Traceable radiographic scale calibration of dimensional CT

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Abstract

Computed tomography (CT) is based on digital radiography. In the last 15 years the use of CT has been established as a coordinate measuring system for dimensional measurements. In order to reach a small uncertainty, correct scaling of the underlying radiography is required. Due to the finite volume of the X-ray source, source degradation, the finite thickness of the detector layer and additional mechanical deviations, a typical industrial cone-beam CT has relative scale deviations of $10^{-4}$ or more. Former work by the authors shows that, by using a thin grid-like structured metal layer, the magnification can be easily determined with a scale resolution better than $10^{-6}$. For this purpose, two radiographs are imaged in 180 degrees opposite directions on the rotary stage of the CT system. The harmonic average of the two magnifications is the magnification belonging to the rotary axis’ centre.

In this article, investigations on the development of a radiographic scale standard are presented. Thin metal foils with holes at equidistant grid positions were fabricated by means of different methods and calibrated on an optical coordinate measuring machine. Depending on the production process, specific contours arise that are imaged differently by the radiographical and the optical method. Based on statistical evaluations of the congruence of both results, the hole centres are determined to better than 1 µm. Further simulations allow an uncertainty to be specified for the radiographic scale determination that is traced back to optical calibration. The result is an SI-traceable determination of the radiographic magnification with a relative uncertainty of a few $10^{-6}$.

1. Introduction

Cone beam CT is normally described by geometrical parameters. These are usually the source to detector distance (SDD) and the source to object distance (SOD), see Figure 1. Both are used to calculate the geometrical magnification as ratio $M = \frac{\text{SDD}}{\text{SOD}}$. Practically, the source position is not exactly known for two reasons. First, the X-ray target is difficult to access inside a vacuum chamber, and second, it is, effectively, a virtual emission volume where the X-rays are originated [1]. In addition, the exact position of the absorbing detector material is not exactly known, as it is encapsulated from the ambience and the effective depth of penetration is spectrum dependent [1,2]. This implies that a cone beam CT’s magnification must be calibrated by a traceable reference standard, normally done by the supplier of the CT. In this procedure the magnification of the standard is measured over the travel path of the z-stage. From a parametric fit, the source and detector position can be concluded. In this contribution, the determination of the radiographic scale $S = \frac{P}{M}$ (in mm per pixel, $P$: detector pitch in mm) and its measurement uncertainty will be considered. Other contributions to the uncertainty of CT measurements – as surface finding, beam hardening effects, Feldkamp artefacts, 2D to 3D problems etc. – are not under scope.
2. Calibration of radiographic scale

The method applied here, was first introduced in a 2015 publication [3] and recently used for the systematic investigation of the spectral dependence of source and detector position in [1]. It is now realised using 50 µm thin metal foils with holes at equidistant grid positions as calibrated standards. These are aligned perpendicular to the z-axis of the CT and two radiographs are taken in 0° and 180° position of the rotary axis. The magnification is traced back to the SI unit of length by determining the calibrated mean grid distance (MGD) of a fitted equidistant grid. This measurement only takes less than five minutes and could therefore also be used for substitutional measurements before and/or after precision experiments. The following sections will show the equipment used, software and contributions to measurement uncertainty.

2.1 Route of traceability

Figure 2 shows the two necessary standards, the necessary equipment (boxes), specially developed software (rounded boxes) and the related uncertainty contributions (dashed lines).

Figure 2. Diagram describing the traceability of S. The measurand is the mean grid distance (MGD) determined with a Levenberg-Marquardt (LM) fit of the determined grid hole positions. The positions are sensed optically and radiographically, the uncertainty is exemplarily given for an experiment shown later.
boxes). Essentially the hole grid standard (HGS) is calibrated on an optical 2D coordinate measuring machine in backlight illumination mode. By substitution using a calibrated Zerodur® length scale at the same position, it is traceable to SI units. The resulting measurand is $\text{MGD}_{\text{opt}}$ (in mm) that is combined with the radiographic measurand $\text{MGD}_{\text{rad}}$ (in pixel) achieved in the CT system. The radiographic scale $S$ (in mm per pixel) is the ratio $\text{MGD}_{\text{opt}} / \text{MGD}_{\text{rad}}$. All relative uncertainty contributions arising from their determination add up quadratically because they are not correlated. The detector pitch $P$ is not relevant, and therefore eliminated. Table 1 lists the single uncertainty contributions including comments. Their values of the later shown measurement of an etched Invar® HGS with 40 $\times$ 40 holes and 1.5 mm nominal pitch are given in Figure 2.

<table>
<thead>
<tr>
<th>Uncertainty Contribution</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_L$</td>
<td>uncertainty of single length graduation marks</td>
<td>from calibration certificate, see section 2.4</td>
</tr>
<tr>
<td>$u_{\text{subs}}$</td>
<td>due to exchange of HGS and substitution from length graduation to hole grid standard</td>
<td>estimation, depends on exact procedure, see section 2.4</td>
</tr>
<tr>
<td>$u_{\text{opt}}$</td>
<td>optical sensing of hole grid</td>
<td>obtained from statistical consideration, see section 2.2 + 2.5</td>
</tr>
<tr>
<td>$u_{\text{thermal}}$</td>
<td>different temperatures during optical and radiographic measurements</td>
<td>depends on linear thermal expansion coefficient $\alpha$, see section 2.3</td>
</tr>
<tr>
<td>$u_{\text{rad}}$</td>
<td>radiographical sensing of grid hole</td>
<td>obtained from statistical consideration, see section 2.2 + 2.5</td>
</tr>
<tr>
<td>$u_{\text{cos}}$</td>
<td>cosine error due to alignment opt./rad.</td>
<td>perpendicular orientation to X-Y plane in CT and CMM, see section 2.3</td>
</tr>
<tr>
<td>$u_{\text{scale}}$</td>
<td>combined uncertainty of scale</td>
<td>$u_{\text{scale}}^2 = \sum_j u_{\text{single}}^2(M_j)$</td>
</tr>
<tr>
<td>$U_{\text{scale}}$</td>
<td>expanded uncertainty of scale (95 % coverage)</td>
<td>$= 2 \cdot u_{\text{scale}}$</td>
</tr>
</tbody>
</table>

2.2 Scaling of precision with grid size

To calculate the uncertainty the radiographically and optically determined $x$-scale $u_j$ ($j = \text{opt.}/\text{rad.}$), the relation to the respective uncertainty of the position determination of a single hole centre position $u_{\text{single}}$ must be investigated. This covers the Levenberg-Marquardt (LM) parametric fit of an equidistant grid into noisy coordinate data.

![Figure 3. MC-Simulation of size dependence of a 2D-coordinate array for error propagation. Left: 40 $\times$ 40 grid with variable noise. Right: variable grid size $N$ with constant noise (1000 runs).](image)
For that purpose, a practical generated \((x_{i,j}, y_{i,j})\) coordinate data set of \(40 \times 40\) points from a radiographical image (Fig. 4, right) was taken and pointwise white noise with standard deviation \(\sigma_{\text{single}}\) each in \(x\) and \(y\) was added. From this data set the LM-fit routine determined the least-squares best fit with 6 parameters: \(x/y\)-position, \(x/y\)-scale, azimuth rotation and trapezoidal distortion. The relevant parameter for scaling is \(x\)-scale. Figure 3, left, shows the resulting standard deviation of the \(x\) scale \(\sigma_{x\text{-scale}}\) of a simulation repeated ten times over variable input noise \(\sigma_{\text{single}}\). Interpreting the standard deviations as uncertainty contributions, it is found that the influence quantity \(u_{j,\text{single}}\) is proportional to \(u_j\). For the example explained in the next section (\(40 \times 40\) grid with 37.8 pixel mean grid distance and \(u_{j,\text{single}} = 0.012\) pixel) it produces a relative uncertainty \(u_{\text{rad}}/\text{MGD}_{\text{rad}} = 1.6 \cdot 10^{-6} / 37.8 / 1.41 = 3 \cdot 10^{-7}\). As two radiographs are used to determine the position of the holes, the uncertainty was reduced by square root of two. Figure 4, right, shows the results for different sized grids for given added noise \(\sigma_{\text{single}}\) with amplitude of 0.2 pixel. With increasing size (and hole number) perpendicular to the \(x\) scale it downscales with the square root as expected for independent experiments. With increasing size in \(x\)-scale direction, it downscales with the power \(-3/2\). Increasing grid size in both directions decreases with the square of the grid size, i.e. proportional to the number of points.

\[
u_j \sim u_{j,\text{single}} N_x^{-3/2} N_y^{-1/2} \quad .
\]

This can be understood intuitively as a gain proportional with the grid length in the scaling direction and additionally with the square root of the number of points. The advantage of the Monte-Carlo-simulation in Figure 3 is that function and stability of the software in use is checked and the real value is proven. Figure 3 right shows outliers to high uncertainty. This might be caused by instability of LM fits to local minima due to the large number of parameters.

For a given size \(L \times L\) of a practical design of an HGS – filling the complete radiographic image – the uncertainty \(u_{\text{rad},\text{single}}\) increases with an increasing number of holes due to decreasing hole size. Considering that the MGD is also decreasing and assuming identical grid distances in \(x\)- and \(y\)-direction \((N = N_x = N_y)\), the scaling of the relative uncertainty is:

\[
u_{\text{rad}}/\text{MGD}_{\text{rad}} \sim N^{-1/2} \quad .
\]

Practically \(N\) is restricted by the size of the hole pattern that must be found with pattern recognition software in the radiograph. Thus, the highest practicable value of \(N\) for a 2000 x 2000 detector, a grid size of \(N = 40\), was used for the HGS design.

\[2.3\ \text{Realisation of an optical calibratable hole grid standard for radiography}\]

To investigate X-ray spectrum dependence [1] standards were fabricated from 50 \(\mu\)m thick metal foils in aluminium, copper and Invar\textsuperscript{®}. The aluminium and copper foils were pressed between two plastic plates and drilled on a CNC machine. The Invar\textsuperscript{®} foils were etched. These foils are placed inside membrane boxes (see Fig. 4, left), whose front and back covers are spared to allow optical calibration. For better evenness, a 0.5 mm thin Makrolon\textsuperscript{®} plate is placed between the lower membrane and the foil. The flatness deviation of the best Invar\textsuperscript{®} foil (No. 2) is \(\pm 50 \mu\)m, the other foils have deviations up to \(\pm 100 \mu\)m. The design used for all measurements consists of a \(40 \times 40\) equidistant circular hole grid with a nominal MGD of 1.5 mm. In the etched Invar\textsuperscript{®} foils (see Fig. 4, middle) nine holes are left closed, so that orientation and position can be recognized even from magnified partial images.
The thickness of 50 µm is selected to ensure that no edge penetration effects in the radiographs are disturbing the circular contours and no beam-hardening is relevant. On the other side, the absorption becomes weak for thin foils, especially for high photon energy and for aluminium as material. With an integration time of one minute, a micro-focus source with 10 Watts of power and a CsI detector sufficient S/N ratios are obtained. Aluminium foils need a little longer integration time and careful “shading correction” in two steps with an additional reference image.

Thickness and flatness practically restrict the perfect orthogonal adjustment of the foil in the CT and on the optical coordinate measurement machine (CMM). In the CT, the foil is turned to 90° and 270° and adjusted to grazing incidence of the central X-rays. Repetitions with different users showed a reproducibility of better than 1/20° for the 50 µm flat foil. The optical CMM measures 3D coordinates and a fit plane is used as z-direction.

It is estimated, that the adjustments to the measurement axis in both, the CT and the optical CMM, are distinct and reproducible to better than 1/20° for a flat HGS. The cosine error is half of the squared angle (in rad) and can be between 0 and 4·10⁻⁷ with U-shaped distribution when considering taking the ratio for calibration. It is approximated as $u_{\cos} / S = 3·10^{-7}$ and normally distributed.

The HGS is preferably manufactured from Invar®, because the linear thermal expansion coefficient $\alpha$ is small and secondly the material (Fe64/Ni36) is radiographically comparable to several typical materials: steel, stainless steel, copper. The HGS is produced from a rolled and annealed metal foil. For the HGS $\alpha$ was measured (±0.2·10⁻⁶/ °C): in x-direction it is +1.8·10⁻⁶/ °C and exactly the value stated by the manufacturer, but in y-direction (transverse to surface micro-scratches) it is -3.8·10⁻⁶/ °C. Practically the temperature of the HGS is known to about ±0.2 °C for the optical and radiographical measurement, so that the relative uncertainty of the MGD in x-direction due to temperature effects is $u_{\text{thermal}} / \text{MGD} = 5·10^{-5}$, if the temperature difference is not greater than 1 °C and is corrected using $\alpha$.

A detailed description of the radiographical determination of the magnification can be found in [1]. It is based on pattern recognition of the holes in two radiographs in which the orientation has been turned by 180 degrees. The deviation chart in Figure 4, right, determined from a single radiograph, shows the vectors of hole centre deviation from...
the ideal position magnified by 200. A vector just reaching the next grid position in the chart has a length of 1/200 of the MGD of 1.5 mm, thus 7.5 µm or 0.189 pixel. The MGD in x-direction is 37.80705 pixel per grid unit and the actual standard deviation of the measured single hole position deviations is 0.06 pixel (in x- and y-direction). This deviation is rather small, as the unevenness of the foil can be observed as modulation, since the geometrical magnification depends on the distance from source to object. In the end, this is eliminated by using two radiographs in 180 degrees rotated orientation of the stage (for the 500× magnified result, see Fig. 6, left). The vertical line distortion is due to a defective detector column.

## 2.4 Optical calibration

For the optical calibration, a Werth VideoCheck® HA was used to sample eight overlapping images around every hole of the grid together with the camera position. A contour recognition software with superposition of the found contours was implemented in Labview® 2015. First an estimated circle is fitted roughly to the coordinates. Outlier contours from dust etc. are removed in seven iterations by removing the 10% most outlying points and fitting a new circle, respectively. In the end about 50% of the points are considered for fitting of the final circle, i.e. radius and centre. Depending on the production process outliers are removed in the inside (drilled holes with splints) or symmetrically (etched contour with crystal boundaries) in each iteration step. Typical samples and evaluated contours are shown in Figure 5. In the diagrams, the radial deviation is plotted over the angle of the full circle. Red dots are found contour points, black dots are considered for the best fit circle.

The substitution measurement was done with a Heidenhain Nano 3 line scale standard. It was measured in x- and y-axis orientation. The position marks were evaluated with a pattern recognition software, similar to the one used for the hole grid evaluation, and the coordinates calculated by addition of the camera position. Comparison of the distances of these coordinates to the calibration certificate of the line scale standard gives each a correction factor for the x/y-direction of the CMM. Dominant for $u_{subs}$ is the machine drift over the time period of two months that is estimated as $u_{subs} / L = 5 \times 10^{-7}$.

Figure 5. Optical contour evaluation. Upper row: drilled hole in copper. Lower row: etched hole in Invar®. Left: Images of one quadrant. Right: Radial contour plot from eight overlapping images.
2.5 Comparison of optical calibration and radiographical results

Figure 6 shows the deviation charts of the radiographical measurement (left) and the optical measurement (middle) of an Invar® HGS and its difference (right). Each of the 1591 points is the superposition of the real, but unknown, grid position and sensor noise – including differences in the definition of the “centre” of a fitted circle. It can be assumed to be statistical and isotropic. Thus, for simplification, in Figure 6 and in the following only the x-direction deviations are evaluated as mean standard deviation $\sigma$ of a single point. There are three independent measurands to calculate the two relevant uncertainty contributions of the optical sensing, the radiographical sensing:

$$u_{rad, single} = \sqrt{\frac{\sigma_{rad}^2 + \sigma_{opt}^2 - \sigma_{rad}^2}{2}}$$

$$u_{opt, single} = \sqrt{\frac{\sigma_{diff}^2 + \sigma_{opt}^2 - \sigma_{rad}^2}{2}}$$ (3)

![Figure 6. Comparison of hole pattern deviations, deviation vectors 500× magnified. Test Object is an etched Invar® foil with 1.5 mm MGD. Left: Radiographic images. Centre: Optical measurement. Right: Difference from both.](image)

For a 40 × 40 HGS with the given MGD the sensitivity of the LM fit on the single hole’s precision can be seen in Figure 3, left. A position uncertainty of one pixel produces 1/1000 pixel per grid unit contribution to the uncertainty in $S$. Thus, $u_j$ can be calculated from the measured $\sigma_j$ (see Fig. 6) and equation (3). Results of different measurements with different HGSs are listed in table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>MGDrad in pixel per g.u.</th>
<th>$u_{rad}/MGD_{rad}$</th>
<th>MGDopt in mm per g.u.</th>
<th>MGDopt corr. in mm per g.u.</th>
<th>$u_{opt}/MGD_{opt}$</th>
<th>$S$ in $\mu m / pixel$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invar® #2 etched</td>
<td>37.882572 20.1 °C</td>
<td>2.6·10^{-7}</td>
<td>1.5001390 X dir., 20.7 °C</td>
<td>1.5001404 20.1 °C</td>
<td>5.1·10^{-7}</td>
<td>39.59784(8)</td>
</tr>
<tr>
<td>Invar® #2 etched</td>
<td>37.882698 20.1 °C</td>
<td>2.4·10^{-7}</td>
<td>1.5001501 Y dir., 20.4 °C</td>
<td>1.5001391 20.1 °C</td>
<td>7.6·10^{-7}</td>
<td>39.59960(8)</td>
</tr>
<tr>
<td>Invar® #1 etched</td>
<td>37.882508 20.5 °C</td>
<td>3.2·10^{-7}</td>
<td>1.5000655 Y dir., 20.3 °C</td>
<td>1.5000558 20.5 °C</td>
<td>3.9·10^{-7}</td>
<td>39.59784(12)</td>
</tr>
<tr>
<td>Cu #1 drilled</td>
<td>37.901927 20.4 °C</td>
<td>10.1·10^{-7}</td>
<td>1.5006166 Y dir., 20.4 °C</td>
<td>1.5006064 20.4 °C</td>
<td>13.3·10^{-7}</td>
<td>39.59209(38)</td>
</tr>
<tr>
<td>Al #1 drilled</td>
<td>37.879869 20.2 °C</td>
<td>25.4·10^{-7}</td>
<td>1.4998765 Y dir., 20.6 °C</td>
<td>1.4998522 20.2 °C</td>
<td>41.8·10^{-7}</td>
<td>39.59561(65)</td>
</tr>
</tbody>
</table>

* repeated measurement after 12 minutes warmup of CT
** after one day more, with a different Invar® specimen and 0.3 mm shifted stage in y-direction
*** X dir. means: HGS is measured in X-axis orientation of optical CMM, measured 2 months before
For comparison, a corrected value of the optically measured MGD is given, where thermal expansion and scale error of the optical CMM is corrected. From that the radiographic scale \( S = \frac{\text{MGD}_{\text{opt}}}{\text{MGD}_{\text{rad}}} \) is calculated. Its expanded uncertainty is given in brackets as its last decimals – for Invar\(^\circ\) HGS #2 the single contributions are given in Fig. 2. When two experimental values of \( \text{MGD}_{\text{opt}} \) are available, the average is used. The given uncertainty contributions of the sensors are only one part of the budget – see section 2.1.

3. Conclusions and outlook

An uncertainty budget for the calibration of the radiographic scale of a cone beam CT under use of a grid hole standard is proposed. Best results were obtained with an etched Invar\(^\circ\) foil of 50 µm thickness in comparison to a drilled copper or aluminium foil – the uncertainty contribution of the optical and radiographic sensing process can be separated and quantified by a statistical comparison of a large number of holes. Interestingly the radiographic determination of hole positions is more precise than the optical (see Table 2) – the optical measurements might be more easily disturbed by dirt. Relative uncertainty improves with the number of holes in a square grid. For that reason, a higher number of holes (40 × 40) were used. Using a grid made from Invar\(^\circ\) foil an expanded relative uncertainty (confidence 95 %) of 2·10\(^{-6}\) can be reached and therefore a length of 60 mm is calibrated with an uncertainty of 120 nm. This requires careful control and measurement of the temperature during all steps in the measurement chain and, moreover, the individual determination of the thermal expansion coefficient and the near-term calibration of the optical CMM as a substitutional measurement. The measurement time for the two necessary radiographic images in the CT is only about five minutes, so that systematic and precise investigations of the accuracy and stability of the CT become possible. Many more investigations are necessary to describe the spectral dependence of the radiographic scale as recently published [1]. The next step is to enhance, develop and implement devices and software for its correction. In future, such grid hole standards could be used by manufacturers and users of CT for tracing back the radiographic scale \( S \) as a part of the traceability chain of dimensional CT measurements.

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References