# Magnetic Leakage Fields as Indicators of Eddy Current Testing

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**Abstract:** With eddy current testing in a homogenous field it is often spoken about hindrances that are represented by cracks to eddy currents that cannot flow so as in non-defective material. Such homogenous condition can be met with the magnetizing in a very long coil. If however the coil is short the field is not homogenous any more and especially with testing of ferromagnetic materials the magnetic leakage field may not be neglected. The demagnetizing effect of eddy currents can be seen also in magnetic leakage field. By proper arrangement of primary and secondary coil it is possible to get more information. It is given in the article how the longitudinal and the transverse cracks can be detected in the proposed arrangement of primary and secondary coils.

If the arrangement of coils with the eddy currents testing is not a standard one it is very important to understand the physics of the new arrangement to be able to interpret the results properly. A very useful tool is the mathematical treatment of field equations. The numerical methods are often applied, since Maxwell equations are rather complex partial differential equations and there are usually real and imaginary components of the field to be taken into account. There are main ideas given how to solve more complicated problems. The actual concrete results will be compared with practical experiments.

**Key words:** numerical solution of partial differential equations, differential coils, magnetic leakage fields, surface cracks, ferromagnetic material.

## 1. Introduction

With the electromagnetic non-destructive testing there are usually two main methods applied.

The basis of detecting surface cracks with eddy currents is in the fact that cracks are preventing eddy currents from flowing across the crack and causing changes in skin effect. In this case cracks being oriented along the magnetic field can be detected.

The second is the method of magnetic leakage fields that is mostly used to show surface changes of the magnetic field of a ferromagnetic part by spreading ferromagnetic fluorescent powder over the surface of the part. In this case cracks being oriented perpendicularly to the field in ferromagnetic material can be detected

At the Faculty of Mechanical Engineering in Ljubljana a special arrangement of coils has been developed where both principles are combined. So it is possible to detect surface cracks in a ferromagnetic bar that can be oriented along or perpendicular to the axis of the bar.

## 2. Eddy currents

### *2.1 Primary and secondary coil*

The basis of the eddy current method is often explained by a simple example of detecting surface cracks in a metal bar:

A long metal bar of circular cross-section is supposed to be put in a homogenous magnetic field oriented along the bar. Because of the demagnetizing effect of eddy currents flowing in the bar (skin effect) the magnetic field in the cross-section is not uniform. It is much stronger close to the surface than in the middle and there are also phase differences in the field among different parts of the cross-section. This effect depends on frequency and on physical properties of the material.

In a secondary coil wound around the bar the induced voltage is directly proportional to the magnetic flux in the cross-section of the bar. There is a phase difference between the primary magnetization and the voltage induced in the secondary coil which depends on frequency and on physical properties of the bar.

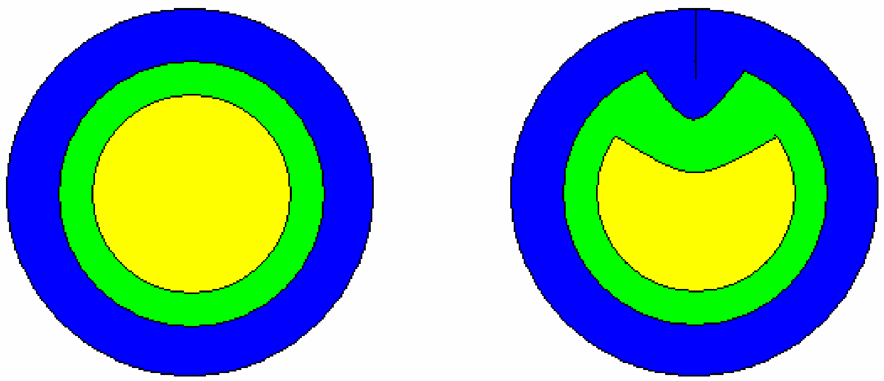

**Fig. 1**: Cross-section of a metal bar with a radial crack and without a crack.

The situation in the secondary coil changes if there is a surface radial crack being oriented along the length of the bar. The demagnetizing influence of eddy currents in the cross-section is diminished because eddy currents cannot flow across the crack. The net magnetic flux in the cross-section is increased in the vicinity of the crack and so is the induced voltage increased in the secondary coil. The difference between two induced voltages is the indicator of the crack in a metal bar. Fig. 1 schematically shows the magnetic field in the cross-section of a metal bar with a radial crack and without a crack. The magnetic flux in the outer regions of the defective cross-section is increased. The darker regions in the cross-section shown in Fig.1 belong to smaller phase difference and to stronger magnetic field.

There are several explanations given in the literature how to distinguish among voltage differences caused by different position and forms of cracks [1].

According to these explanations it would not be possible to detect difference in the induced voltage if a surface crack is not hindering the flow of eddy currents. It would not be possible for instance to detect a very thin surface crack in the direction perpendicular to the axis of the bar. In this case the eddy currents namely would not be forced to change their direction at all.

*2.2 Differential arrangement of secondary coils*

Changes of voltage induced in a secondary coil (according to Fig. 2) are normally very small and the so called differential arrangement of two secondary coils is used very often. Two differential coils are wound around the bar and their contacts are connected so that the induced voltages are subtracted. If there is no difference in the induced voltages the bar is supposed to be without a defect. As shown in Fig. 2 a sudden beginning of a defect or a sudden end of a defect would cause a signal from the pair of coils. This arrangement is a very sensitive means for detecting longitudinal surface cracks in a metal bar.

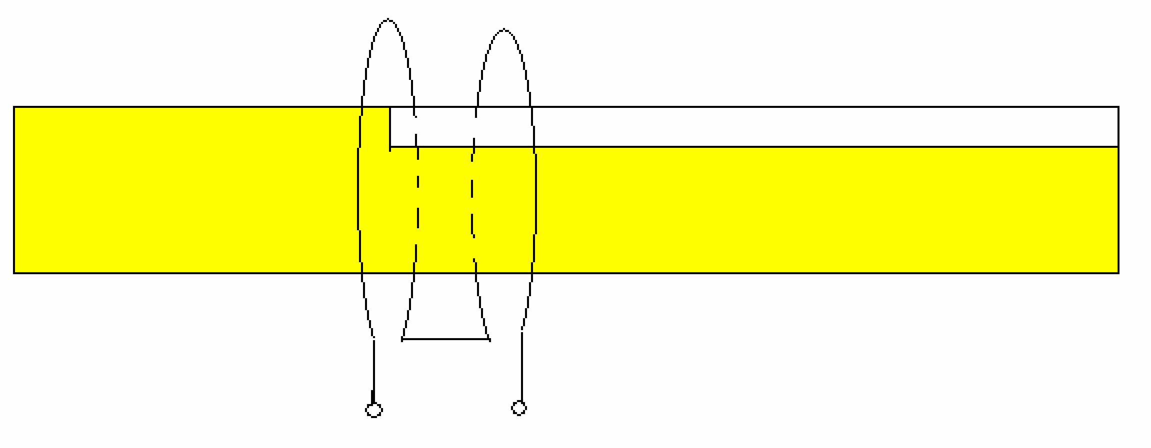

**Fig. 2**: Two differentially connected loops of pair of secondary coils.

This very promising method has some drawbacks. For example if the crack is uniform along all the length of the bar the magnetic flux would increase uniformly and the differential arrangement of coils could not detect it since the induced voltage in both secondary coils would be the same.

Even if the crack has a good defined start and good defined end it is also not possible to tell its position on the circumference of the bar.

*2.3 Radial magnetic flux*

There are however some other effects observed in the case of a metal bar with a uniform crack which starts at a certain point and which ends at a certain point. A differential arrangement of secondary coils can detect both points. According to the electromagnetic theory the increase in the magnetic flux in the cross-section that is caused by smaller demagnetizing effect of eddy currents, can be seen also in a different way. If the magnetic flux is increased due to a surface crack the magnetic flux lines must enter the bar somewhere from outside. The theory and the practical measurements showed that there is a strong radial magnetic flux flowing from outside of the bar into the bar localized just at the point of the beginning of the crack. This fact was theoretically and practically verified years ago [5]. The measured radial flux would be namely proportional to the signal from the pair of two secondary coils being connected differentially. The measurement of the radial magnetic flux as indicated in Fig. 3 is giving an additional information. Not only the position of the start and the end of the crack but also the circumferential position of a crack could be detected.

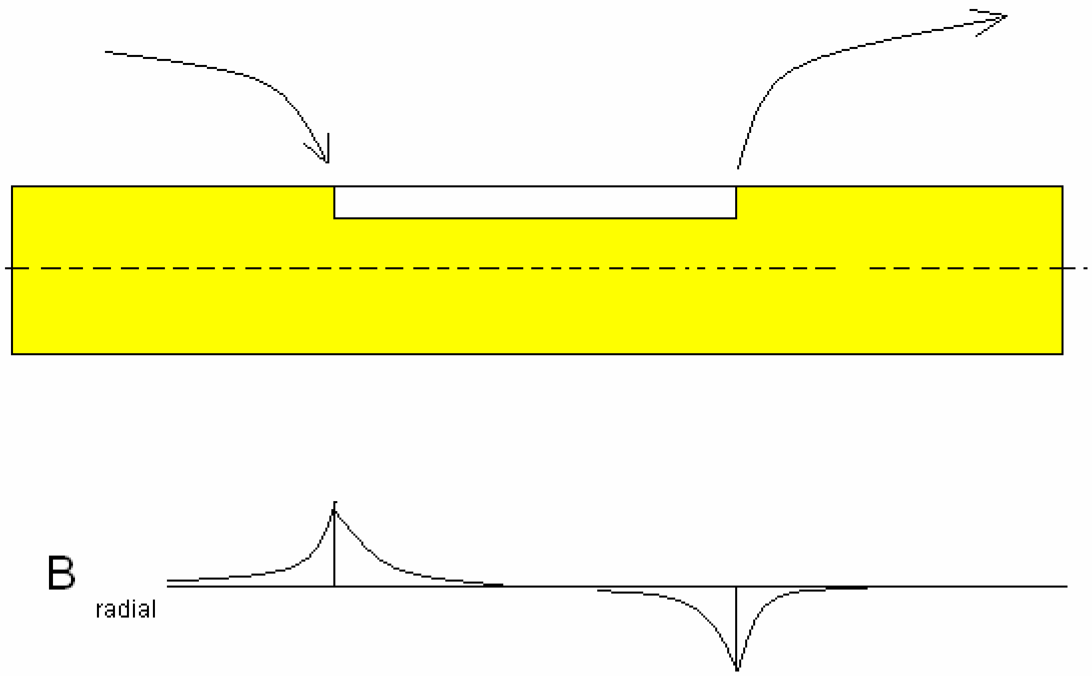


**Fig. 3**: Radial flux entering the bar at the beginning and leaving it at the end of the crack.

### 3. Magnetic leakage flux

As mentioned above there is another effect of the flow of magnetic flux used in non destructive testing although it is more often mentioned with the D.C. magnetic fields. It is so called magnetic leakage flux used for indicating surface cracks in magnetized ferromagnetic parts.

The explanation of the principle is very often given for two dimensional case of detecting surface crack when the homogenous or not homogenous filed is crossing a surface crack in a ferromagnetic plate (Fig. 4).

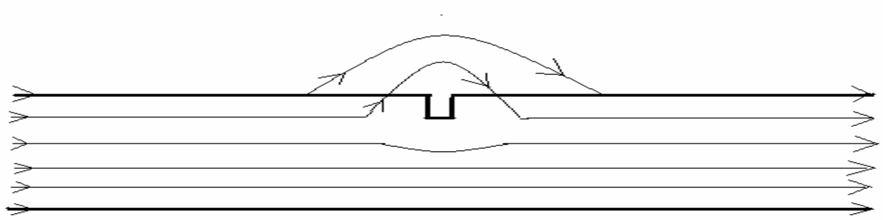

**Fig. 4**: Magnetic leakage field in two dimensions in the cross-section of a ferromagnetic plate.

Suppose the homogenous magnetic field flows from the left to the right. Near the crack the flux is divided to two parts. One part tries to avoid the crack by flowing under the bottom of it and another part flows out of the material up into the air and then it again enters the plate on the right side of the crack. This effect is observed in ferromagnetic materials and if the surface of the plate is covered by ferromagnetic powder it is possible to view the emerging field around the defect outside by naked eye.

The situation with the A.C. magnetic field in a ferromagnetic material is very similar. Instead of magnetic powder an appropriate secondary coil can be applied to detect emerging magnetic flux.

There were some attempts made to calculate and to measure the emerging field and to connect it with the depth of the crack [2].

### 4. Two parts of a primary coil and one radial secondary coil

In the Faculty for Mechanical Engineering in Ljubljana a special arrangement of coils for the NDT control of ferromagnetic bars of circular cross-section was developed where both methods were joined together.

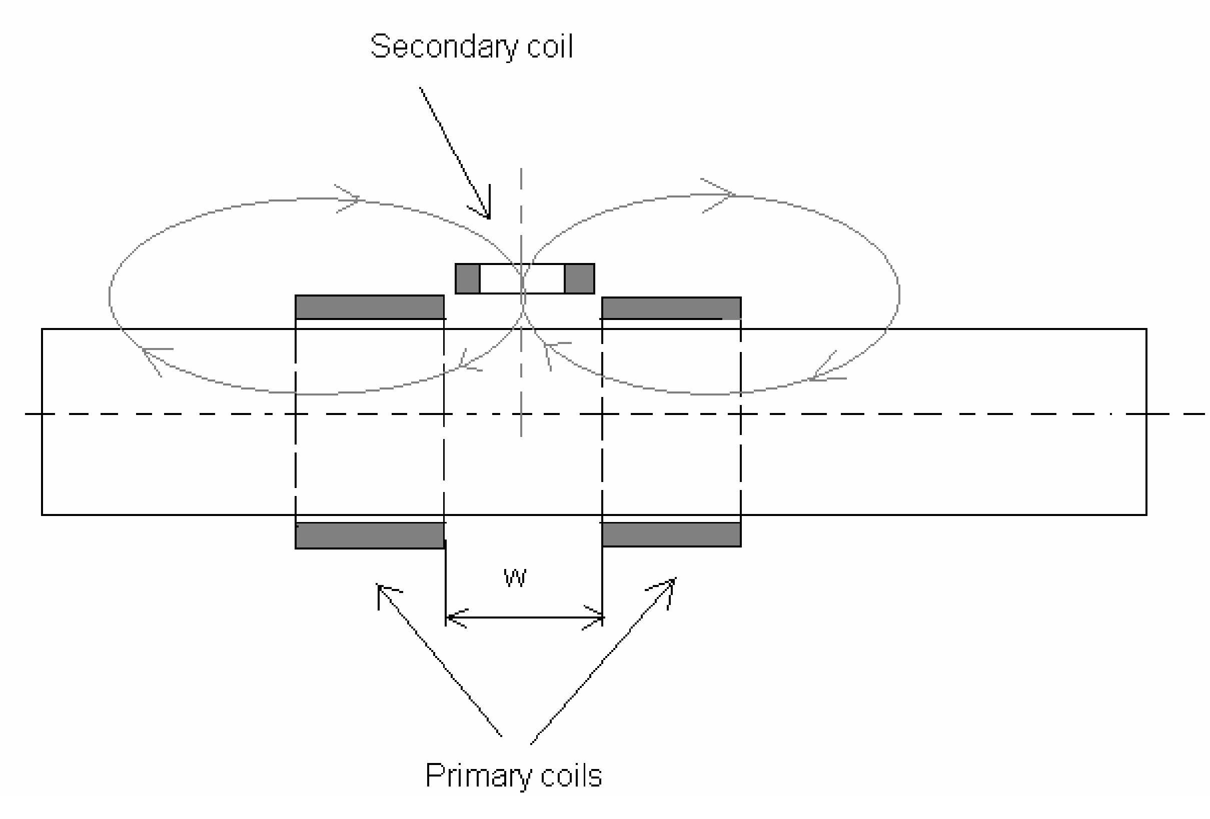


**Fig. 5**: Two parts of a magnetizing coil with one secondary coil in the middle.

The magnetizing primary coil is made of two equal parts being separated by so called critical distance (w). In the place between them there is one secondary coil oriented so that the induced voltage is caused only by the radial magnetic flux flowing through it.

In the secondary coil the magnetic fluxes emerging from the two parts of the primary coil are subtracted and if the bar is homogenous there is no net radial magnetic flux and no induced voltage in the secondary coil. If however there is some crack present in the bar being placed in one part of the primary coil, both parts of magnetic flux flowing through the secondary coil are not equal any more, and a net induced voltage indicates the presence of the defect.

In this way i*t is possible to detect a thin surface crack in a ferromagnetic material in longitudinal or in perpendicular direction.*

*4.1 Critical distance (w)*

It is very important to choose the critical distance properly. If the distance between the two parts of the primary coil is too small, the stray magnetic flux is also small and so are all the changes in the radial magnetic flux. If it is too large there is no direct impact on magnetic radial fluxes in the secondary coil.

The critical distance must be chosen so that the information in the longitudinal magnetic field in the bar can be transferred optimally to the radial sensing coil.
The optimal dimension of the region w from Fig. 5 can be evaluated from the calculations taking into account practical limitations of coil construction and the electric parameters being used [3].

## 5. Mathematical calculation

In the arrangement of coils from Fig.5 it is not possible to speak about homogenous magnetic field. The magnetic field in the neighborhood of all three coils can be calculated assuming some idealization of coils. Computer simulation of different positions and dimensions helps a lot with construction of actual arrangement. It is shown in due text how it is possible to assess the necessary separation in the primary coil and how it is possible to assess the induced voltage in the secondary coil caused by a defective bar moving through the whole arrangement.

*5.1 Maxwell equations for the magnetic field*

For the case of a ferromagnetic bar with the circular cross-section it is very convenient to start with the calculation of vector potential in cylindrical coordinate system [4].

$$\nabla^2\vec{V} - \sigma.\ \mu.\mu_0.\frac{\partial\vec{V}}{\partial t} - \mu.\mu_0.\varepsilon.\varepsilon_0.\frac{\partial^2\vec{V}}{\partial t^2} = 0 \qquad (1)$$

By introducing new variable

$$\vec{V} = \vec{W}.e^{i\omega t}$$

we can have the real and the imaginary component of the vector potential : $\vec{W} = \vec{A} + i.\vec{A}^*$.
It is possible to write two separated equations for the real and for the imaginary component.

$$\nabla^2\vec{A}\ + \omega.\mu.\mu_0.\sigma.\vec{A}^* = 0 \qquad (2)$$

$$\nabla^2\vec{A}^* - \omega.\mu.\mu_0.\ \sigma.\vec{A} = 0 \qquad (3)$$

The vectors $A$ and $A^*$ have generally three components in space and so we have a system of 6 partial differential equations to solve, for each space component and for the real and imaginary part.

To illustrate the procedure let us limit to the system of two dimensions only. Namely the magnetizing coil has the form of a cylinder and if we choose the source of the coordinate system in the axis of the coil, the problem can be much simplified. If the problem is rotationally symmetrical, only one component $A = A_\varphi$ is different from zero.

The following pair of equations is to be solved in cylindrical coordinate system assuming that the problem is rotational symmetrical:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}.\frac{\partial A}{\partial r} - \frac{A}{r^2} + \frac{\partial^2 A}{\partial z^2} + F.A^* = 0 \qquad (4)$$

$$\frac{\partial^2 A^*}{\partial r^2} + \frac{1}{r}.\frac{\partial A^*}{\partial r} - \frac{A^*}{r^2} + \frac{\partial^2 A^*}{\partial z^2} - F.A = 0 \qquad (5)$$

$$F = \omega.\mu.\mu_0.\sigma.a^2$$

On the other hand it is possible to write the corresponding expression for the two components of the magnetic field density $B$:

$$B_r = -\frac{\partial A}{\partial z} \qquad B^*{}_r = -\frac{\partial A^*}{\partial z}$$

$$B_z = \frac{1}{r}.\frac{\partial}{\partial r}(r.A) \qquad B_z^* = \frac{1}{r}\frac{\partial}{\partial r}.(r.A^*)$$

*5.2 Boundary conditions*

For the points that are lying on the boundary between the air and the material, the basic boundary condition must be fulfilled. One must keep in mind that when crossing the boundary the normal component of the magnetic field density ($\vec{B}$) must be preserved. On the other hand the tangential component of the magnetic field strength ($\vec{H}$) must be preserved as well.

It is rather complicated to solve the system of equations generally. For some special simplified cases it is possible to find maybe even analytic solution, but much more often it is necessary to use some numerical methods.

There are several algorithms available but it depends on the experience of the research worker which method should be used. It is not necessary to calculate the vector potential to some great precision. A more or less rough assessment is usually good enough.

*5.3 Method of finite differences*

We have solved this problem by the method of finite differences.
The coordinate system was chosen as shown on Fig. 6. Instead of looking for the general solution for the unknown vector potential we wish to find the solution in discrete mesh points as shown in Fig. 6.

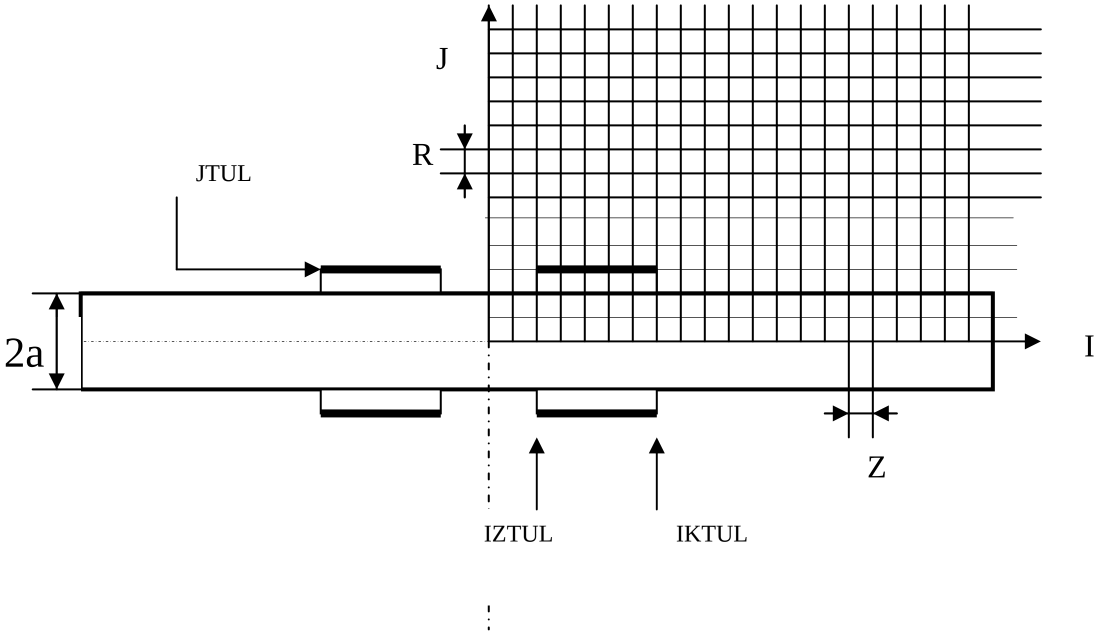


**Fig. 6**: The mesh points where the vector potential is to be calculated.

For any mesh point (I, J) a linear numerical expression corresponding to the partial derivatives from Equation 4 or Equation 5 can be written.

For example: Instead of Equation 2 and Equation 3 the following linear expression can be written for the point (I, J):

$$\frac{A(I,J-1)-2*A(I,J)+A(I,J+1)}{R^2}+\frac{1}{R*(J-1)}*\frac{A(I,J+1)-A(I,J-1)}{2*R}-$$
$$-\frac{A(I,J)}{(R*(J-1))^2}+\frac{A(I-1,J)-2*A(I,J)+A(I+1,J)}{Z^2}+F*A^*=0 \qquad (6)$$

$$\frac{A^*(I,J-1)-2*A^*(I,J)+A^*(I,J+1)}{R^2}+\frac{1}{R*(J-1)}*\frac{A^*(I,J+1)-A^*(I,J-1)}{2*R}-$$
$$-\frac{A^*(I,J)}{(R*(J-1))^2}+\frac{A^*(I-1,J)-2*A^*(I,J)+A^*(I+1,J)}{Z^2}-F*A=0 \qquad (7)$$

In Equations 6 and 7 R means the mesh distance in radial direction and Z in longitudinal direction.

If there are IKON mesh points chosen in the direction I and JKON mesh points in the direction J, it is necessary to find the solution to IKON*JKON*2 linear equations with the same number of unknowns. From the solutions in discrete mesh points it is also possible to calculate the values of the real and of the imaginary component of magnetic field density.

The problems how to write the corresponding numerical difference equation in the corners and on the lines of symmetry can be avoided by application of commonly used algorithms in methods of finite differences [6].

In some cases it is also convenient to use unequally spaced mesh. Far from the coils where nothing is being changed any more, the logarithmic mesh is very often applied. Also the mesh points inside the material are sometimes chosen denser close to interesting spots. All these modifications of the mesh represent some minor additional difficulty and some mathematical experience is needed.

*5.4 Explanations of symbols used in Equations 1-7.*

| | |
|---|---|
| $A, A^*$ | real and imaginary components of the amplitude of the vector potential |
| $a$ | radius of the bar |
| $B_r, B_z$ | real components of the magnetic field density |
| $B_r^*, B_z^*$ | imaginary components of the magnetic field density |
| $F$ | dimensionless frequency |
| $r$ | radius |
| I, J | coordinates of a mesh point |
| IZTUL | beginning of the magnetizing coil |
| IKTUL | end of the magnetizing coil |
| JTUL | radius of the magnetizing coil |
| R | mesh distance in the radial direction |
| $t$ | time |
| $\vec{V}$ | vector potential |
| $\vec{W}$ | amplitude of vector potential |
| $z$ | coordinate z |
| Z | mesh distance in the longitudinal direction |
| $\sigma$ | electric conductivity |
| $\mu_0$ | permeability of empty space |
| $\mu$ | relative permeability |
| $\omega = 2\pi * f$ | frequency |

*5.5 Radial field between the two halves of the primary coil*

The arrangement of coils was simulated according to Fig 6 and the distance W between two parts was varied. There were two equal parts of magnetizing coil simulated with a ferromagnetic rod in the middle. The radial component of the magnetic field strongly depends on this distance and on the frequency and on the gap between the secondary coil and the surface of the bar. The relative permeability of the ferromagnetic bar in the middle is also of decisive importance. All these data together are giving the necessary information to calculate the distribution of the magnetic field in the vicinity of the defective spot. From the computer calculations also the radial magnetic flux could be evaluated. On basis of these simulations we could construct a very sensitive apparatus for detection of surface cracks of both kinds in a ferromagnetic bar.

The most important issue is that the separation of the primary coils must not be too small. It must be big enough to “bring” the field from inside of the bar across the surface to the outside where the radial secondary coil can “catch” the flow lines emerging from the interior. Since both parts of the primary coil are as equal as possible the secondary coil acts as the differential arrangement of a pair of secondary coils. The only difference is that in this case there are not two induced voltages subtracted but the two parts of the magnetic field that are flowing in opposite direction.

It is interesting to simulate different geometry and different physical properties on the distribution of the magnetic field at different frequencies. The results are part of a project where the region around coils mentioned above will be investigated thoroughly.

### *5.6 Practical example*

We solved a simple rotational symmetrical case and here only one fourth of the whole cross-section is taken into account due to the symmetry according to Fig. 2. The length of one part of magnetizing coil is chosen 6*Z. The radius of the rod a= 6*R, the radius of the coil is 9*R, where R and Z can be chosen deliberately. The number of points in radial direction was chosen JKON= 30, the number of points in longitudinal direction IKON= 30, the points on the surface of the rod in radial direction JK= 7, the points on the surface of the coil in radial direction JTUL=10.

For practical investigations we used a ferromagnetic bar with the diameter of 27 mm, the length of one part of the primary coil was 50 mm and the diameter of the secondary coil was 20 mm. At the frequency of 5-7 kHz the arrangement of coils was extremely sensitive for the longitudinal and perpendicular surface cracks of the bar [7].

The calculations of the radial component of the magnetic field were performed also for the rod moving through the coils bearing uniform radial longitudinal surface crack with a good defined start and good defined end. All these calculations are part of a more complex project that is not yet complete.

## 6. Additional comments

Since there are in the abstract “*only main ideas given how to solve more complicated problems”* it is necessary to add some additional comments.

Suppose we have the arrangement of two magnetizing coils and a radial sensing coil as shown in Fig.5. If inside the arrangement a ferromagnetic bar with a thin surface radial crack is placed the distribution of the magnetic field around the beginning of he crack is not rotationally symmetrical any more. In this case the vector potential has three components depending on three spatial coordinates. There are 6 equations with 6 unknowns in each mesh point to be solved (three real and three imaginary components of the vector potential). However the corresponding difference equations can be written a similar way as above.

Since there is not just one component of the vector potential to be taken in account the number of equations to be solved is increased considerably. To keep this number within some reasonable limits the introduction of logarithmic coordinate system in the radial and the longitudinal direction is advisable. However this makes the difference expressions a little less transparent.

The boundary conditions are to be expressed in the same way as above. The only difficulty is how to find the corresponding difference equation for the boundary points lying in a corner and at the same time taking into account the nonlinearity of the lattice of mesh points. Some approximation formulas are available and it is the question of experience which one should be applied. Finally several thousands of linear difference equations with several thousand of unknowns are to be solved. According to our experience some relaxation method is to be used.

A simple principle was successfully used years ago [5]. It was started with some initial values for the unknowns at all lattice points. Then it was investigated which unknown at which lattice point was not chosen good enough. This was found by looking for the equation where between the left and the right side there was the biggest difference. The value of the unknown contributing the biggest share to this difference was corrected so that the difference equation was fulfilled i.e. that the left and the right side were made equal. After this correction the new search for the worst fulfilled difference equation was carried on and the correcting procedure was repeated again.

The whole procedure was repeated several thousand times and it was checked if the differences between the left and the right side of the difference equations were diminishing. At the beginning of the relaxation process the values of the vector potential at all lattice points were converging satisfactorily but after a while the convergence became very slow till it was nearly stopped. The values of the vector potential $A$ at specific lattice points started to vary around some middle value. We stopped the relaxation procedure and the 6 components of the magnetic field density $B$ were evaluated from the last evaluated results for vector potential.

Finally also the magnetic flux in the sensing coils can be evaluated and compared with the measured voltage being induced in that coil.

Now the sensing coil is a radial coil and the induced voltage is proportional to the difference in fluxes coming from the first and the second half of the magnetizing coil.

For the mathematicians such problem is interesting and they also have probably developed better procedures for solving the system of difference equations. For the research work in the field of NDT it is not necessary to calculate magnetic fields in detail at this stage. The most important task in the research work is first to connect the theoretical results with practical measurements and with practical use of the principle. Then a detailed analysis would follow.

## 7. Conclusion

The mathematical methods for solving Maxwell equations are an excellent tool to verify new ideas and when looking for new possibilities.

Using the arrangement of two equal parts of a primary coil and one secondary coil for detecting radial net flux it is possible to construct a device for detecting all kinds of surface cracks in ferromagnetic material. It would be possible to detect the precise position of defect along the circumference if there were more secondary coils placed around the bar. The idea of using method of two part primary coil and a secondary coil in the middle is very promising especially for the development of NDT devices for controlling ferromagnetic tubes.

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