Taking into Account Uncertainties of Non-Destructive Tests in the Estimation of Stochastic Degradation Processes

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Abstract. Controlling the ageing of passive components and facilities implies a good knowledge of the various degradation phenomena and of their evolution. To achieve that goal, a mathematical model that describes degradation kinetics (such as a crack growth model) and enables the assessment of the component's remaining lifetime and its uncertainties is obviously useful. Such a tool is for instance needed to optimize the maintenance policy while preserving safety.
Stochastic processes may be used to describe the random evolution of a degradation indicator around an average trend. Their construction, that is to say the estimation of the trend and of the intrinsic variability around it, relies on the statistical treatment of degradation measures. Expert's knowledge remains of course a part of the estimation process, for instance to choose a relevant trend type (e.g. a linear trend that depends on several physical variables).
In most real applications, the available data is provided by non-destructive tests (NDT). Of course, this source of information is not perfect, since it is affected by measurement uncertainty. Previous studies of such data-sets suggest that this uncertainty can lead to a biased assessment of the degradation kinetics, at least if it is not properly taken into account in the construction of the stochastic degradation process.
In this paper, we propose some statistical methods that explicitly make use of the available knowledge on NDT uncertainties to overcome this difficulty. The approach relies on variations of the Expectation-Maximization algorithm, which has been historically developed to process data-sets that include hidden information (in our situation, the hidden information is the difference between the measurement and the true value of the degradation indicator). The proposed statistical tools can be applied to several types of stochastic processes. The practical interest of this work is finally illustrated through a simulated application.

1. Context: Stochastic Degradation Models

Improving the knowledge of degradation phenomenon is a key industrial issue for the control of ageing of a facility. In this paper, the focus is placed on degradation mechanisms that affect passive components (e.g. tubes, pipes), and that can be measured through a scalar indicator (such as crack length, corrosion loss of material, etc.).
Let X_t denote the value of the degradation indicator at time t; in the following we assume that the higher X_t, the more critical the degradation. In a decision process, it may be useful at time t_0 to forecast a future value X_t of this indicator ($t>t_0$). Suppose for instance that a company has to plan a preventive maintenance act on a critical component or structure, affected by an identified degradation phenomenon. The underlying goal here is to optimize maintenance costs (i.e. to avoid over-frequent maintenance) while preserving safety (i.e. planning the intervention soon enough to avoid failure). Roughly speaking, the "optimal" moment T is the time at which the degradation indicator comes too close to a safety/failure threshold s_T. In mathematical terms, this means that:

$$T = \inf\left\{t > t_0 \middle| X_t \geq s_T - \varepsilon\right\}$$

where ε denotes a margin more or less close to 0 depending on the application considered. Nevertheless, in some applications, the knowledge of the underlying physics may not be sufficient to build a reliable physical model i.e. a set of mathematical equations that describe precisely the evolution of X_t in a given environment (e.g. fluid pH, temperature, material type or any influent variable). Therefore, the value of T remains quite fuzzy and is not of much help for the decision-maker.

Stochastic processes provide an interesting way of dealing with this uncertainty. As we shall see, these models rely on very simple considerations on physics (e.g. linear or non-linear trend), and complete this information by a statistical treatment of a degradation measures dataset, generally obtained via *non-destructive tests (NDT)*. In such models, the degradation indicator becomes a *random variable* (see figure 1). For a given environment, the aim is not to forecast the exact value of X_t, but rather the range of possible values weighted by a *probability distribution function (pdf)* denoted by f_{Xt} : the more important $f_{Xt}(x)$, the more likely $X_t = x$.

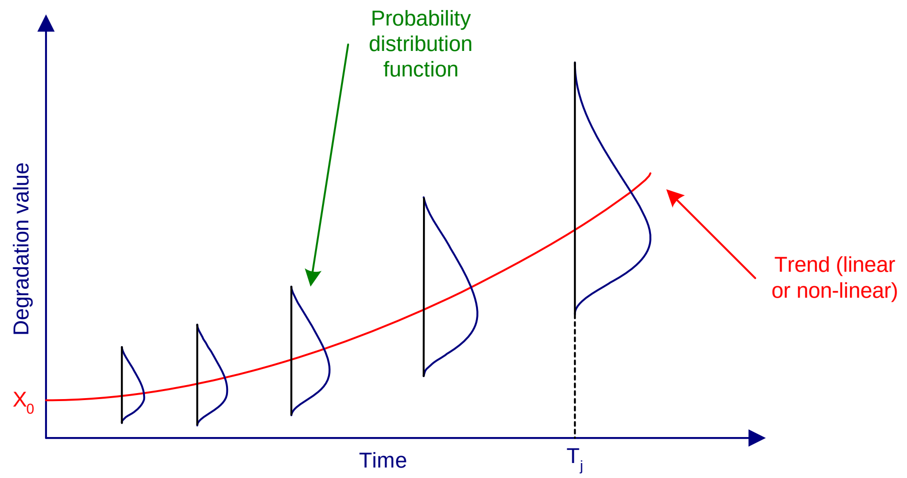

Figure 1 : principle of a stochastic process

This pdf can be used to quantify various quantities of interest for the decision-maker, by using the basic definition:

$$\Pr(a \leq X_t \leq b) = \int_a^b f_{Xt}(x)dx$$

For instance, one can compute the probability $\Pr(X_t \geq s_T - \varepsilon)$ that the degradation is too important at time t (the "failure probability"), the mean value $E(T)$ of the optimal maintenance time, or the probability $\Pr(T>t)$ that it is not necessary to plan maintenance before t :

$$\Pr(X_t \geq s_T - \varepsilon) = \int_{s_T - \varepsilon}^{+\infty} f_{Xt}(x)dx \,,\; E(T) = \int_{t=0}^{+\infty} \int_{x = s_T - \varepsilon}^{+\infty} f_{Xt}(x)dxdt \,,\; \Pr(T > t) = \Pr(X_t \geq s_T - \varepsilon)$$

(note that the last two equations assume that degradation is a non-decreasing function of time, which is often quite reasonable). Another interesting feature of stochastic processes is that they enable the actualization of the forecast. More precisely, if a NDT indicates at a time u that $X_u = z$, then this information can be taken into account to adjust in consequence the pdf of X_t for any $t>u$.

From an industrial point of view, a major difficulty encountered while using such an approach is to determine, for a given degradation mechanism, which stochastic process should be used among the numerous ones proposed in literature (step 1 of the process

described in figure 2).[1] The first part of the article addresses this problem; it does not provide a final answer to this complex question, but rather some simple elements that may either justify or invalidate the use of a very common stochastic process: the *gamma process*.
The second part of the article focuses on the second step described in figure 2, in the particular context of a *gamma process*. As for most stochastic processes, the pdf of X_t belongs to a family of probability distributions (here the gamma distribution functions), and is parameterized by a few set of parameters θ (here two parameters, $\theta = (\alpha, \beta)$). The statistical problem consists in choosing the values of θ that are the most relevant to describe a degradation dataset obtained by NDT. Of course, this source of information is not perfect, since it is affected by measurement uncertainty (inherent with the measurement process and the surface considered). We shall show that this uncertainty can lead to a biased assessment of the degradation kinetics and therefore to a biased maintenance decision, at least if it is not properly taken into account in the calibration of the gamma process. Finally, a statistical method that explicitly makes use of the available knowledge on NDT uncertainties is proposed to overcome this difficulty.

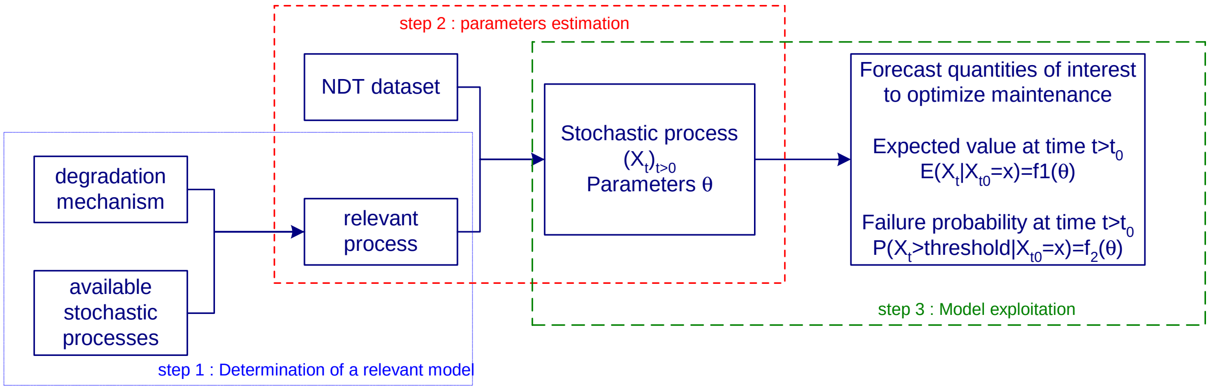

Figure 2: the different steps of a study

2. Definition and Properties of a Gamma Process

Among the list of stochastic models proposed in literature to describe degradation mechanisms, the gamma process seems to be an interesting compromise between modeling capacity and mathematical tractability. It has already been used for instance to model dyke erosion [1,2] or NDT data [6]. Its mathematical properties enable the optimization of complex maintenance policies for reparable systems [3-5].

Let us first recall the definition of this model. A stochastic process $\{X_t \geq 0\}$ is called gamma process if:

- $X_0 = 0$: the degradation is equal to 0 at the origin of time;
- For all $u<v\leq s<t$, the random variables $X_t - X_s$ and $X_v - X_u$ are independent: the increment of degradation on the time interval $[u,v]$ does not provide any information on the increment on a future period $[s,t]$;
- For all $0<s<t$, the probability distribution of the random variable $X_t - X_s$ is a gamma distribution with parameters $\theta=(\alpha(t-s),\beta)$; this means that the degradation increments on $[s,t]$ is non-negative, and that its pdf is given by:

$$f_{\alpha(t-s),\beta}(x) = \frac{\beta^{\alpha(t-s)} x^{\alpha(t-s)-1} e^{-\beta x}}{\Gamma(\alpha(t-s))},$$

[1] Academic literature on stochastic processes applied to reliability mainly focuses on a further step, which is the optimization of a complex maintenance policy once the relevant stochastic process has been chosen.

where Γ denotes Euler's gamma function: $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$.

What are exactly the physical implications of this mathematical framework? At least two features could be considered as restrictive. The first one results from the use of a gamma distribution function, which implies that:

$$E(X_t - X_s) = \frac{\alpha(t-s)}{\beta}$$

(where $E(\ .\ )$ denotes the mean value of the random variable). Thus, a gamma process always assumes a *linear trend*. For instance, if it is applied to model crack growth, the evolution of each flaw can wander from linearity (due to the stochastic nature of the model). But if a large number a flaws is considered, the mean value of the crack size among this population grows at a constant rate. This characteristic – *which can often be validated or invalidated by experts* – turns out to be relevant for several real applications (e.g. crack growth in the propagation phase, corrosion-erosion problems), particularly if we note that extensions of the gamma process allow to consider the parameters α and β as functions of influent physical parameters (e.g. α and β may depend on fluid pH for corrosion modeling) [11,12].

The second particularity of the gamma distribution function that should be questioned is the expression of the standard deviation (which is a measure of the variability of a random variable around its mean value):

$$\sigma(X_t - X_s) = \frac{\sqrt{\alpha(t-s)}}{\beta}, \text{ and therefore } \frac{\sigma(X_t - X_s)}{E(X_t - X_s)} = \frac{1}{\sqrt{\alpha(t-s)}}$$

In the first expression, we can see that the standard deviation increase with the length of the time interval $[s,t]$. This means that the variability of the degradation indicator become more and more important as time grows, which is quite realistic in many cases. But if this variability is expressed as a percentage of the average degradation (second equation), we see that the relative uncertainty on the value of X_t tends to zero when t tends to infinity. *Such a property seems quite difficult to validate through expert judgment in practice, and a graphical analysis of NDT measures* $(t-s, X_t - X_s)$ *is then the only way to decide if this particularity is relevant or unrealistic*.

Thus, the gamma process is obviously not always a good solution to model a degradation mechanism, but expert's knowledge and NDT datasets may provide some first elements of validation / invalidation. However that may be, such questions should always be addressed before moving to the next step of the study, which is the subject of the last part of the article: the calibration of the parameters of a gamma process.

3. Parameters Estimation and NDT Measurement Uncertainty

From now on, we assume that we dispose of a dataset

$$\{(t_i - s_i, X_{t_i} - X_{s_i})\}_{1 \le i \le n}$$

composed of n degradations increments on various time intervals, measured via NDT. We shall see briefly, and without giving thorough theoretical details, how to use this dataset to choose the most relevant values of the two parameters α and β of a gamma process. Taking into account influent physical parameters [11,12] is out of the scope of this paragraph: we will assume that our dataset has been obtained in "homogeneous environments". Nevertheless, this problem is very important in practice and will be the next step of our research on this topic.

Two situations will be considered. In the first one, we assume that the measurement uncertainties that "blur" the values $X_{t_i} - X_{s_i}$ are negligible, which is the easiest situation to handle. Then, we shall see that if this uncertainty becomes non-negligible, an inappropriate statistical treatment could lead to a poor calibration and to over-pessimistic maintenance decisions. Finally, we will show how to avoid this problem, by taking into account experts' knowledge on NDT uncertainties.

3.1 Estimation with maximum likelihood on perfect measures

The maximum likelihood method is a very common statistical tool [7], which relies on the notion of compatibility between a measure and a given set of values of α and β. The likelihood $L_i(\alpha,\beta)$ of the measured degradation increment $X_{t_i} - X_{s_i}$ is given by the pdf of the gamma distribution:

$$L_i(\alpha,\beta) = f_{\alpha(t-s),\beta}(X_{t_i} - X_{s_i})$$

In the context of independent degradation increments, the likelihood $L(\alpha,\beta)$ of the whole dataset is defined as the product of the likelihood of each increment:

$$L(\alpha,\beta) = \prod_{i=1}^{n} L_i(\alpha,\beta)$$

Then, a classical optimization algorithm is carried out to find the values $\hat{\theta} = (\hat{\alpha},\hat{\beta})$ that maximize the dataset likelihood. These numerical values are those that should be used afterwards to forecast quantities of interest for maintenance problems, such as failure probabilities.

To illustrate this method, let us carry out a purely theoretical exercise. We consider a fictitious crack growth model that is described by a gamma process with parameters $(\alpha,\beta) = (1E^{-4},1)$. We shall simulate randomly a dataset from this gamma process (n=14 degradation increments, given if table 1), carry out the maximum likelihood method, and study the difference between our calibrated gamma process and the theoretical one. The comparison will be made on the basis of two quantities of interest:

- the mean value of the degradation increment during 10000 hours:

$$E(\Delta X_t) = \frac{\alpha(\Delta t)}{\beta} = 1mm$$

- and the failure probability that the degradation reaches the threshold $s_T = 6mm$ before 30000 hours:

$$P(X_{30000} \geq s_T) = 1 - \int_0^{s_T} \frac{\beta^{\alpha(30000)} u^{\alpha(30000)-1} e^{-\beta u}}{\Gamma(\alpha(30000))} du = 6.2\%$$

	Time *t* (hours)		
	10000	20000	30000
Flaw size X_t (mm)	-	1,32	1,50
	-	2,01	2,53
	-	3,88	8,64
	2,29	2,43	4,23
	2,16	2,91	4,52
	3,18	4,54	5,18
	-	3,49	4,37
	-	1,40	1,46
	1,99	2,13	2,96
	-	3,71	4,33

Table 1 : Simulated perfect dataset

For the dataset given in table 1, we obtain $\hat{\theta} = (\hat{\alpha}, \hat{\beta}) = (1.34E^{-4}; 0.97)$. The estimates of the quantities of interest are then:

$$\hat{E}(\Delta X_t) = \frac{\hat{\alpha}(\Delta t)}{\hat{\beta}} = 1.07\,,\ \hat{P}(X_{30000} \geq s_T) = 1 - \int_0^{s_T} \frac{\hat{\beta}^{\hat{\alpha}(30000)} u^{\hat{\alpha}(30000)-1} e^{-\hat{\beta}u}}{\Gamma(\hat{\alpha}(30000))} du = 7.1\%\,.$$

These results nearly match the real values, but of course they would have been slightly different for another dataset. Nevertheless, the theoretical properties of the maximum likelihood method ensure that this "statistical variability" remains as low as possible. Moreover, no significant bias would affect the maintenance decision based on these estimated quantities of interest: we are ensured that the average estimates over a large number of different datasets are almost equal to the unknown true values.

3.2 The Danger of Ignoring Measurement Uncertainty

Let us now consider the case in which NDT measurement uncertainties are non-negligible. We shall introduce a new variable Y_t, which represents the measure provided by a NDT. Because of measurement uncertainty, Y_t may be more or less different from X_t. In the following, this will be expressed through the following model:

$$Y_t = X_t + \varepsilon_t\,,$$

where ε_t is a "white noise" i.e. a random variable that describe measurement error. Table 2 gives a dataset that has been obtained by adding a random white noise to the perfect measures of table 1 (the standard deviation σ_ε of the white noise that we used is equal to 0.5mm).

	Time *t* (hours)		
	10000	20000	30000
Flaw size X_t (mm)	-	1,01	1,00
	-	2,57	2,64
	-	4,02	8,88
	2,24	2,94	4,84
	2,93	4,08	5,01
	1,91	3,81	5,21
	-	3,70	4,33
	-	1,32	0,69
	1,79	1,96	2,50
	-	4,07	5,72

Table 2. Simulated measured dataset

A first possible approach to process this dataset is to forget that these measures are uncertain, and to inject the raw dataset in the statistical procedure described in the previous paragraph. But several tests have shown that this first approach is particularly dangerous in term of maintenance decision [8]. For instance, here, we obtain.

$\hat{\theta}_1 = (\hat{\alpha}_1, \hat{\beta}_1) = (1E^{-4}; 0.68)$

Then:

$$\hat{E}(\Delta X_t) = \frac{\hat{\alpha}_1(\Delta t)}{\hat{\beta}_1} = 1.47\,, \hat{P}(X_{30000} \geq s_T) = 1 - \int_0^{s_T} \frac{\hat{\beta}_1^{\hat{\alpha}_1(30000)} u^{\hat{\alpha}_1(30000)-1} e^{-\hat{\beta}_1 u}}{\Gamma(\hat{\alpha}_1(30000))} du = 22.8\%\,.$$

These estimates of the mean degradation rate and of the failure probability are far too pessimistic. This particular dataset illustrates in an extreme way what can be observed with less intensity on average. For instance, if we simulate a very large number of datasets of size n=14, we observe that the average overestimation of the failure probability is nearly equal to 50% of the true value! Thus, one would on average take an over-pessimistic maintenance decision, and plan a preventive maintenance act far too early.

Though, measurement uncertainty can be handled statistically through a much more efficient way, *at least if experts are able to identify the value of the standard deviation* σ_ε *that characterize the uncertainties of NDT measurements*.

3.3 Estimation with EM Algorithm on Imperfect Measures

If the standard deviation σ_ε of measurement uncertainty is known, the maximum likelihood method can still provide a solution to our estimation problem. But to be rigorous, the likelihood of a degradation increment should not be evaluated via the gamma distribution, but through the convolution of the gamma pdf with the white noise pdf. The Expectation-Maximization (EM) algorithm [9,10] can be used to solve efficiently this problem. Indeed, it has been developed to process datasets that include hidden data; our problem lies within this framework since measurement error is nothing but a hidden information.

Applying the EM algorithm to the dataset given in table 2 leads to the following results:

$$\hat{\theta}_2 = \left(\hat{\alpha}_2, \hat{\beta}_2\right) = \left(1.42E^{-4}; 1.2\right),\ \hat{E}\left(\Delta X_t\right) = \frac{\hat{\alpha}_1(\Delta t)}{\hat{\beta}_1} = 1.18\,,$$

$$\hat{P}\left(X_{30000} \geq s_T\right) = 1 - \int_0^{s_T} \frac{\hat{\beta}_1^{\hat{\alpha}_1(30000)} u^{\hat{\alpha}_1(30000)-1} e^{-\hat{\beta}_1 u}}{\Gamma(\hat{\alpha}_1(30000))} du = 8.9\%\,.$$

The conclusions are clearly better than with the previous approach, even if the situation is still not as good as for perfect measures (but this is natural since we are working with a degraded information). Large-scale simulations of datasets confirm that the EM algorithm solves the bias problem mentioned previously; this ensures a better confidence in the maintenance decisions that could be taken on the basis of the calibrated gamma process.

4. Conclusion and Perspectives

The aim of this article is to illustrate the potential industrial interest of stochastic degradation models for passive components. Such models could be useful in a decision process if the knowledge of the physical phenomenon is not sufficient to build a reliable degradation forecast model. But for the moment, these mathematical tools are still hardly compatible with the constraints of real-case industrial studies. A solution has been proposed to overcome one of the obstacles: the rigorous statistical treatment of NDT measurement uncertainty, which could otherwise lead to over-pessimistic maintenance decisions. But many are still to be adressed, such as the inclusion of influent physical parameters in the calibration of the stochastic degradation process [11,12]. Several real-case studies are currently in progress at Electricité De France; this test phase will help us to assess more precisely the difficulties and the possible contributions of such stochastic approaches.

References

[1] J. Van Noortwijk, HE . Klatter. Optimal inspection decisions for the block mats of the Easter-Scheldt barrier. Reliability Engineering and System Safety, 65:203-211, 1999.

[2] MJ. Kallen, J. Van Noortwijk. Optimal maintenance decisions under imperfect inspection. Reliability Engineering and System Safety, 1-9, 2004.

[3] A. Grall, C. Bérenguer, L. Dieulle. A condition-based maintenance policy for stochastically deteriorating system. Reliability Engineering and System Safety, 76(2):167-180, 2002.

[4] C. Bérenguer, L. Dieulle, A. Grall, M. Roussignol. Sequential condition-based maintenance scheduling for a deteriorating system. European Journal of Operational Research, 150(2):451-461, 2003.

[5] C. Bérenguer, A. Grall, M. Roussignol. Maintenance policy for a continuously monitored deteriorating system. Probability in the Engineering and Information Science, 2002

[6] K. Simola, U. Pulkkinen. Models for non-destructive inspection data. Reliability Engineering and System Safety, 60:1-12, 1998.

[7] G. Saporta. Probabilités, analyse des données et statistique. Edition Technip,1990.

[8] M.Desgrouas. Aide à la modélisation des dégradations. Master Diploma Report, Université de Marne La Vallée, 2003.

[9] SK. Ng, T. Krishnan, G.J. McLachlan. The EM Algorithm. Center For Applied Statistics and Economics, 2004.

[10] JL. Foulley. Algorithme EM : principe et illustrations. Journée Biopharmacie-Paris, 2003.

[11] V. Bagdonavicius, M. Nikulin. Accelerated Life Models : modeling and statistical analysis. Chapman & Hall - CRC, London, 2001

[12] V. Bagdonavicius, M. Nikulin. Modèle statistique de dégradation avec des covariables dépendant du temps. Académie des Sciences de Paris, 330(1), 131-134, 2000.