Simulation of Ultrasonic Continuous Wave Fields in Homogeneous Media with Soft Curved Interfaces

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Abstract. This work presents an acoustic field simulation method derived from the Green's functions based monochromatic solutions of the wave equation in homogeneous media. Following this approach, for a given emitting aperture and for an arbitrary set of target points, it is possible to define a spatial monochromatic transfer matrix (MTM). The amplitude and phase of the acoustic field at the target points are computed by multiplying this matrix by the excitation vector formed by a spatial sampling of the acoustic field on the aperture. Interfaces are considered as secondary sources from which the incident wave is re-emitted. This way, it is possible modelling reflection, transmission and changes of vibration modes at interfaces.

Introduction

The growing complexity of ultrasound based inspections for non-destructive evaluation of structures makes advisable the use of field simulation tools for a right design of inspections and for a correct interpretation of results. This is especially true for phase array based inspections, where it is necessary to compute complex focal laws for emission and reception.

Different methods have been developed to study the characteristics of radiated fields both in time and space. Numerical methods, such as low-order finite difference and finite element methods suffer from the need of extremely dense spatial discretization and lead to overwhelmingly large computational tasks. Between the most used methods are those based on the concept of spatial impulse response [1], which makes possible to compute the time dependent pressure at a point of an isotropic and homogeneous media excited with a broadband signal. When changes of medium are involved, as it is usual in NDE, this method can still be applied [2] but the successive convolution operations at interfaces greatly increase the necessary computing time. For NDE, ray-tracing based methods [3] permit predict sound trajectories in complex media, and compute focal laws, nevertheless, in complex media may become very time consuming and difficult to apply [4]. A simulation method based on monochromatic continuous wave solutions of the wave equation has been used to model propagation problems through interfaces in NDE, and to interactively evaluate changes in the field due to different emission patterns. The basis of the method is described below.

Basic Theory

1.1 Harmonic Solutions of the Wave Equation

Let us assume a complex time-harmonic plane source, surrounded by an infinite baffle, radiating into a homogeneous medium. The solution of the wave equation is given by the integral equation [5]:

$$U(P_0) = \frac{1}{4\pi}\int_S \left(\frac{\partial U}{\partial n} G - U\frac{\partial G}{\partial n}\right) ds \qquad (1)$$

which provides, at the point P_0 in the medium, the complex amplitude U of the acoustic field (usually acoustic pressure, velocity potential, density…). G is the Green's function of the problem and S is the surface containing the radiating aperture Σ.

Equation (1) is obtained using the Sommerfeld radiation condition at infinity. If the Dirichlet boundary condition: $U=0$, is applied to the baffle (1) yields:

$$U(P_0) = \frac{-jk}{2\pi}\int_\Sigma \frac{\exp(-jkr)}{r}\cdot\cos(\alpha)\, U(r) ds \qquad (2)$$

where $\vec{r}$ is the vector from P_0 to the points on the radiating surface Σ, α is the angle defined by $\vec{r}$ and the normal to the surface pointing to the outside of the medium, and $U(r)$ is the excitation function at the aperture. When U is the acoustical pressure (2) is known as the Sommerfeld equation and the boundary condition as the soft-baffle condition.

In the same way, for the Neumman boundary condition: $\partial U/\partial n = 0$, $U(P_0)$ is given by:

$$U(P_0) = \frac{1}{2\pi}\int_\Sigma \frac{\exp(-jkr)}{r}\cdot\frac{\partial U(r)}{\partial n} ds \qquad (3)$$

from which it is easily obtained the Rayleigh equation for the rigid-baffle condition when U represents the velocity potential and considering its relation with the acoustical pressure and with the velocity of particles. A common form for the equations (2) and (3) is:

$$U(P_0) = \int_\Sigma h(P_0, r)\cdot E(r) ds \qquad (4)$$

which provides the acoustic field at point P_0, given the excitation function $E(r)$ at the radiating aperture. This integral can be solved only for a few simple forms of Σ and E(r), but can be used as the basis for numerical simulation of acoustic fields.

2. Simulation Method

2.1 The Monochromatic Transfer Matrix (MTM)

To numerically solve equation (4), the radiating aperture can be divided in a set of n cells s_j, small enough to considerer that both, $h(P_0, r)=h_j(P_0)$ and the excitation function $E(r)=E_j$, are constant for every element. Figure 1 illustrates the geometrical relations between the involved elements. Substituting the integral of equation (4) by a sum over all surface cells results:

$$U(P_0) = \sum_j h_j(P_0)\cdot E_j \cdot s_j = \sum_j m_j(P_0)\cdot E_j \qquad (5)$$

Now, for a set $\{P_i\}$ of m target points it can be defined a *field vector* **U** in which each term U_i represents the complex amplitude of the acoustic field at the target point i, and a *excitation vector* **E** with the components E_j being the complex amplitude of the excitation

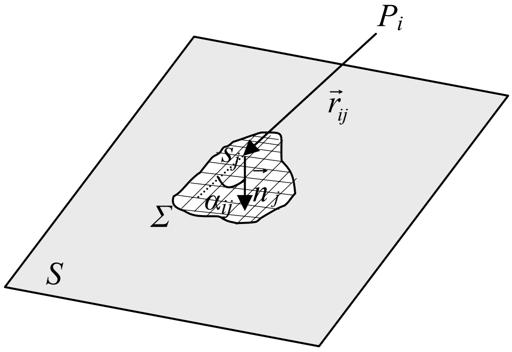

Figure 1. Radiating aperture and target point

applied to the j-element of the radiating aperture Σ. By applying (5) to the whole set of targets, and using matrix notation, the relation between **U** and **E** is given by:

$$\mathbf{U} = \mathbf{T} * \mathbf{E} \qquad (6)$$

where * means matrix multiplcation. We call *monochromatic transfer matrix* (MTM) to the operator **T** which computes the field at the targets as a superposition of the fields propagated from each source. **T** takes the form of a mxn matrix whose elements are:

$$m_{ij} = h_j(P_i) \cdot s_j = \begin{cases} \dfrac{s_j}{2\pi} \dfrac{\exp(-jkr_{ij})}{r_{ij}} & \text{Neumman boundary condition} \\ \dfrac{-jks_j}{2\pi} \dfrac{\exp(-jkr_{ij})}{r_{ij}} \cos(\alpha_{ij}) & \text{Dirichlet boundary condition} \end{cases} \qquad (7)$$

which are only function of the source-target distances and of the wavelength in the medium, and it is not dependent on the acoustical magnitudes (pressure, density, velocity potential…) used to represent the field and the excitation.

Equations (6) and (7) have been used as the basis of an efficient field simulation computer program, which is fast enough to interactively modifying the excitation vector and observing the effects at the targets. The program operates in two stages:

a) Modelling stage: in which, for a given arrangement of sources, interfaces and targets, the MTM is computed using (7).
b) Field computing stage: by evaluating (6) for each specified excitation vector.

Modelling is, usually, more time consuming than field computing stage. Moreover, large apertures in relation to the wavelength, the huge number of source cells within the aperture can result in an overflow of the available computer memory. However, the matrix form of (6) makes possible dividing the aperture, and/or the target points, in separate subsets that can be independently evaluated (sequentially or in parallel on a computing grid). A most effective approach, which can be combined with the former, consists on reducing the whole size of the MTM.

2.2 MTM of Piston Sources with Apodization

Ultrasonic transducers can be considered as radiating elements excited by a single driving signal. A usual approach is to considerer that the transducer behaves like a piston, or in a most realistic situation, as a piston modulated by a -possibly complex- apodization function which reproduces differences, in amplitude and phase, of the excitation at different positions on the transducer surface. This way, for a transducer modelled by n radiating surface elements with apodization coefficients A_j at each element, the excitation vector is written:

$$\mathbf{E} = (A_j \cdot E) = (A_j) \cdot E = \mathbf{A} \cdot E \qquad (8)$$

where the complex scalar E is the common -piston- part of the excitation, and **A** is the *apodization vector of the aperture*. Substituting (8) in (6):

$$\mathbf{U} = \mathbf{T} * \mathbf{E} = (\mathbf{T} * \mathbf{A}) \cdot E = \mathbf{V} \cdot E \qquad (9)$$

in which the mxn matix **T** has been reduced to the m elements column vector **V**, and the excitation vector **E** is replaced by the scalar E.

For transducer arrays with N elements, each single transducer obeys (9) and the field at targets is the sum of contributions of each single element. This superposition can, again, be written as a matrix equation formally identical to (6):

$$\mathbf{U} = \sum_{j=1,N} \mathbf{U}_j = \sum_{j=1,N} \mathbf{V}_j \cdot E_j = \mathbf{T} * \mathbf{E}$$

where, now, each column of the resulting MTM **T,** is the column vector defined in (9) for each array element, and the excitation vector **E** is the applied *emission focal law*.

2.3 Propagation Through Interfaces

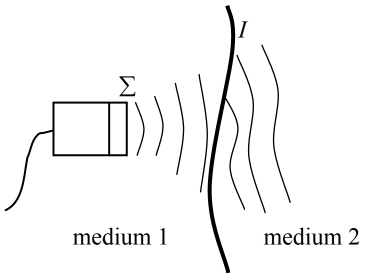

Figure 2. Field propagated through an interface I

In ultrasonic non-destructive evaluation of components, it is common to use solid or liquid wedges, with diverse geometry, for coupling the transducer to the part under inspection. It is also frequent to inspect structures with discontinuities. The distortion in the acoustic beam produced by such interfaces must be evaluated to correctly perform the inspection, and to interpret its results. Integrals solutions, like (4), of the wave equation have been used to model propagation through soft curved interfaces [6]. This way, for an acoustic source Σ in medium 1 which is separated of an acoustically different medium 2 by the interface I, as it is shown in Figure 2, the interface is considered to be a source radiating into medium 2 with an excitation field $\mathbf{E_I}$ produced by the field radiated from the source Σ. Let be **T1I** the MTM which computes the field at interface propagated through medium 1, and **TI2** the MTM which yields the field at targets in medium 2 from the field at interface. Using (6) the field vector **U** at targets is given by:

$$\mathbf{U} = \mathbf{TI2} * \mathbf{E_I} = \mathbf{TI2} * (\mathbf{T1I} * \mathbf{E}) = (\mathbf{TI2} * \mathbf{T1I}) * \mathbf{E} = \mathbf{T12} * \mathbf{E} \qquad (10)$$

stating that the MTM, which directly computes the field **U** at targets in medium 2 from the excitation **E** of sources in medium 1, is the product of MTM from interface to targets by the MTM from sources to interface. Equation (10) naturally gives account of refraction, reflection and mode changes at interfaces if the right value of the wavenumber k is used in **T1I** and **TI2**.

In order to accomplish with energy conservation and field continuity at both sides of interfaces, matrices of transmission **Tr** and reflection **Rf** can be defined for each vibration mode. The element t_{ij} of **Tr** is the transmission coefficient at point i of the interface for the field propagated from the source j, and the same way for reflection matix. For example, to compute the refracted shear wave at targets for an incident longitudinal wave, equation (10) is modified as:

$$\mathbf{U} = \mathbf{TI2} * \left(\left(\mathbf{Tr}^{\mathbf{ls}} \circ \mathbf{T1I} \right) * \mathbf{E} \right) = \left(\mathbf{TI2} * \left(\mathbf{Tr}^{\mathbf{ls}} \circ \mathbf{T1I} \right) \right) * \mathbf{E}$$

where $\mathbf{Tr}^{\mathbf{ls}} \circ \mathbf{T1I}$ denotes element by element matrix product.

2.4 Phase Conjugation

A straightforward way of calculating emission focal laws, in the case of transducer arrays, or apodization functions of transducers to obtain a given emission pattern is to use phase conjugation [7]. In this approach, the desired focal points are considered punctual sources, from which the field at the transducer surface is computed. By emitting with the phase conjugated of that field (in fact the amplitude is not relevant) a beam which reverses the path of the former is obtained. Another possibility is using the pseudo-inverse matrix method [8] to solve the matrix equation (6).

3. Simulation Examples

As an example of the capabilities of the method, Figure 3 shows the results of simulating the field produced by a 12λ circular transducer on the Z=0 plane with its centre at the origin of coordinates radiating into an homogeneous medium (λ is the wavelength in the medium). To carry out the simulation the transducer has been divided in 491 surface elements, which are managed as independent radiators. The field has been computed in a grid of 201x291 points on the plane Y=0 at the intervals: [-10λ, 10λ] in the X axis and [1λ, 30λ] in the Z axis, with a pitch of 0.1 λ in both directions. The MTM is a 58491x491 elements complex matrix and has been computed in 6.4 seconds on an AMD Opteron 248 based workstation running MATLAB 7.1 with a set of specialized mex-functions.

Figure 3 (a) shows the phase-apodization function applied to have a focus at point (3 λ, 0, 15 λ). To calculate this function a spherical wave is propagated from the focus to the transducer grid, phase conjugated and re-emitted to the medium. Figure 3 (b) is a snapshot of the vibration state, the amplitude has been normalized. Figure 3 (c) is the radiation pattern in logarithmic scale, and Figure 3 (d) shows the constant-phase curves (frontwaves).

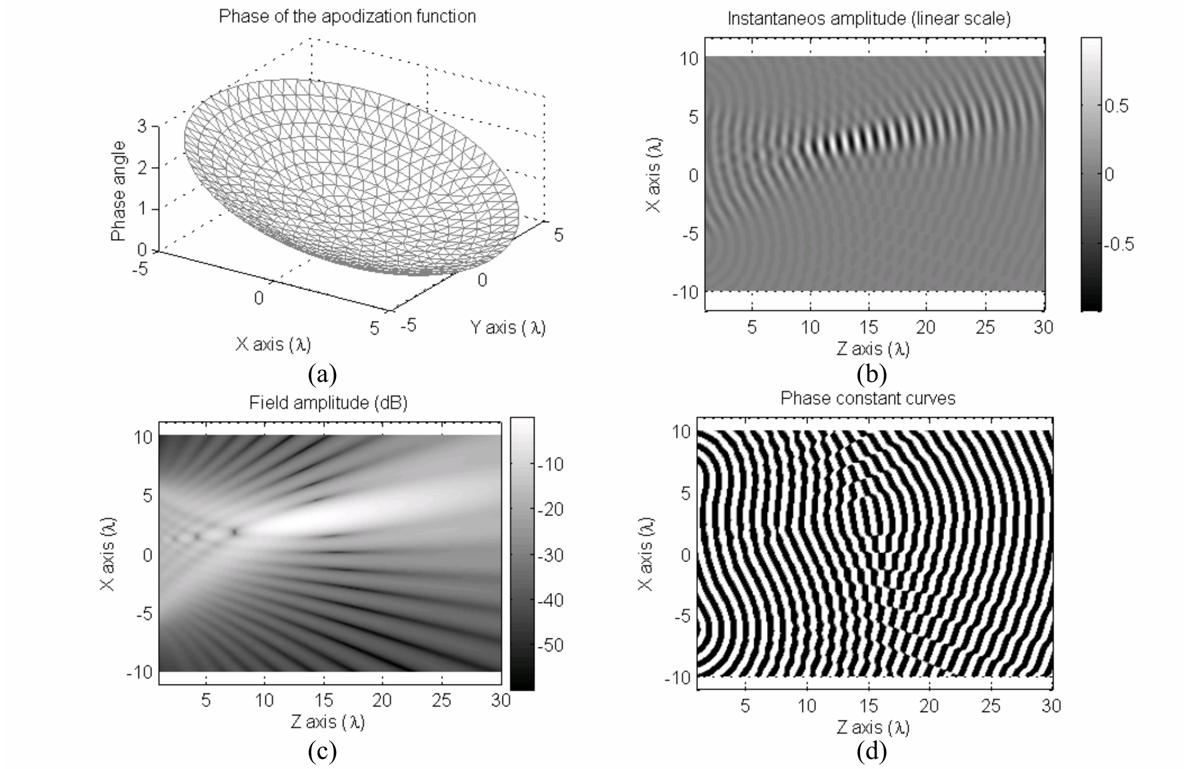

Figure 3. Simulated field of a circular transducer of 12λ diameter with a phase apodization function to produce a focus at (12 λ, 3 λ). (a) Applied phase apodization function. (b) Instantaneous value of the amplitude. (c) field radiation pattern. (d) phase-constant curves

The whole time used to complete this calculus and perform the graphic representation has been 0.96s.

Acknowledgements

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