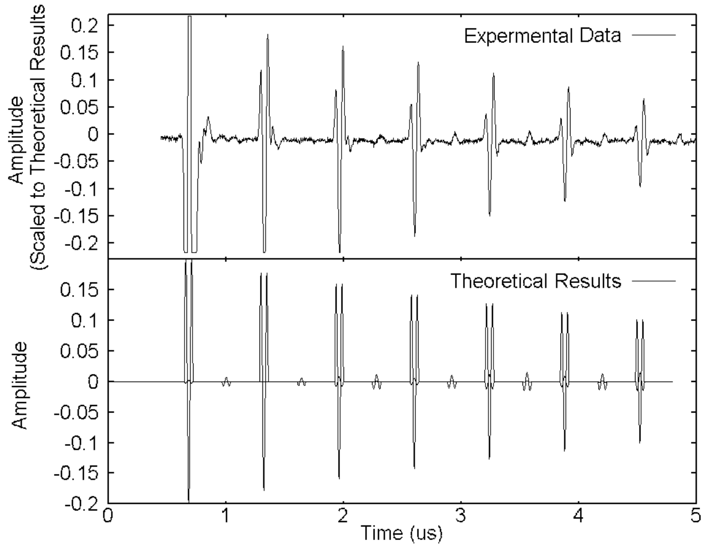


Figure 6: Experimental and Theoretical results for multiple reflected waves in a steel plate.

6. Conclusion

By considering the acoustic waves traveling in a plate structure as rays, and a careful examination of the paths these rays travel, allows for the reduction in the number of waves to consider at each interface, thus lowering the number of required reflection and transmission calculations, in turn creating a problem which can be solved in a time efficient manner. The calculation of transmitted waves as rays allows the properties of more realistic acoustic waves to be considered via a separate set of calculations, allowing the results to be useful for a variety of acoustic sources (in the case of a constant sample structure, and incident wave angle).

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