Probabilistic Design System (PDS): A Realistic Approach of Finite Element Modelling for Capacitive Micro-machined Ultrasonic Transducers (cMUTS)

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Abstract. This paper represents a realistic approach for modeling a cMUT device. CMUTs have become very popular over the last decade because of the comparable bandwidth, sensitivity and dynamic range with its piezoelectric counterparts. The ease of fabrication is an added advantage. Modeling of CMUT is a coupled physics problem, which involves solving Electrostatics-Structural-Fluid interactions simultaneous. Finite Element models of the CMUT are constructed using the commercial code ANSYS (10.0). In the standard approach of modeling, from existing literature assumes deterministic values for design parameters, however fabrication of the device introduces some amount of variation in the design parameters. In this paper, the PDS FEM approach is discussed to account for the variability in fabrication. The results from the PDS approach on the performance characteristics like resonance frequency; collapse voltage and electromechanical coupling coefficient will be discussed.

Keywords: CMUT, Ultrasound, Couple Physics Analysis, Reduced Order Model.

1. Introduction

CMUTs have gained great attention recently due to the advantages over the conventional transducers. In characteristics like bandwidth, acoustic matching properties, fabrication and sensitivity, CMUTs have an edge over their piezoelectric counterparts. These projected advantages have resulted in researchers across the world working in increasing the efficiency of the mass fabrication and functional properties of CMUT [1-3].

This increased interest in CMUT technology has fuelled modelling efforts for these devices and several approaches were suggested [1], which includes equivalent circuit models and numerical models. The equivalent circuit model in its simplest form can be represented as mass spring model attached to a capacitor (figure 1) [4,5].

$$m\frac{d^2x}{dt^2} - \frac{\varepsilon_0 A V^2}{2(t_a - x)^2} + kx = 0 \qquad (1)$$

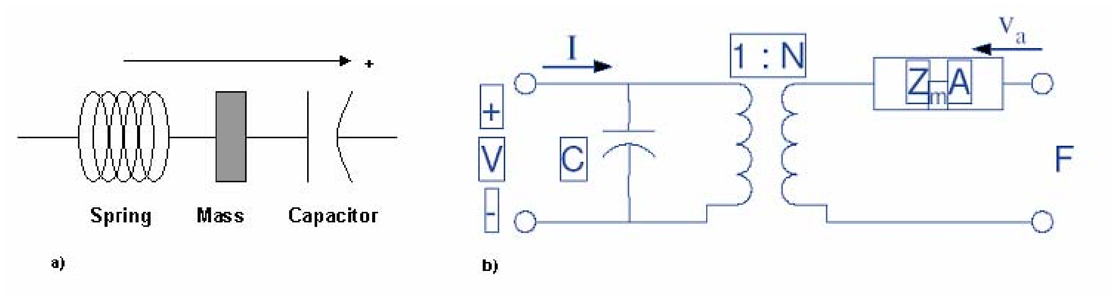

Figure 1: a) Mass spring model b) Equivalent circuit model

Equation 1 represents the first order model of CMUT (figure 1a). The equivalent circuit model as shown in figure 1b is a complex representation which includes the clamped capacitance in the electrical port, spring softening effect due to electromechanical interaction and mechanical impedance effects of the membrane with the surrounding medium [7-9]. However these two approaches have limitations like low accuracy, limited output parameters and inability to model cross talks and fringing.

Numerical methods like Finite Element Modelling (FEM) or Finite Difference Methods are also used in modelling of CMUTs. FEM method by far has contributed the most to the development of the CMUTs and enhanced the understanding of the device. Using customized and commercially available FEM codes, most of the studies were aimed at understanding the output parameters like collapse voltage, system capacitance, radiated pressure and membrane deflections. These are some of the critical parameters required to design and optimize cMUT for improved performance.

In this paper, commercially FEM package ANSYS® was used for our analysis. Different ways of modelling a CMUT in ANSYS® includes Sequentially Coupled Field (SCF) model, Direct Coupled Field (DCF) model and Reduced Order Model (ROM). Authors have reported on the characteristics and advantages of these models [8]. The DCF model takes least amount of time for a numerical simulation (axisymmetric model) when compared to the SCF and ROM models as shown in figure 2.

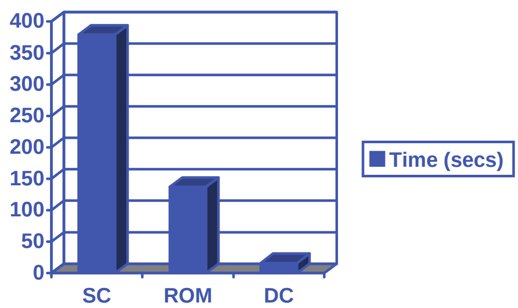

Figure 2: Time Comparison [8]

The standard FEM modelling approach is deterministic, considers input variables like radius of the membrane, thickness of the membrane etc, as constant variables and the output parameters obtained using both static and dynamic analysis. The authors have reported an extensive study on the linear variation of critical input parameter to understand its effect on key output parameters like collapse voltage, resonance frequency, capacitance and efficiency etc [8]. However, the effect of random variation in multiple input parameters is not considered, for evaluation of sensitivity. The parameters of interest like collapse voltage can have significant impact with small variations in input parameters like radius and thickness of the membrane. Deterministic models do not capture these variations as needed in many cases.

In reality, the fabrication of these devices produces some amount of random variation in the geometric parameters and material properties. As these devices accumulate service, they undergo ware and tear and partly due to usage in harsh environments. Furthermore, mishandling also contributes to change in model parameters. Hence, deviation from optimum performance characteristics of the device is seen.

In this paper, we introduce the concept of Probabilistic Distribution System (PDS), where we consider a random distribution for all the critical input variables and also compare their sensitivity towards key output parameters. The PDS module is the ideal method of simulation for accounting both fabrication and functionally related variations.

2. Design Parameters

Standard dimensions were considered for the design parameters for the FE model of CMUT. The typical DCF model is as shown in figure 2. In this study, commercially available FEM package ANSYS® version 10.0 was used for the analysis.

Geometric Parameters

- Radius of the membrane = 20 µm
- Thickness of the membrane = 1 µm
- Air gap = 0.5 µm
- Radius of top electrode = Radius of membrane
- Insulation thickness (SiO_2) = 0.3 µm

Material Properties of Membrane (SiN)

- Young’s modulus = 320 GPa
- Poisson’s ratio = 0.263
- Density = 3270 Kg/m^3
- Relative permittivity = 5.7

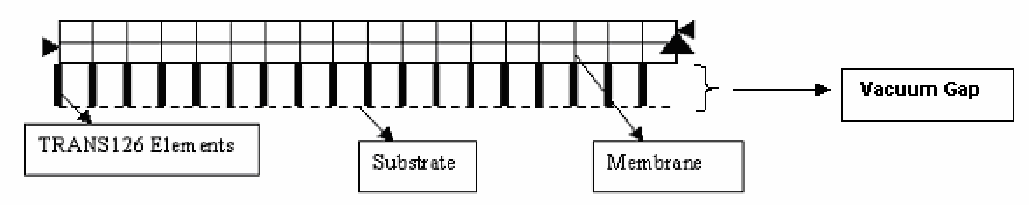

Figure 3: Direct coupled field finite element model

3. Probabilistic Design System [9]

The Probabilistic Design System (PDS) analyzes a component or a system involving uncertain input parameters (Figure 4). These input parameters could be anything ranging from geometry and material properties to different boundary conditions. These parameters are defined as random input variables and are characterized by their distribution type and variables (mean, standard deviation). The key outputs of the simulation are defined as random output parameters.

During a probabilistic analysis, multiple analysis loops are executed to compute the random output parameters as a function of the set of random input variables. The values for the input variables are generated either randomly (using Monte Carlo simulation) or as prescribed samples (using Response Surface Methods).

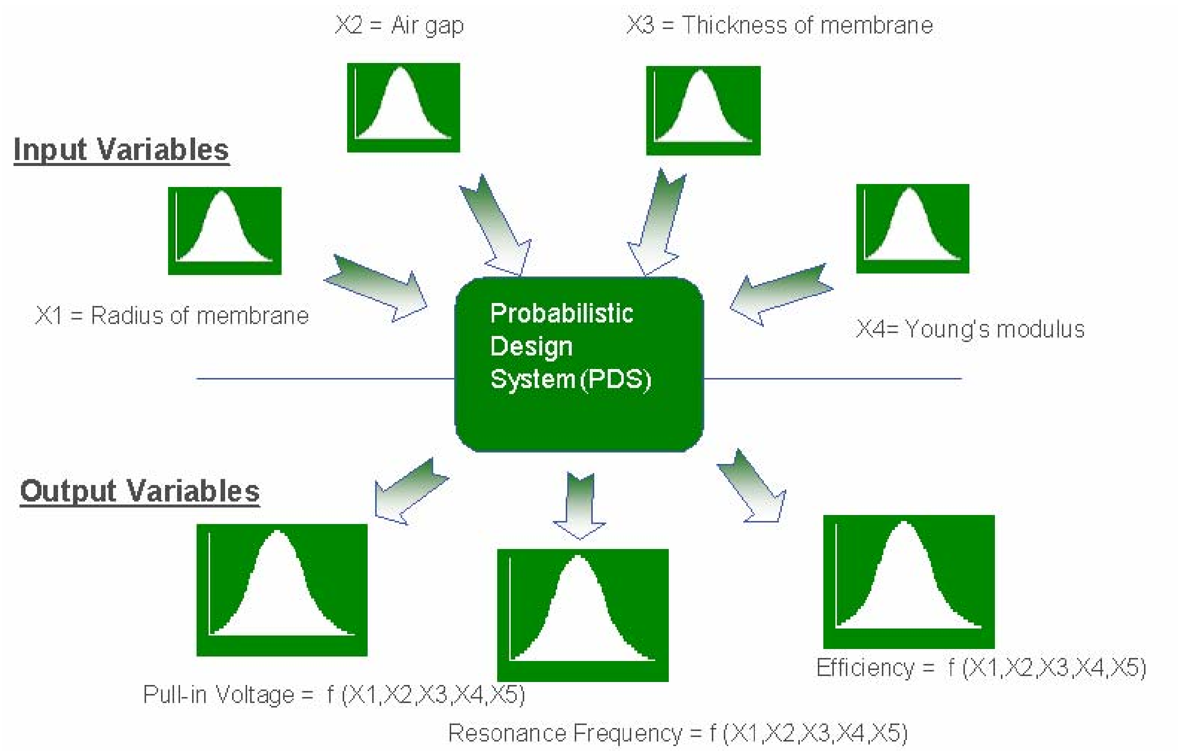

Figure 4: Schematic on the working of PDS

3.1 Deterministic model

The analysis file containing the deterministic model (Figure 3) gets executed or "loops through" multiple times during the probabilistic analysis. In this deterministic model four parameters were considered as random input variables. Table 1 illustrates the random input variables, the distribution they are subjected to, and their distribution parameters. The discrete sampling characteristics and the probability density functions for the input variables are shown in Figure 5. The variance on geometrical and material input parameters was gathered from MEMS fabrication experts. Accordingly, a 10% variation for each of these input parameters was considered assuming a Gaussian distribution for all input variables.

Table 1: Input random variables for probabilistic design

Name	Mean	Standard Deviation	Minimum	Maximum
Airgap	0.50	0.02	0.453	0.548
Radius of Membrane	20.0	0.75	17.98	21.80
Thickness of Membrane	1.0	0.04	0.891	1.097
Young’s Modulus	1.50e05	9e3	1.28e5	1.74e5

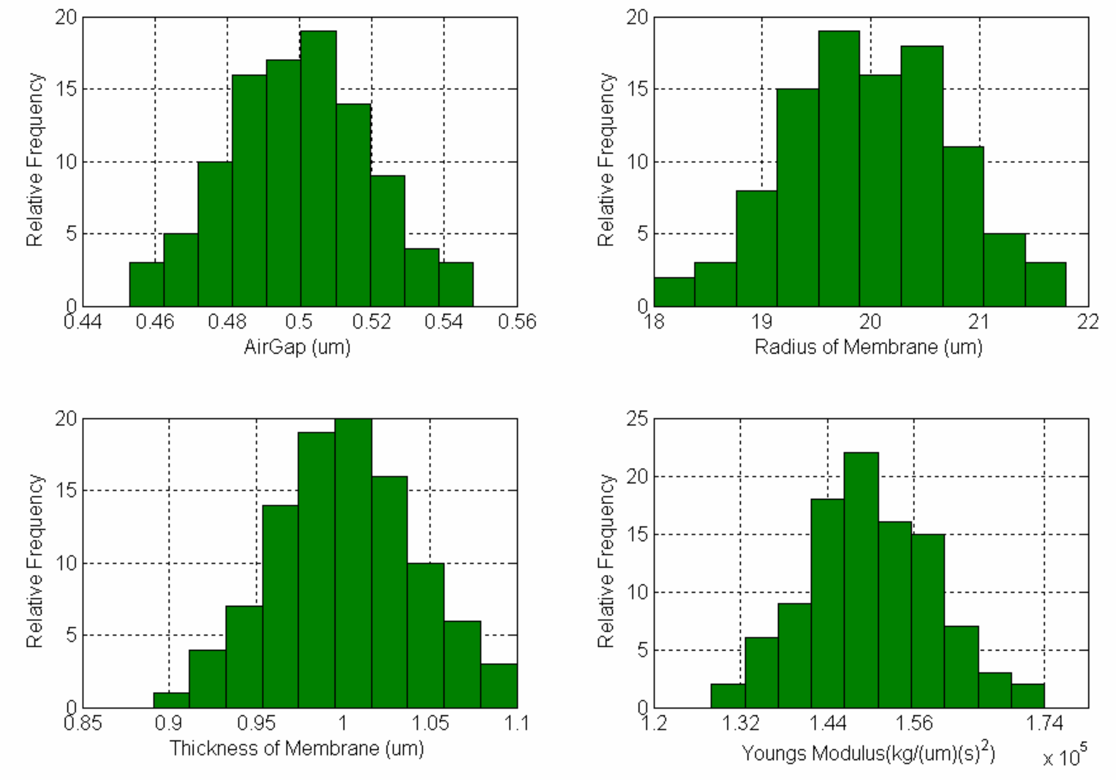

Figure 5: Probability density function for input random variables

3.2 Analysis summary and results:

The PDS analysis was looped through 100 sample points considering the variation defined in the input variables. Tables 1 and 2 show the sample point characters for both input parameters and the outputs of the simulation. Figure's 5 and 6 illustrates the histogram distribution on the sample characteristics of input variables and the output variables. Figure 7, illustrates the sensitivity plots for the output variables with respect to the randomised inputs. It can be observed that all the parameters considered in the simulation have a contribution to the prediction of collapse voltage. Collapse or pull-in voltage is directly proportional to thickness, airgap and Young's modulus of the membrane and inversely proportional to radius of the membrane. Thickness and radius of the membrane have a more sensitivity to the collapse voltage when compared to the Airgap and Young's modulus.

Table 2: Output variable Collapse Voltage characteristics in the probabilistic design simulations

Name	Mean	Standard Deviation	Minimum	Maximum
Collapse Voltage (V)	220.8	28.16	150.7	294.0
Efficiency (K_t^2)	6.69e-02	3.48e-3	6.09e-02	7.72e-02
Resonance Frequency (MHz)	8.26	.752	7.04	10.24

The prediction of resonance frequency shows that it is sensitive to variations in thickness, radius and Young's modulus of the membrane. Airgap is not a significant factor for estimating resonance frequency. Furthermore, resonance frequency is directly proportional to thickness and Young's modulus of membrane and inversely related to radius of membrane. In these simulations, the resonance frequency is estimated at 90% of the collapse voltage. From the deterministic model it can be observed that variation of bias voltage slightly around the collapse voltage can result in a significant variation in resonance

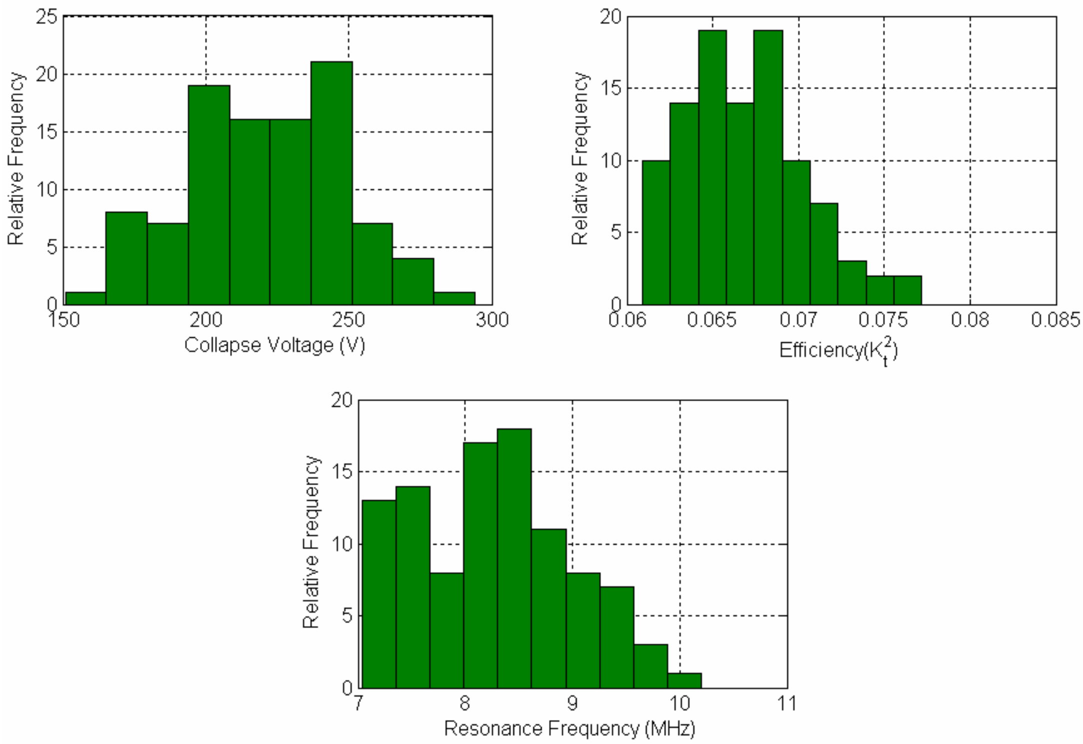

Figure 6: Histogram distribution of output variables Collapse Voltage, Resonance Frequency and Efficiency (K_t^2)

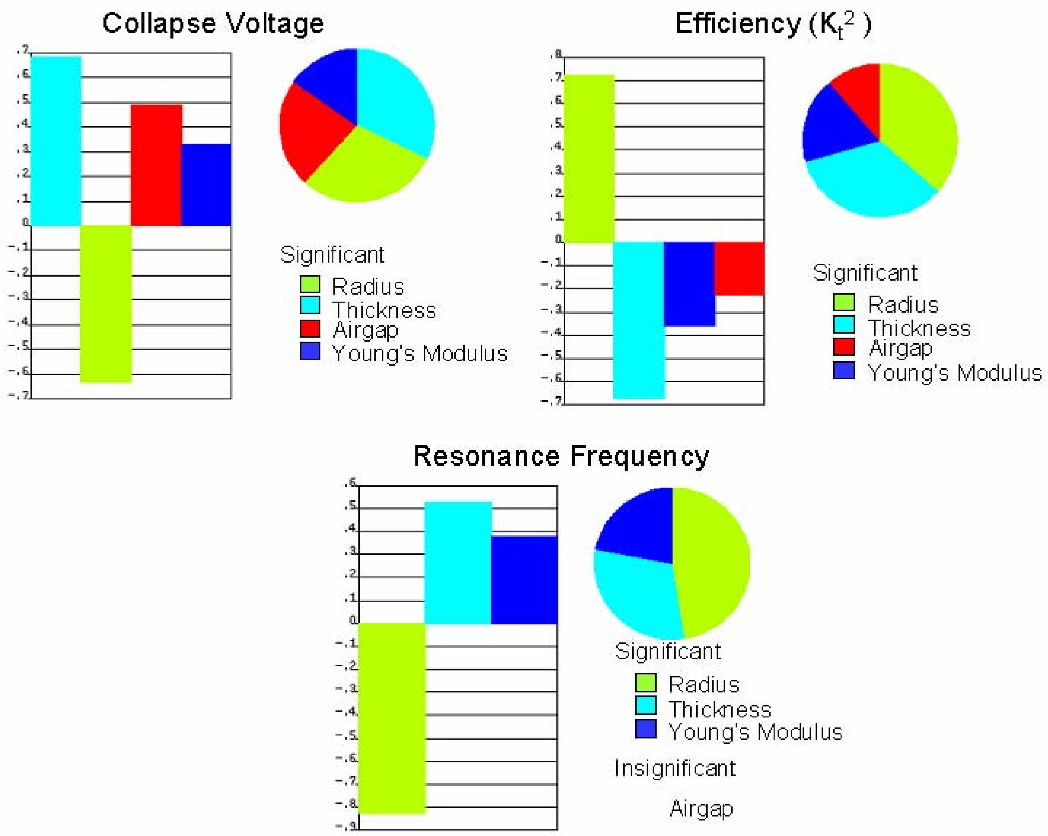

Figure 7: Sensitivity of Collapse Voltage, Resonance frequency and Efficiency with respect to the input parameters

frequency prediction. Small variations in airgap will result in change in collapse voltage and hence results in variation of resonance frequency. However, the PDS shows that under the given conditions airgap is insignificant parameter when estimating resonance frequency.

Mechanical efficiency is defined as the ratio delivered mechanical energy over the stored total energy. The radiated pressure from the membrane is a measure of the delivered mechanical energy. Efficiency of the device is observed to significantly increase with the bias voltage and is expected to reach unity near to the collapse voltage. The efficiency shown in the graph is estimated at bias voltage equals 90% of collapse voltage. The parameters radius, thickness, Young's modulus and airgap show sensitivity to efficiency in the decreasing order of significance. The efficiency shows a direct relationship with radius and an inverse relationship with rest of the input parameters. The result obtained considers a single cell for in the PDS analysis. In reality, a CMUT consists of few thousands of cells, when multiple cells are in operation, cross talk between these cells has to be considered and the performance (efficiency) may have a different behaviour.

With increase in radius of the membrane the collapse voltage is observed to fall exponentially. This can be observed from figure 8. This is quite an expected behaviour when other input parameters are kept constant as the stiffness of the membrane drops. With increase in radius the impact on the collapse voltage prediction is expected to be more significant. From the figure 8, we can observe this as percentage variation in collapse voltage from the mean increases with increase in radius.

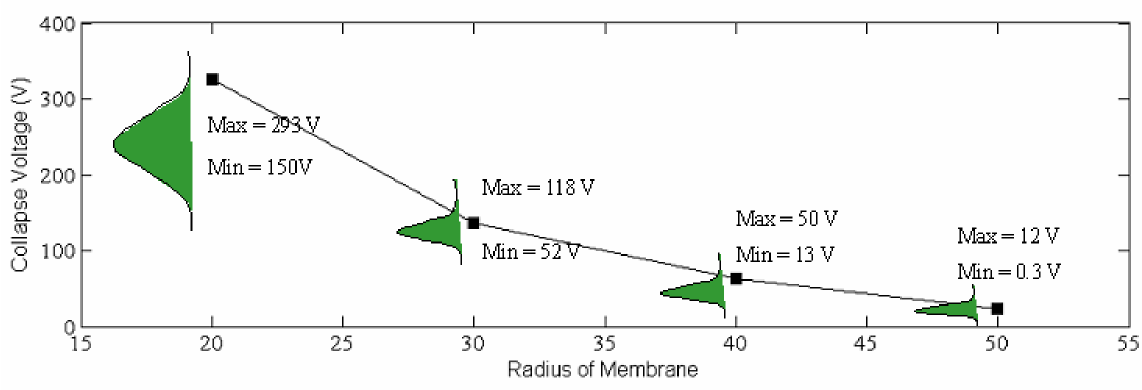

Figure 8: Comparison of Probabilistic Design with deterministic model

4. Summary

Depending on cMUT cell dimensions and associated parameters we have to choose an appropriate FE model (linear or non-linear) for understanding physics of the system and optimization of the device. Fabrication introduces an inherent variation in the critical parameters of the cMUT, which leads to system output variations. The PDS approach is ideal for such scenarios to explain the deviation from the observed output parameters in experiments. Furthermore, it identifies the critical/sensitive parameters for optimization during fabrication to achieve the desirable output.

In this study, we have considered simulated variations in geometric and material parameters for a simple axisymmetric model to illustrate the effectiveness of PDS. The author's recommend considering ground truth data for the input variables when evaluating a design using PDS.

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