Introduction.

It is well known the use of guide waves in ultrasonic NDT. The fundamental property is the geometrical dispersion phenomena in which the phase and group velocities are depending of lateral dimension as plate thickness and the frequency or wavelength. For the case of plates there are symmetrical and antisymmetrical modes known as Lamb waves. These modes are solution of Lame equation where it is assumed time harmonic solutions and also free boundary conditions. Fig. 1 shows the dispersion curves for a phase velocity in a steel plate of 4.7 mm thickness that was used in this paper.

As we can be seen in the last figure there are several modes, where only the fundamental modes S₀ and A₀ have values at zero frequency. However higher order modes have cut-off frequencies. The previous results were obtained according to the Disperse software from the Imperial College UK. [1].

In Fig. 2 it is plotted the group velocity dispersion curves obtained with the same software and steel plate conditions.
The pulse propagation in plates can be represented by harmonic components in the context of Fourier transform. Hence each infinite harmonic component travels through a plate with a phase velocity according to the dispersion curves. Nevertheless a pulse can also be represented as a sum of “narrow band pulses” components, each travelling with a group velocity [2]. Then the propagation of pulses in guided waves is a complex problem because, in dispersion phenomena, both the phase and group velocities have frequency dependence and present different values each other. For the case of a classical ultrasonic transducer, the pulses excited can be considered as narrow band, then the centroide propagates at group velocity but according to the fact that the phase velocity has a different value relative to the group velocity, there is an internal movement inside the pulse travelling forward or reversal depending of the mode presented.

According to the previously expressed, in an experiment with two transducers working in transmission-reception mode, it is possible to make measurements of both the phase and group velocities, when the receiver is placed at different positions over a plate with a fixed transmitter position. For the group velocity measurements one follows the centroide of the pulse, meanwhile for the phase velocity one have to construct a time distance line following a point at a fixed phase. Moreno and Acevedo [3] show this procedure which is known as a phase velocity method (PVM) in pulses.

However the pulse propagation, it does not have only Lamb components. It is possible to observe a small amplitude component which travels at a head velocity that corresponds to longitudinal velocity [4].

A question arises depending on, if it is possible to use the PVM in pulse echo mode. The previous work only described the use of this method in pulse-transmission. Hence the objective of this paper is the use of PVM method in pulse-echo. An example of a steel plate is presented with a perpendicular notch over the surface.
For the excitation of Lamb waves a variable angle transducer has been used to obtain a specific mode through a Snell law, which relates the angle of plane incident waves (i.e. in acrylic). Viktorov [5] and Rose [6] explain a review of the theory for the excitation of Lamb and Rayleigh waves for these cases. Simulations are presented by the use of Finite Element Methods simulation (FEM) in a similar way as described by Castiangs [7].

**Experimental procedure.**

It was used a variable angle transducer from Krautkramer UWB 1-N at 1 MHz over a steel plate of 4.7 mm of thickness. The transducer was excited with a Panametrics Pulser-receiver 5900 PR, and the echoes were displayed with a digital oscilloscope connected to a PC. The Fig 2 shows details of the transducer and notch positions. This last one has a width close to 15 mm and 1 mm of thickness and aperture respectively.

Fig. 2. Transducer and notch details over the steel plate at initial conditions.
As can be seen in the previous figure the transducer was located at few centimeters from the notch. Different signals were obtained when the transducer is swapped over 5 positions with 5 mm of distance each other. At each position, time values of the first maximal and minimal were obtained according to Fig. 3 (left). The same Fig. 3 (right) shows the scale used in these experiments where a relative zero was situation at the edge of the plate. The notch was at 15.7 cm at the initial position and the transducer head was at 20 cm.

Fig. 3. (Left) Pulse time measurements. (Right). Plate geometry with notch and transducer positions. The arrow shows the swap direction of the transducer. Plate thickness 4.7 mm.

For the experiments two angles $\alpha$ of 40 and 60 degree were selected with the transducer. From the Snell expression (1) these correspond to 4247 m/s ($40^\circ$) and 3152 m/s ($60^\circ$) respectively.

\[ C_{\text{phase}} = \frac{C_{\text{acrylic}}}{\sin \alpha} \]  

According to Fig. 1 the velocities obtained by expression (1) seem to correspond to $A_1$ ($40^\circ$) and $S_0$ ($60^\circ$) respectively. Fig. 4 shows the results of Disperse software for the incident angle versus frequency for this plate. The angle selected corresponds approximately with de $S_0$ and $A_1$ modes at 1 MHz. There are some differences in the values on angle at 1 MHz, which are attributed to the error in angle transducer and steel elastic parameters.
Fig. 4. Angle in acrylic (Perspex) versus frequency in steel plate of 4.7 mm of thickness.

Results

Fig. 5 shows the results of simulation of pulse propagation in a plate with angle of $40^0$. This was obtained by FEM method and represents an $A_1$ antisymmetrical mode. The simulation was based according to boundary condition given by Viktorov [5].

Fig. 5. Cross section of a plate with $A_1$ mode, emitted from transducer at $40^0$.

Finally, fig. 6 shows an example of the signal obtained in these experiments. The arrow shows the echo from the notch with the main pulse at the beginning and a second echo which corresponds to the final edge of the plate.
Fig. 6. Echoes obtained in pulse echo experiments. The arrow shows the echo from the notch after the excitation signal. The second echo corresponds to the edge of the plate.

Table I and II show time measurements at different points of pulses according to Fig 3 (left). Details can be obtained from reference [3]. With these values phase velocity measurements were accomplished using the inverse of the slope in distance-time diagrams. At the bottom of each table it is possible to observe the velocity results that should correspond to the expression (1) (Snell velocity). Nevertheless as a matter of fact the case of Table II could have components of antisymmetrical modes. Fig. 1 shows that at 1 MHz, $S_0$ and $A_0$ are very close. Also the notch represents an antisymmetrical reflector so that it is expected and stronger influence in $A_0$ component than in the $S_0$ component. If we compare the velocities obtained in Table I and II with expression (1), we can observe that the Snell velocity is lower than the symmetrical case and higher than antisymmetrical case. The Snell velocity is a model that assumes many physical factors like a plane wave among others. Pulse propagations and echoes from a real transducer could be described at different ways as described by Rose [6] in his description of “source influence” using a normal mode expansion technique. The idea in this paper is to present another way for studying pulse echoes in Lamb waves.

### Table I. Distance and time ($\mu$seg) at $40^0$ angles. At bottom velocity results

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>17,1</td>
<td>15,3</td>
<td>19,6</td>
</tr>
<tr>
<td>205</td>
<td>19,4</td>
<td>17,6</td>
<td>21,9</td>
</tr>
<tr>
<td>210</td>
<td>21,8</td>
<td>20,0</td>
<td>24,0</td>
</tr>
<tr>
<td>215</td>
<td>24,1</td>
<td>22,3</td>
<td>26,3</td>
</tr>
<tr>
<td>220</td>
<td>26,3</td>
<td>24,5</td>
<td>28,6</td>
</tr>
</tbody>
</table>

| Velocity m/s | 4324   | 4317   | 4471   |
Table II. Distance and time (μseg) at 60° angles. At bottom velocity results

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>t₁</th>
<th>t₂</th>
<th>t₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>16.2</td>
<td>16.4</td>
<td>13.1</td>
</tr>
<tr>
<td>205</td>
<td>19.1</td>
<td>19.4</td>
<td>16.5</td>
</tr>
<tr>
<td>210</td>
<td>22.0</td>
<td>22.7</td>
<td>19.7</td>
</tr>
<tr>
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<td>25.3</td>
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<tr>
<td>220</td>
<td>28.7</td>
<td>29.1</td>
<td>26.4</td>
</tr>
</tbody>
</table>

**Velocity m/s**

|     | 3190 | 3132 | 3014 |

**Conclusions.**

Pulse propagation in guided waves is a very complex physical problem. Even more for the case of pulse reflection in defects. For the case of plate with Lamb dispersion laws it is possible to use the phase velocity method in pulse as a tool for this purpose. The values obtained show some differences relative to Snell velocities which is a consequence of the complex dispersion of reflected pulses.

**Future work**

In this moment some works are developing in the field of simulation in guide waves structures different of the case of plate. Semi analytical finite element method (SAFEM) is one of the directions of the research for the dispersion and propagation on pulse in non symmetrical cross section. Fig. 7. Shows an example of the software that can be used in plate and in more complex geometries like railroad case.

![Fig. 7. SAFEM software (in Matlab) for dispersion curves in non symmetrical cross section [8].](image-url)
References.

[1] http://www.me.ic.ac.uk/dynamics/ndt


