1.8.19. DAMAGE DIAGNOSTICS IN A VERTICAL BAR HANDED ON THE ELASTIC SUSPENDER WITH CONCENTRATED MASS

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Using two natural longitudinal vibration frequencies we can determine both place and size of an incision in a vertical bar hanged on the elastic suspender and stretched by its own weight and the gravity of the load at the upper end of the bar. A stress-strain state in the limits of flexibility for a thin bar is under consideration. The incision is a model for damage in the bar, particularly for a lateral open crack.

Considering that the crack appears as a result of the growth of a small nucleus (not necessarily in the most stressed place) we assume that the incision might be located in any place along the length of the bar. In the case of the bars of finite length for detecting defects we can use changes in the natural frequency spectrum of flexural vibrations [1] or those in natural longitudinal vibrations [2]. In [3] the solution is given for determining the variable cross-sectional area on the longitudinal coordinate by the dependence of the bar’s free end movement on the perturbing force frequency. Paper [4] deals with the solution of inverse problems on longitudinal travelling waves in bars of finite length. We consider the stress-strain state of a straight bar with the upper end hanged on the elastic suspender with rigidity $c_1$ and stretched out by its own weight and the gravity of the load with mass $M$.

It is assumed that there is a short portion in the bar (small when compared to its total length) with a lesser cross-sectional area. This incision does not cause any bar bending and serves as a model for damage in the bar, particularly for a lateral open crack. Our goal is to determine the coordinate of this incision and its size in the approximation of plane cross sections. Within the bounds of this incision of comparatively short length $l$ and in its close proximity there is a complex spatial stress-strain state [5].

However, for the sake of simplicity we assume uniaxial tension-compression and also take no account of inertial forces. As shown by experimental data [6], on impact at the lower end, the average value of the longitudinal damping coefficient for the hanged bar with incision is 20% greater than the same coefficient for the bar without incision.

We examine the dynamic problem [2]. Parameter $m$ and coordinate $x_c$ are thus involved in the simplest model of incision. In parameter $m$ the ratio of the cross sectional area to the length of the bar $F/L$ is considered to be known. In case of the direct problem the ratio of the length of incision to its area is also known, the inverse problem necessitates the determination of this ratio. Variables $l$ and $f$ by themselves are not determined in the model [2]. As for the bar without incision ($m = 0$) and at $c_1 \to \infty$ from the equation $\cos \alpha L = 0$ natural frequencies are equal [2] to $\alpha L = (2k - 1)\pi / 2$ ($k = 1, 2, \ldots$) or $\omega_k = (2k - 1)a / 2L$.

The numerical solution to the frequency equation was made for the following parameters: $E = 2 \cdot 10^{11}$ Pa, $\rho = 7800$ kg/m$^3$, $L = 10$ m, $F = 0.01$ m$^2$, $M = 50$ kg, $c_1 = 10^7$ N/m. Sound velocity $a = 5063.6$ m/s. In this case the first, second and third natural frequencies of the bar without incision are $\omega_1 = 116$ rad/s, $\omega_2 = 1705$ rad/s, $\omega_3 = 3390$ rad/s, respectively.

Fig. 1 shows the dependences of parameter $m$ on circular frequencies of longitudinal vibrations $\omega_1$, $\omega_2$, $\omega_3$ (rad/s) at different $x_c / L$. These dependences for small
values of \( m \) are linear. Further analysis of the curves shows that it is quite difficult to determine parameter \( m \) from the first natural frequency. Fig. 2 gives the dependences of ratio \( x_c/L \) on circular frequencies \( \omega_1, \omega_2, \omega_3 \) of longitudinal vibrations at different \( m \). One can see the periodical dependence of circular frequencies of longitudinal vibrations \( \omega_1, \omega_2, \omega_3 \) on \( x_c/L \).

![Fig. 1. Dependence of parameter m on circular frequencies of bar’s longitudinal vibrations \( \omega_1, \omega_2, \omega_3 \) (rad/s) at different \( x_c/L \)](image1)

![Fig. 2. Dependence of \( x_c/L \) ratios on circular frequencies of bar’s longitudinal vibrations \( \omega_1, \omega_2, \omega_3 \) (rad/s) at different \( m \)](image2)

It is also correct to conclude that determination of the coordinate of incision from the first circular frequency presents difficulty because of accuracy. Big changes in the coordinate of incision and parameter \( m \) correspond to insignificant ones in the first circular frequency.

If we write the frequency equation for two frequencies of free longitudinal vibrations, using the obtained set of equations we can determine the coordinate of incision \( x_c \) and parameter \( m \). The performed research shows the possibility of determining the coordinate of incision \( x_c \) and parameter \( m \) by two natural frequencies of longitudinal vibrations. This work has been done under RFFR grant 08-01-97008-r_povolzhie_a.

References: