OPTIMIZATION OF A DESIGN AND PARAMETERS OF GEOMETRICALLY SIMILAR MAGNETO – INDUCTIVE TRANSDUCERS WITH THE OUTER FERROMAGNETIC SHIELD WITH USE OF A METHOD OF FINITE ELEMENTS

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Article is devoted to the principles of optimization and development of the primary magneto-inductive transducers for coating thickness gauges.

Magneto-inductive method of coating thickness measuring is based on determining the change in magnetic resistance of sub-circuit (the primary measuring transducer - ferromagnetic controlled component), which depends on the coating thickness $h$, on the amplitude or area of the EMF (Electro-Motive Force) induced in the measuring winding of the primary transducer, the exciting winding of which feeds the AC (alternative current) harmonic or pulsed low-frequency current $i(t)$. Generalized block diagram of magnetic induction coating thickness gauge is shown in Fig. 1.

![Generalized block diagram of magnetic induction coating thickness gauge](image)

Fig. 1. Generalized block diagram of magnetic induction coating thickness gauge

Primary magneto-inductive measuring transducer (PT), in the general case, contains several series-connected primary windings and series/parallel secondary windings, wound on a common or separate cores. The simplest, most common in practice, absolute PT consists of a primary $W_1$ and secondary $W_2$ windings, wound on a common ferromagnetic core. As a primary information, the magneto-inductive method involves the use of the amplitude or area of the EMF excited in the secondary winding — $e(t, h)$, depending on $h$.

Magneto-inductive transducers quality depends on:

1. The sensitivity of the PT, which defines measurement error $\Delta h$ and achieved (in accordance with this measurement error) range of measured thickness $h_{\text{max}}$;
2. The minimum possible diameter of the measurement area $\Omega_m$ on a flat surface for given $\Delta h$ and $h_{\text{max}}$;
3. Design response of the transducer (with a flat fixed measurement system, with a spring-loaded measuring system, "pencil"-type, etc.);
4. Mass, size and ergonomics;
5. Durability;
6. Temperature and time stability of the data;
7. Simplicity and ease of operation.

To ensure the required quality it is necessary to look for a solution that takes into account the interrelated indicators: \( h_{\text{max}}, \Delta h \) and \( \Theta_m \). For example, an increase \( h_{\text{max}} \), at the same \( \Delta h \), inevitably increases the size of the transducer (diameter), which entails an increase \( \Theta_m \).

Currently the most widely are absolute and differential primary transducers with an outer ferromagnetic shield with axial-symmetric magnetic field (Fig. 2).

Fig. 2. PT with an outer ferromagnetic shield (\( W_1 \) - primary winding, \( W_2 \) - secondary winding, \( W_k \) - compensation winding): \( a \) - an absolute with a radial arrangement of windings, \( b \) - absolute with a radial - axial arrangement of windings, \( c \) - absolute with an axial arrangement of windings, \( d \) - differential with an axial arrangement of windings.

Geometrical model of the absolute primary measuring transducer with an external shield is shown in Fig. 3.

Fig. 3. Geometrical model of the absolute PT with an outer ferromagnetic shield

In analyzing the model we take the diameter of the core \( \Theta_c = \text{const} \) (homogeneous smooth rod). \( \Theta_1 \) and \( \Theta_2 \) - outer diameter of windings \( W_1 \) and \( W_2 \), respectively, inside diameter of the shield - \( \Theta_s \). Wall thickness of the shield will take constant and equal to 0.6 mm; \( l_1, l_2, l_5 \) - "air" gap between the base and the beginning of the primary winding, the beginning of the secondary winding and the shield, respectively; \( H_1, H_2, H_c \) - the height of primary and secondary windings and core, respectively; \( R_{sp} \) - radius of the sphere contact surface of the core. Inside diameter of the shield \( \Theta_s \), in a first approximation, determines \( \Theta_m \) and \( h_{\text{max}} \), and the other geometrical characteristics determine its sensitivity. There are relationships of geometric characteristics of the PT, which would allow a minimum \( \Theta_s \) receive optimal in terms of sensitivity and range of controlled thicknesses \( h \) Insertion Flow-coupling of the secondary winding \( \Psi(h) \).
Flow-coupling of the secondary winding $\Psi_{12}(h) = \Psi_{12}(h = \infty) + \Psi_I(h)$.

In general, the excitation of the primary winding by harmonic current $i(t)$, as the primary information parameter of the magnetic induction transducer, you can choose the amplitude of EMF $e(t, h)$, induced by the secondary winding

$$e(t, h) = -\frac{d}{dt} (\Psi_{12}(h = \infty)) - \frac{d}{dt} (\Psi_I(h)) = e(t, h = \infty) + e_I(t, h)$$

For constant magnetic fields

$$\varphi(h) = 1 + \Psi_I(h) / \Psi_{12}(h = \infty) = 1 + M_I(h) / M_{12}(h = \infty)$$

is called relative coefficient of introduced mutual induction ($M_I(h)$ - coefficient of introduced mutual induction, $M_{12}(h = \infty)$ - coefficient of mutual induction). Ratio $(\Psi_I(h_1) - \Psi_I(h_2)) / (h_2 - h_1)$ will be called the sensitivity of the PT in the range of coating thicknesses $h_2 \ldots h_1$.

When measured set of influencing factors causes some deviation (scatter) of $N(h)$ at the output of secondary measurement transducer (ST) on the value of $\delta N$, because of the network, pulse and high-frequency interference, as well as its own internal noise, nonlinearities, time and temperature instability of analog - digital and digital - analog transducers, sampling frames and storage, microprocessor devices. In the first approximation we can assume that the magnitude of deviation $\delta N$ does not depend on the thickness of the coating being measured, and is a constant. Value sensitivity of the transducer and $\delta N$ determines $h_{\text{max}}, \Delta h$.

To calculate the sensitivity of the PT in the range of measured thicknesses necessary determine the coefficients of mutual induction for various $h$, which, in turn, requires determining the values of flow of the magnetic induction vector.

A number of analytical methods of calculating the un-shielded primary magneto-inductive transducers without ferromagnetic cores are developed. The basis of calculation methods is the representation of transducers windings as an equivalent to a single - turn windings. Picture of the magnetic field generated by the primary winding of the transducer is considered as a set of magnetic fields created by the system $j$ equivalent single-turn windings. Then one of the way, such as the mirror method, the Insertion flow-coupling $\Psi_{2j}(t, h)$ of the $j$-th winding is calculated.

While excitation of the primary winding by the harmonic low-frequency current, electric field of the eddy currents, arising in the core, shield, basis and electro-conductive coatings is negligible. Use as a primary information parameter amplitude $e(t, h)$, while analyzing the electromagnetic field, allows to consider the primary winding and core as a permanent magnet.

Analytic solution of the equations describing the field of shielded magnet, is a complex system of differential equations of the first and second order. Application of these methods is not possible because of the great mathematical complexity of obtaining equations.

At the present time to meet the challenges posed by the propagation of electromagnetic fields the numerical methods are widely used. An effective and widely used method is the method of finite elements. With two - dimensional problem space model is divided by straight or curved lines into separate pieces (finite elements) that are sufficiently small, but finite size. Finite elements do not overlap each other. Singular points of finite elements (in these points values of the unknown parameters are calculated) are called nodes or nodal points. Scalar magnetic potential of each finite element is a polynomial with constant coefficients within that element $\varphi^M = a_i + b_j x + c_j y$. The main
task of calculating using the finite element method is to determine the coefficients $a_i$, $b_i$, $c_i$. After finding the coefficients it is possible to calculate the magnetic potential at any point in space model. Forming the system of equations to calculate the field using finite element method can be performed by minimizing a certain functional.

Reference data, supplemented by boundary conditions, and the energy dependence lead to a system of algebraic equations, which allows to calculate the unknown coefficients of all finite elements. After determining $\phi^M$ anywhere in the field we can determine the magnetic field intensity, magnetic induction and other parameters.

It is proved that any boundary problem can be associated with some so-called variation problem - finding the function that minimizes the corresponding functional. It is necessary to solve two problems: to find the appropriate functional and find function, it is minimized. This method is based on the fundamental principle of least action.

For the purpose of calculation of static magnetic fields with the boundary conditions of the first type (Dirichlet conditions), the minimized functional is proportional to the stored magnetic energy in space:

$$W_M = 0.5 \int \mu \mu_0 H^2 dv,$$

where $\mu$ - magnetic permeability of the medium, $H$ - magnetic field intensity.

Since $H = -\text{grad} \phi^M$, then the functional to be minimized can be written as:

$$W_M = 0.5 \int \mu \mu_0 (\text{grad} \phi^M)^2 dv,$$

and desired (minimizing) function will be $\phi^M(\xi, \zeta, \eta)$, where $W_M\{\phi^M\} \Rightarrow \min$. As functional acts amount of magnetic energy, accumulated in all elements. Since the elements in contact with each other, they have common points. The energy of the elements is expressed through the magnetic potentials of these common points $W = W\{\phi_1, \phi_2 \ldots \phi_N\}$, where $N$ - number of points. Such values of the magnetic potentials of common points in which $W_M$ is minimal are calculated. Then the problem reduces to the formation and solution of algebraic equations in which unknown quantities are the magnetic potentials of common points of elements. The magnetic induction and magnetic field intensity are calculated using the found magnetic potentials. This method allows to determine the value of the Insertion flow-coupling $\Psi_{2j}(t,h)$ and coefficient of introduced mutual induction $M_I(h)$, required to calculate the sensitivity, for the primary transducers with complex geometry of the external and internal borders that have sub-domain models with different magnetic properties.

Under optimal geometric characteristics of the primary magneto-inductive transducer we mean those characteristics and their relationships at which the maximum sensitivity in the range of measured thicknesses and the lowest possible diameter of measurement area are reached.

In software products that implement the finite element method, we can formulate the following statement of the problem of calculation of the primary magneto-inductive transducer: two-dimensional, axially symmetric, stationary, in the general case nonlinear, with open borders.

When considering the model we make the following assumptions:

1. Model completely stationary (no time and temperature drift of the physical characteristics of the PT).
2. If needed, the winding of the PT can be replaced by a single ampere-turns or a set of single
ampere-turns.
3. There is no hysteresis in ferromagnetic parts of the model of the PT, the magnetization
characteristic of the material is linear, without saturation.
4. Relative permeability of the shield, core and base, respectively $\mu_s = \mu_c = \mu_b \approx 2000$
(corresponding to soft magnetic steels), the permeability of air and winding $\mu_a = \mu_w = 1$, all are
isotropic.

As boundary conditions for models of considered PT, assign the boundary conditions of the first
kind (Dirichlet conditions). In our setting this boundary condition is applicable to set zero normal
component of magnetic induction at the axis of symmetry and to specify the complete decay of the
field to the boundaries, conditionally infinitely distant from the PT. To consider the overall picture of the field of the PT and its changes while the measured coating thickness is increased, we choose the most convenient for the analysis physical quantities characterizing the magnetic field of the PT and its sensitivity, as well as the point of model space where the observations will be made.

To analyze this variant of the model of the transducer with the following basic relative to the
geometric characteristics: $\varnothing_c / \varnothing_s = 0.2; H_c / \varnothing_s = 1.25; l_1 = l_2 = l_3; R_{sp} = 2\varnothing_c$ (Fig. 4).

![Fig. 4. Alternative models of the PT and the point pattern analysis of the field](image)

Primary winding is represented as the coil, wound infinitely thin wire along the entire length of the
core, i.e. $H_1 = H_c - l_1$ and $\varnothing_1 \approx \varnothing_c$. We assume that the winding has a cross-sectional area $S_1$ and
number of turns $W_1$. The density of the excitation current of the primary winding $j = W_1 I / S_1$, where $I$
- the total current, $S_1$ – cross-sectional area of the winding.

The flow-coupling $\Psi_{12}$ is selected as a physical quantity to analyze the sensitivity of the transducer.
In his calculation:

1. The secondary winding with the number of turns $W_2$ decomposed into many single axially
symmetric secondary windings. Radii $R_j$ of the single secondary windings are taken to be: $R_1 \approx
\varnothing_c / 2; R_\Pi = (\varnothing_s - \varnothing_c) / 4; R_{III} \approx \varnothing_c / 2$. 
2. Single winding of the same radius are adopted as distributed uniformly in the height of the core and located very close together. Consequently, the calculated flow-coupling values of each individual winding is possible to construct the intrinsic function of changes in flow $\Psi_{ij}$ depending on the coating thickness $h$ and the height of the core $H_c$.

Initially, the calculation of flow-coupling $\Psi_{ij}$ of each single measuring winding is performed (with different values of relative thickness $h'=h/\varnothing_c$ for each of the radii $R_j = (R_I, R_{II}, R_{III})$ and the relative height $z'=z/H_c$ of the contact surface of the core).

Then, you calculate the function of the relative flow-coupling of single winding $\phi(h', R_j, z')$ for $z'$, changing in the general case, from 0 to 1. Calculations of the functions are performed for fixed values of $h' = 0; 0.1; 0.2; 0.3$ and radii $R_I$, $R_{II}$, $R_{III}$. In other words, we define the change of the relative flow-coupling depending on the changes of coating thickness of the single measuring windings of different radius at different heights from the base of the core.

Of particular interest is the function $\phi(h'=0, R_j, z')$, which determines the range of variation of the relative flow-coupling of the single secondary winding, depending on its position on the core (height) and radius. The maximum value of $\phi(h'=0, R_j, z')$ is obtained for a single measurement windings, located directly at the base ($z'=0$). It should be noted that in a physically realizable design of the PT it is impossible to place measuring winding at the base of the core, and there is always an air gap $l_2$, so the calculation of this dependence should be started at nonzero values of $z'$. The nature of change and absolute values of function $\phi(h'=0, R_j, z')$ for a series of measuring windings of different radius are virtually identical. Thus, since $z'=0.1$ the results of calculations for various $R_j$ differ by no more than 3%, while for $z'>0.5$ difference in calculation results do not exceed 2%. As a rule, in practice the gap $l_2$ corresponds to $z' = 0.1$. This gives us an opportunity further to consider only one of a number of single measuring windings, while assuming that the other ranks of the results obtained will be identical.

Calculations were performed only for the single measuring winding with radius $R_{II}$, so in terms $\phi(h', R_j, z')$ variable $R_j$ was lowered.

With the increase of the thickness of measured coating $h'$ the sensitivity of each single measuring winding is reduced, also the slope characteristics decreases. $\phi(h'=0.6, z')$ does not exceed 1.04 even for a single measurement windings at the base of the core. If $z'>0.5$, $\phi(h', z'>0.5) = 1$, which corresponds to the theoretically attainable maximum thickness measurement of coatings.

The calculations focused on the analysis of the influence of geometric characteristics of the primary transducer on the relative flow-coupling and its sensitivity. It was performed optimization of the relation $\varnothing_c/\varnothing_s$ with the location of primary and secondary windings.

According to the analysis prior to developing design magneto-induction transducers were divided into three groups:

1. Small-size transducers for measuring plating on small-size products. They are designed to measure coating thickness up to 300 microns (mostly non-ferromagnetic electrically conductive protective coating, applied by the electric, electrochemical or chemical method for products of different sizes and shapes). The most difficult to measure are the coatings, applied to small-sized products curvilinear shape with small radii of curvature of the surface;

2. Transducers of general purpose with the range of measured thicknesses is up to 2 mm (paint and other coatings, applied to products of relatively large size, with large radii of curved surfaces);

3. Transducers with a range of measured thicknesses up to 30 mm. Transducers of this group are designed to measure the thickness of bitumen, plastic, film, plastic, rubber, composites and
various heat-protective coatings on articles of large dimensions, with quasi-flat surfaces. Table 1 shows the best estimates of the relative values of geometric characteristics for groups of transducers.

Table 1. Optimum values of relative geometric characteristics for groups of transducers

<table>
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<tr>
<th>Relative characteristics</th>
<th>Transducer</th>
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<tr>
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<td>1 group</td>
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<tr>
<td>∅c/∅s</td>
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<tr>
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<tr>
<td>H/∅s</td>
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**Transducers of the first group**

Basic requirements to a group of transducers for measurement data coatings:

1. The minimum possible diameter of the measurement area and the small diameter of the shield ∅s
2. Reduced radius of the sphere of the core base as compared with the other groups of transducers (to reduce the error caused by "swinging" of the transducer during the measurement - "pencil" type of transducer);
3. Increased wear resistance of the core. The requirement relates to the need for measurements of galvanic coatings with high hardness.

Transducer of "pencil" type (without the outer spring-loaded shell) with a reduced radius of the sphere of the core reasons for the minimum possible impact of "swing" of the transducer during the measurement. The primary transducer is shown in Fig. 5.

![Fig. 5. Primary measuring transducer of the first group (a). Relative geometric characteristics of the transducer ∅c/∅s =0.4; Hc/∅c=4.1; Hs/∅s =0.2; l2/∅c=0.28; H2/∅s =0.6; Rsp/∅sp=0.5=min; ∅1=∅2=∅s. Relative flow-coupling \( \phi(h') \) measured from the relative thickness \( h' \) (b).](image)

The core of the transducer, covered with chrome 7 microns thick, is made of structural steel. It provides its high wear resistance and low probability of cleavage coating. The symmetry of the magnetic system enables measurements with a deviation of its axis from the normal to the surface to 4 degrees.

**Transducers of the second group**

Basic requirements to a group of transducers for measurement data coatings:
1. The diameter of the measurement area of about 5 ... 8 mm, allowing the measurement of products with a diameter of more than 10 mm;
2. The radius of the sphere of the core base of large diameter, providing a better sensitivity in the region of large thickness;
3. Increased wear resistance of the core. The requirement relates to the need for measurements of surface scanning in accordance with the latest techniques to ensure greater reliability of measurement results.

Transducer design, executed in accordance with the principles of optimization of the geometric characteristics presented in Fig. 6.

Fig. 6. Primary measuring transducer of the second group (a). Relative geometric characteristics of the transducer $\varnothing_c / \varnothing_s = 0.42$; $H_c / \varnothing_s = 1.7$; $l_s / \varnothing_s = 0.13$; $l_2 / \varnothing_s = 0.18$; $H_2 / \varnothing_s = 0.2$; $R_{sp} / \varnothing_{sp} = 0.7$; $\varnothing_1 = \varnothing_2 = \varnothing_s$. Relative flow - coupling $\phi(h')$ measured from the relative thickness $h'$ (b).

**Transducers of the third group**

Measurement of coating thickness on large quasi-flat products allows to design transducers with almost optimal geometric characteristics.

Basic requirements to a group of transducers for measurement data coatings:

1. The diameter of the measurement area must ensure the minimum possible edge effect;
2. The radius of the sphere of the core base, approaching the flat, providing a better sensitivity for large thicknesses.

To solve the problems of this group consider the construction of the transducer, performed in accordance with the principles of optimization of geometrical characteristics. The transducer has a spring - loaded outer housing, providing installation perpendicular to the surface of controlled items. The primary transducer is shown in Fig. 7.
Fig. 7. Primary measuring transducer of the third group (a). Relative geometric characteristics of the transducer $\frac{\varnothing_c}{\varnothing_s} = 0.45; \frac{H_c}{\varnothing_s} = 0.9; \frac{l_1}{\varnothing_s} = 0.09; \frac{l_2}{\varnothing_s} = 0.13; \frac{H_2}{\varnothing_s} = 0.08; \frac{R_{sp}}{\varnothing_{sp}} = 1.3; \varnothing_1 = \varnothing_2 = \varnothing_s$. Relative flow - coupling $\phi(h')$ measured from the relative thickness $h'$ (b).

On the basis of the calculations was developed by a new series of wear-resistant transducers for electromagnetic coating thickness gauges series "The Constant", cover the range of controlled thicknesses from 1 mm to 30 mm, providing the measurement error of 1 ... 2 percent of the measured values of thickness.