Mathematical Methods of Thermal Nondestructive Testing

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Abstract. An overview of methodises for the thermal nondestructive testing (TNDT) of buildings is presented. A new technique for TNDT of buildings is considered which provides qualitative investigation of building and calculation of reduced thermal resistance of the outdoor guarding constructions of the building. A procedure of investigation of the building is described. The input data processing procedure with use of the plausibility functional approach is presented.

1. Introduction

Energy saving is a modern and actual tendency in industry. It is caused by numerous reasons. They include rising prices for different energy carriers, ecological monitoring and growing standards of living. This tendency also involves building sector and requires the energy saving abilities of buildings to be enhanced. The problem is mostly important in northern countries where cold weather lasts for about half a year and more.

The building receives heat in different ways, for example, as the electrical energy, by a stream pipe. There are also several ways for the building to loose the heat. One of the primary energy loss paths is the heat conductivity through the building’s envelopes. The insulation of the envelope stands up against wasteful loss.

The quality control for the insulation must be carried out several times. For the first time the procedure should be done just after the construction is built. This should discover errors that could take place during the building. Then the quality monitoring should take place periodically to ensure the building provides its owners with the same energy preservation capabilities.

2. The thermal nondestructive testing overview

The general overview of the testing procedure is presented at Fig. 1. The procedure consists of several steps. It usually takes about a week to accomplish all the testing procedures. The procedure is nondestructive because no influence to the building structure takes place. But at this time the procedure cannot be discussed as noncontact since the reference measurements can only be carried out using contact sensors.

The laboratory that carries out the experiment must be equipped with an infrared camera, a number of contact temperature sensors and a number of air temperature sensors. It is preferred that temperature measuring devices have a data logging ability. Also it is useful to equip the laboratory with heat flux, air speed and moisture measuring devices as their information might be useful during the experimental data analysis. It is important that all the used devices support the measurement at the selected circumstances: the outdoors air temperature may be -20°C or less.

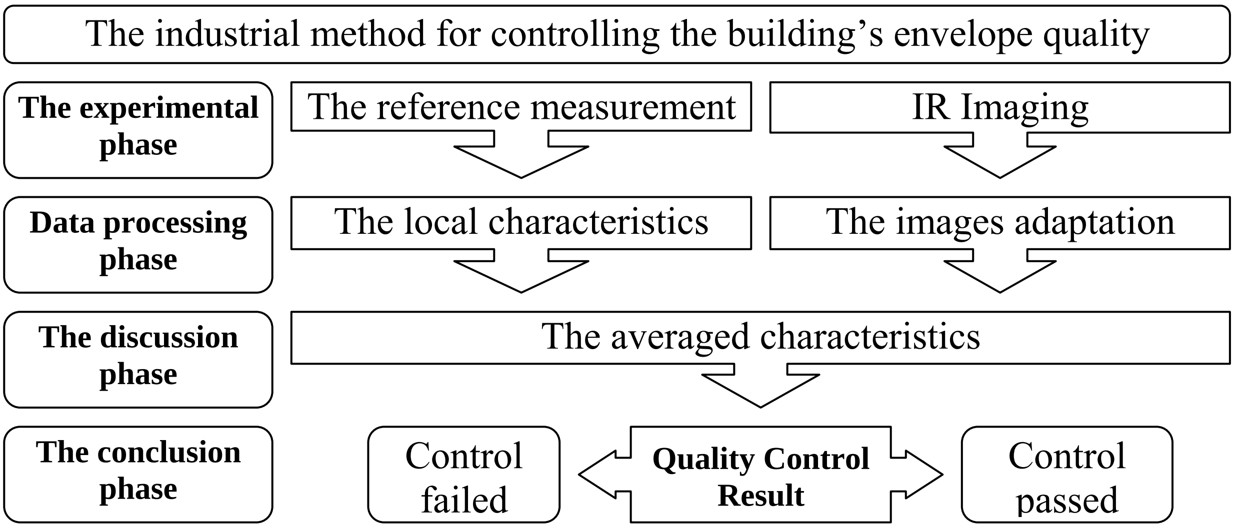

Fig. 1. The quality control overview.

The first step of the procedure is the collecting of the experimental data. The reference measurement uses the temperature sensors with logging ability. This is the longest procedure. The used mathematical approach requires a non-stationary heat transfer process to take place. This way the reference measurements must be carried out for at least 5 days to collect enough information. Another experimental task is the infrared imaging. This task should be accomplished within the shortest possible time to ensure the building's envelope temperature do not change noticeably. This procedure usually requires about half an hour or a little more. The preferable time for this procedure is night because of minimal external influence to the surface of the coating.

The second step of the control is the general data processing. It includes two separate tasks which conform ones of the first step. The reference measurements are used to restore local heat engineering characteristics of the envelope reference zone. The IR images of each envelope's plane are "sewn" together to create a complex infrared (IR) image of the total plane. This task is usually carried out using special software produced by the IR camera vendor. Since the images were taken using an IR camera at the previous step a special procedure is carried out to take into account the emissivity of the outdoors envelope surface.

At the next step the data collected during the reference measurement and the data collected during the IR imaging procedure is joined. The IR image of a part of the envelope is used to estimate the local heat engineering parameters of the zones other than the reference one. This procedure can be named a "calibration". It uses both results of the reference zone data processing and of the IR imaging. The characteristic that is usually determined is the averaged heat transfer resistance of the envelope.

The last step is the conclusion about the quality of the building's envelope and its insulation. There are two conclusions that take place usually. The envelope either correspond the energy saving requirements or not. But more precise scale is also often used. For example there are several grades of the building's energy preservation capabilities introduced in Russia.

3. Experimental tasks

The primary experimental task is to select a reference zone of the building envelope. The operator should consider the envelope's technical documentation and do a complete IR inspection of the envelope. The inspection should discover homogenous zones that could be considered as reference. The reference zone should satisfy several conditions. It is necessary that the corresponding part of the envelope can be discussed as plane-like. Its dimensions lengthwise to its surface must be about 10 times greater than the dimensions lengthwise to the normal of the surface. In this case the one-dimensional approximation used below is a good one. The reference zone should be examined with the IR camera to find possible defects. If ones exist, a choice of another zone should be considered. It is preferred that more than one reference zone is selected.

Ones a reference zone is chosen the temperature sensors must be installed. At least four sensors must be installed at the zone: two contact ones and two air ones. Their position is presented at

Fig. 1, right. The longer the measured temperature time series is the more accurate result can be achieved. The measured air temperature dependencies on time are marked below $T_{in}^{a}(t)$ and $T_{ex}^{a}(t)$. And the measured surface temperature dependencies on time are marked $T_{in}^{w}(t)$ and $T_{ex}^{w}(t)$. Subscripts "in" and "ex" mean the value is measured at the internal or at the external surface of the envelope. Usually 5 days is enough to complete the measurements. It is also useful to measure heat flux at the reference zone. The measured series are presented at

Fig. 2, left.

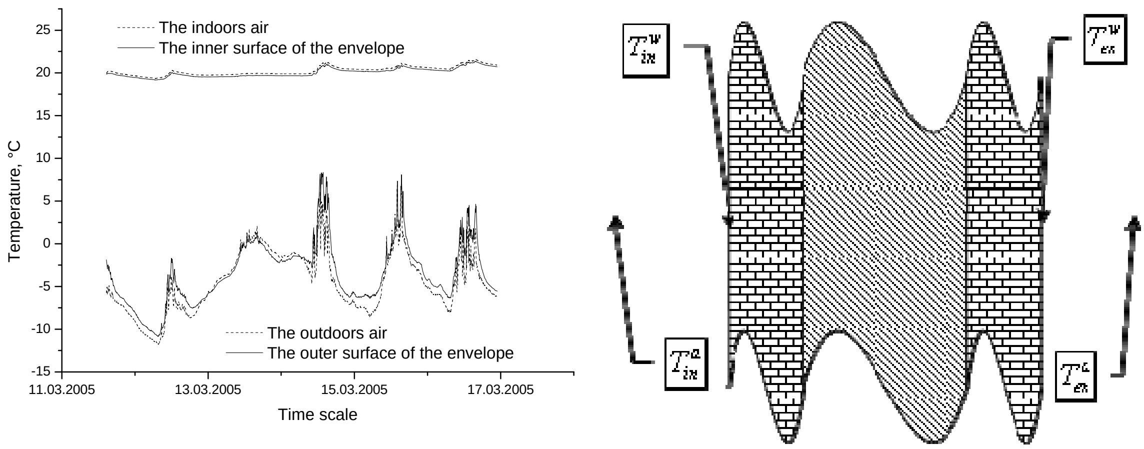

Fig. 2. Typical temperature time series of the reference zone (left). Sensor position at the reference zone (right).

A time interval between successive measurements must be few time shorter than characteristic time of outer air temperature variation. In our investigation we use 5 min. interval between two successive measurements.

Cause to different reasons electronic logger malfunction is an ordinary event. In this case the operator must either go on with the calculations with insufficient data or repeat the measurements. A special analysis shows that if three the mentioned above four sensors have produced successful measurements it is possible to get accurate result.

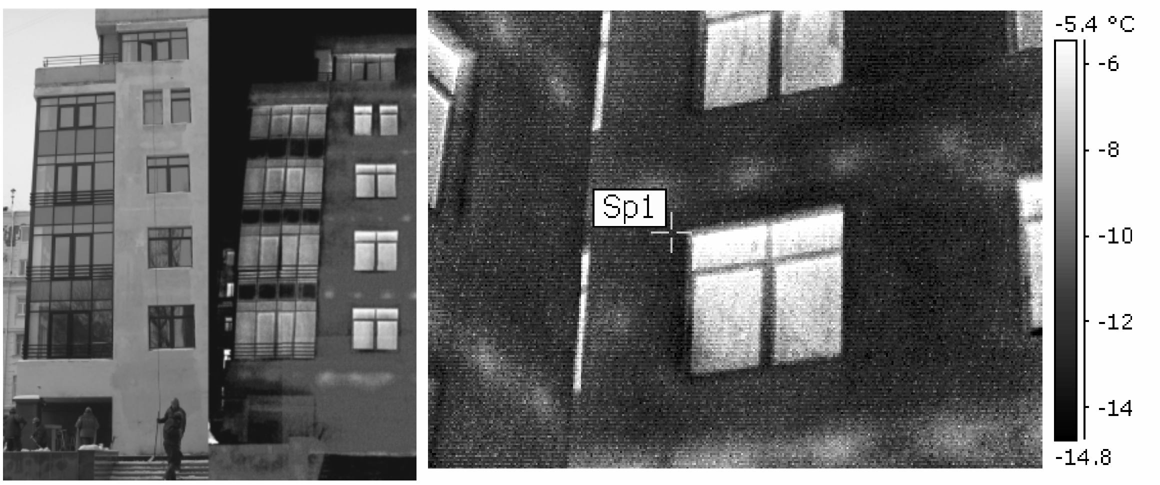

Fig. 3. A photo of a typical building envelope and its IR image (left). A more detail IR image of a zone with defects (right).

After the reference measurements are carried out an IR examination take place. During a short period of time IR images of each part of the envelope must be done. Later these images are combined to create an IR image of greater parts of the envelope. Such image is presented at Fig. 3, left. It is usually impossible to present the entire envelope at a single IR image because of geometric reasons. The IR camera operator should take this into account.

The IR image cannot be used along as a measurement result. Since the emissivity of different surfaces deferrers dramatically a special attention should be paid to it. When discussing the resulting IR image it is necessary to select the correct emissivity value. In some cases the image should be divided into several smaller images which correspond surfaces with constant emissivity.

It is an important question how to find out the correct value for the emissivity. If the material of the envelope is well-documented it is sometimes useful to use the corresponding parameter of the used material. The other way is to calibrate the IR image using additional reference contact measurement data. After all of the calibration procedures the finished the result can be discussed as the $T(\vec{r})$ dependency, the temperature distribution at the external surface of the building envelope.

Several additional values are measured at the time of the IR imaging. These values include outdoors and indoors air temperatures and the surface temperature at the reference zone. They are marked T_{in}^{a} and T_{ex}^{a} below.

4. The application of the general Inverse problem approach

The experimental data of the reference zone is used to solve the inverse heat transfer problem (heat transfer inverse problem (IP)). Let's discuss the general approach to solve the heat transfer IP. It is based on the search of the extreme of a quadratic discrepancy (1). The functional $\Phi[U]$ is sometimes called a plausibility functional. The quadratic discrepancy depends on functions $U(\tau)$ and $U_0(\tau)$; τ is time. The IP solution can be used to find out any number of the unknown system parameters.

$$\Phi[U]=\int_0^t \left(U_0(\tau)-U(\tau)\right)^2 d\tau \qquad (1)$$

In the equation (1) the $U_0(\tau)$ function is usually a measured function, which is also called the reaction function. The $U(\tau)$ is a calculated function of both known and unknown system parameters. This function corresponds $U_0(\tau)$ thus these functions become close one to another if $U(\tau)$ is calculated using actual system parameters. In general case the quadratic discrepancy is only one path to implement the plausibility functional $\Phi[U]$, many others can be suggested. They are not discussed here since the quadratic discrepancy satisfies the designated needs and is easy to calculate.

When solving the IP one first should prove that the solution of the IP exists and is unique. The existence and the uniqueness of the IP solution inside a bounded region is proved in [2]. The solution of the IP in the (1) form is based on the solution of the direct heat transfer problem [1]. A number of the corresponding direct problems need to be solved until one reaches the $\Phi[U]$ functional extreme. This way the $U(\tau)$ function becomes dependent on the unknown system parameters Θ. The (1) functional becomes:

$$\Phi(\Theta) = \int_0^t \left(U_0(\tau) - U(\tau,\Theta)\right)^2 d\tau \qquad (2)$$

Function (2) extreme can be found using numerous methods. The choice depends on the number of parameters should be found. If this number is greater than 1 the gradient downhill method used previously in [3] and [4] would be useful:

$$\frac{d\Theta_i}{d\tau} = -\frac{\partial\Phi(\Theta)}{\partial\Theta_i} \qquad (3)$$

In the (3) equation Θ is an aggregate of the unknown system; τ is time. Let's discuss the solution existence conditions. As to the oscillation theory classification [6] the (3) equation describes an autonomous dissipative dynamic system. The solution existence means the system (3) has a stationary attracting critical point. Therefore the right part of equation (3) can be factorized up to the first term near the critical point:

$$\frac{d\Theta_i}{dt} = -H_{ij}\left(\Theta_j - \Theta_j^0\right), \quad H_{ij} = \frac{\partial^2\Phi\left(\Theta^0\right)}{\partial\Theta_i\partial\Theta_j} \qquad (4)$$

In the equation (4) the summation is carried out by the iterant indexes; the upper index 0 marks the coordinates of the critical point. The Hessian H_{ij} is symmetrical. Therefore his proper values are real. The global minimum existence means the Hessian is nonsingular and positively defined. The system dissipation (4) is positive and equals to the sum of Hessian's proper values, taken with the inverse sign.

Each iteration all of the (3) derivatives should be determined to accomplish the extreme search procedure. When solving the problem numerically the procedure requires lots of time. A special attention must be paid to the choice of the parameters that should be verified to reduce their number. Since the (4) Hessian is nonsingular and positively defined the compression of the phase volume can be done using connection superposition.

Until now we did not discuss the explicit $U(\tau,\Theta)$ dependency on the parameters Θ and even did not consider whether it is known or not. Let's take into account that the explicit dependencies of $U(\tau,\Theta)$ on some of the Θ are known. In this case the system (4) can be replaced by some other system of the lower dimension. This can be done if an explicit dependency between different system parameters Θ_i can be built.

Let's divide the Θ parameters in two sets: φ and Ψ. Let's imply that each parameter Ψ explicitly depends on the φ set of parameters. The $\Phi(\Theta)$ function becomes $\Phi(\varphi,\Psi(\varphi))$. At the stationary point the following statement will take place:

$$\frac{d\varphi_i}{d\tau} = -\frac{\partial\Phi(\varphi,\Psi(\varphi))}{\partial\varphi_i} = 0 \qquad (5)$$

This method allows reducing the system (3) dimension by a number of variables, analytical dependency of which is known.

5. The non-stationary heat transfer equation

To solve the heat transfer IP one needs to solve the direct problem. The statement of the direct heat transfer problem is bases on the law of energy conservation. The law can be written in form of the continuity equation [5]:

$$\frac{\partial Q(\vec{r},t)}{\partial t}+\nabla\vec{J}(\vec{r},t)=0 \tag{6}$$

The equation (6) means that there are no any energy sources inside the discussed body. $Q(\vec{r},t)$ is the specific thermal energy; $\vec{J}(\vec{r},t)$ is the specific thermal flux. These values are determined by (7).

$$Q(\vec{r},t)=\rho(\vec{r})c(\vec{r})T(\vec{r},t),\quad \vec{J}(\vec{r},t)=\lambda(\vec{r})\frac{\partial T(\vec{r},t)}{\partial\vec{r}} \tag{7}$$

Here $T(\vec{r},t)$ is the temperature, $\rho(\vec{r})$ is the specific density, $c(\vec{r})$ is the specific thermal capacity, $\lambda(r)$ is the heat conductivity. The three last values are the material's thermalphysic characteristics which depend on coordinate. The linear approximation of the heat transfer equation means they do not depend on time or other parameters, e.g. temperature. Equation (6) is usually considered at approximations of 1 or 2 dimensions, the 3-dimensional solution is rather difficult. Which approximation is the best is usually determined by the system symmetry.

The full description of the system means that the initial and boundary conditions are set. In practice they must be determined experimentally. Within the used linear approximation they look like the Newton's Law [5]:

$$-\lambda_0\left.\frac{\partial T(\vec{r},t)}{\partial n}\right|_0=\alpha_0\left(T(\vec{r},t)-T_0(t)\right) \tag{8}$$

n is the normal to the surface of the body. The left part of the (8) equation is the heat flux, α_0 is named the thermal emission coefficient, $T(t)$ is the known temperature of the environment air, $T_0(t)$ is the object surface's temperature. In practice the initial conditions mostly have no meaning: on the expiry of the specific system time there contribution into the general solution (6) would be insignificantly small. This specific time for a single-component structure is determined using formula:

$$T_d=\left(\frac{L}{\pi}\right)^2\frac{\rho c}{\lambda} \tag{9}$$

This formula for a multi-component structure would be much more complex and is not presented since the (9) formula is rather demonstrative. Excluding the specific system time from the total variable time series makes special concideration about the initial conditions needless. Therefore the initial conditions may be calculated using the stationary solution of the corresponding problem. It is possible to solve the direct heat transfer problem using the Fourier transform method. A sample of the method application is described at [7].

6. The calculation of the local Heat engineering parameters

The determination of the local heat engineering parameters is based on a model of a multilayer object. Samples of such determination have been discussed earlier in [8]. Due to some reasons not all of the four temperature time series can be measured. A special analysis shows that sometimes it is possible to solve the direct problem using 3 temperature time series.

Let's mark $T_0(t)$ the measured temperature at the envelope surface, either $T_{ex}^{w}(t)$ or $T_{in}^{w}(t)$; $T_a(t)$ – the measured air temperature near this surface, either $T_{ex}^{a}(t)$ or $T_{in}^{a}(t)$; $J_n(\lambda,t)$ – the calculated heat flux. The calculated air temperature can be determined by formula:

$$T_n(\lambda,\alpha_0,t)=\frac{J_n(\lambda,t)}{\alpha_0}+T_0(t), \qquad (10)$$

Substituting this dependency into the initial equation (1) allows to build the quadratic discrepancy. The functional becomes the discrepancy between the measured air temperature and the air temperature calculated using equation (6):

$$\Phi\left[T_n(\lambda,\alpha_0,t)\right]=\int_0^t \left(T_n(\lambda,\alpha_0\tau)-T_a(\tau)\right)^2 d\tau\,. \qquad (11)$$

For multicomponent constructions the analytical temperature profile expression tends to be ponderous. In practice numerical heat conductivity equation solutions is preferred. The boundary condition (8) provides with the explicit functional dependency of the functional Φ on α_0. Taking this account the equation (11) is written the following way:

$$\Phi\left[T_n(t)\right]=\frac{1}{\alpha_0^2}\int_0^t J_0^2(\lambda,\tau)+\frac{2}{\alpha_0}\int_0^t J_n(\lambda,\tau)\left(T_0(\tau)-T_a(\tau)\right)d\tau+\int_0^t\left(T_0(\tau)-T_a(\tau)\right)^2 d\tau \qquad (12)$$

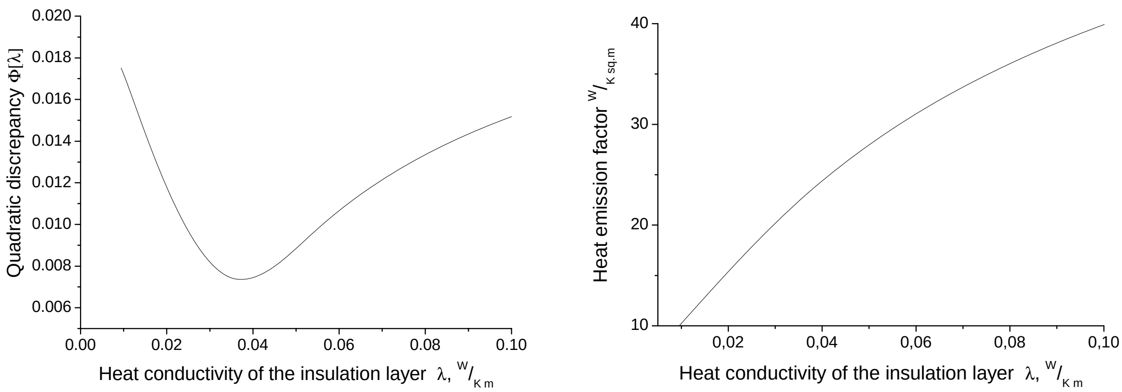

Fig. 4. Quadratic discrepancy (11) (left). The heat emission factor dependency on the heat conductivity (right).

Let's determine (12) extreme of variable α_0 (this extreme is conditional in fact) by means of equating the corresponding argument's partial derivative to zero in compliance with the (5) statement. This way the α_0 dependency on λ is determined:

$$\alpha_0=-\frac{\int_0^t J_n^2(\lambda,\tau)d\tau}{\int_0^t J_n(\lambda,\tau)\left(T_0(\tau)-T_a(\tau)\right)d\tau} \qquad (13)$$

The initial task been reduced to $\Phi\left(\lambda,\alpha_0(\lambda)\right)$ function's minimum search. The global minimum is achieved by the solving direct heat conductivity problem for sufficiently large set of λ values. Dividing the initial measurement while into several parts allows discussing each calculation result as an independent measurement. This is one of approaches for error estimation.

Using the given algorithm the 3 system parameters can be determined. When the heat emission coefficient dependency on λ is determined the quadratic discrepancy becomes a function of λ. A sample α_{ex} dependency on λ is presented at Fig. 4, right. The sample calculated quadratic discrepancy is presented at Fig. 4, left. This dependency has been calculated for an envelope of 3 layers. The minimum search procedure gains the actual λ value, 0.047 $^{W}/_{K\,m}$ in the sample case. Than the dependency at Fig. 4, right is used to determine the corresponding α_{ex} value.

Implementation of the analogous algorithm with the data of the opposite surface of the envelope gains α_{ex} value. The local value of the heat transfer resistance is calculated using formula:

$$R = \frac{1}{\alpha_{in}} + \frac{1}{\alpha_{ex}} + \sum_{n=1}^{N} \frac{l_n}{\lambda_n} \tag{14}$$

7. The calculation of the averaged heat engineering parameters

In order to calculate an average thermal resistance of the building envelope the temperature distribution of the envelope surface can be used. This distribution $T(\vec{r})$ has been obtained during the IR imaging. The thermal resistance $R(\vec{r})$ of an arbitrary fragment of the envelope can be estimated using the formula:

$$R(\vec{r}) = \frac{1}{\alpha_{ex}} \frac{T_{in}^{a} - T_{ex}^{a}}{T(\vec{r}) - T_{ex}^{a}} \tag{15}$$

After this estimation the averaged heat transfer resistance R_{av} is calculated using formula:

$$R_{av} = \frac{1}{S} \int R(\vec{r})\, ds,\ S = \int ds \tag{16}$$

The integration is carried out through the total surface S of the envelope.

8. The survey results

The method has been tested on numerous objects. By now it has been used to test quality of envelopes of more than 500 buildings. Typical objects are residential structures; many industrial buildings have been tested too. The method proved to be reliable. In spite of quite a long measurement procedure it also shows high productivity because the real man-hours expense to carry out all the measurements are about 10 times less then the time of the entire testing procedure. This means that one team can carry out several experiments in one time. The data processing can easily be automated by special software.

The main result of the control – the averaged heat transfer resistance – is calculated with acceptable error. A special analysis has been done to discover the primary source of the error. The greatest error input comes from the temperature measurement devices and the IR camera. The error input that comes from the given initial thermalphysic parameters is several times smaller. The total error for the calculated values usually does not exceed 15%.

The primary answer given by the quality testing procedure is whether the examined envelope correspond the designed value or not. Also if it does the detailed IR examination can discover existing or potential defects in the envelope structure. If it does not the IR examination can sometimes tell why.

Several reasons can lead to the inconsistency between designed and measured value of the heat transfer resistance. If the inconsistency is causes by mistakes during the building, it is useful to use the heat transfer IP problem solution to verify geometrical parameters of each layers of the envelope. And if the inconsistency is caused by material ageing, the heat transfer IP should primarily be used to verify physical parameters of the envelope's insulation.

Most of the surveys were carried out by the request of Moscow Government. The surveyed buildings include civil structures, many-storied buildings, palaces, industrial buildings and some others. The corresponding methods were certified by "State Standard", others were agreed with Federal Agency of Science and Education of Russian Federation and "State City Technical Supervision".

Conclusion

A state-of-art quality controlling method has been developed. The method equally matches for surveying buildings both just after the construction and after prolonged building exploitation. The method has proved its efficiency and allows calculating the averaged heat transfer resistance of the building's envelope with an acceptable accuracy. The local heat engineering parameters calculated during the survey are also useful.

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