Correct sizing of reflectors smaller than one wavelength

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Abstract

In the field of ultrasonic testing there are two key questions: Which defects can be found and – in the case indications are found – do they restrict the use of the part? Regarding both questions, the prerequisite is a method for defect sizing. Over the last decades sizing methods were established like DGS (Distance Gain Size) or DAC (Distance Amplitude Correction) for defects smaller than the beam profile. Those methods utilize the echo amplitude and provide results which are proportional to the defect area. However, those approximations are only accurate for defects larger than one wavelength even that experience shows it can be applied for slightly smaller defects.

With the progress of material technology and ultrasonic inspection the need to detect and size smaller defects is growing. Therefore, both for flat bottom holes and disc shaped reflectors the usability for small defects needs to be checked.

In this publication, it is investigated how to correctly size small defects below one wavelength. Utilizing a grid-based simulation method the echo signals of cylinder and disc shaped reflectors of various sizes are calculated. By properly choosing the simulation method and grid it is ensured that all physical wave modes are included in the simulation and that the discretization error is negligible.

A good correspondence between the simulation and classical defect sizing for defects larger than one wavelength is found. In the region between one quarter of a wavelength and one wavelength resonance effects are found, which result in classical defect sizing methods giving conservative results. In the region below one quarter of a wavelength classical DGS and DAC sizing leads to undersizing. This is discussed in detail and a formula for defect sizing is derived, which is applicable to small as well as large defects.

1. Introduction

Indication sizing using ultrasonics is usually performed by echo dynamic or area amplitude based sizing. For indications larger than the beam spread, echo dynamic sizing (sizing by probe travel, e.g. -6 dB drop method) is used and for indications smaller than the beam spread area amplitude based sizing methods are used (like DAC or DGS).

Area amplitude based sizing methods compare the reflection amplitude of an indication to the reflection amplitude of reflectors with known size, e.g. artificial reflectors like flat bottom holes (FBH) or side drilled holes (SDH). After a correction for different sound path lengths to the indication, the size of the indication is denoted in form of the equivalent reflector size. Different area amplitude based sizing methods are used depending on qualification of techniques on a location-by-location basis.
The most traditional methods used from the early days of ultrasonic testing employed Distance Amplitude Correction (DAC) based on calibration using multiple flat bottom holes machined into calibration blocks and some extrapolation based on the inverse square law.

Already in 1950, five years after the start of ultrasonic testing (1), Kinsler and Frey published their book about the fundamentals in acoustics (2) with a theoretical calculation of the sound field of a piston source in the far field. Seki et al (3) took this work a few steps further. Their work is the basis for all theory-based sizing methods.

In 1959 J. Krautkrämer published the Distance Gain Size (DGS or AVG) Method (4) which is a method widely used in Europe. The DGS diagram (see figure 1) gives the amplitude loss (gain) of the backwall and of different disc shaped reflector sizes (KSR – German: Kreisscheibenreflektor) and is plotted against the soundpath (logarithmic scale). This is the gain necessary to bring reflector echoes to the same screen height. The different curves within this coordinate system start with the backwall curve on the top (low gain values) and the different KSR sizes in decreasing order. In the farfield the backwall curve has a slope of 6 dB for doubling the sound path ($1/\left(s^2\right)$ dependency) and the KSR curves have a slope of 12 dB for doubling the sound path ($1/\left(s^2\right)$ dependency). The individual KSR curves are spaced by 12 dB for dividing the KSR size in half ($D_f^2$ dependency). With this diagram it is not only very easy to calibrate the equipment but also to size indications and to determine the sensitivity.

Equation 1 is the theoretical basis for the farfield information in the DGS diagrams, for the inverse square law, and for indication sizing using DAC with FBHs. For a monochromatic situation the gain $V$ can be derived analytically by integrating over the surface of a piston transducer (5, 6):

$$V = 20 \log_{10} \left( \pi^2 \frac{D_f^2 N^2}{D^2} \left(\frac{s^2}{4}\right) \right) \cdot 20 \log_{10} \left( \frac{D_f^2}{\lambda^2} D^2 \left(\frac{\pi}{4 \cdot s}\right)^2 \right).$$

(1)

Hereby $D_f$ denotes the diameter of the FBH or KSR, $s$ the soundpath, $D$ the effective diameter of the transducer, $N$ the nearfield, and $\lambda$ the wavelength.
This derivation uses a simplified analytic point source approach which assumes that the Kirchhoff approximation holds. The Kirchhoff approximation basically assumes that the scattered field of a reflector is generated by sources on the “illuminated” side with the inverse amplitude of the incident field, and no sources on the shadow side. Because of the Kirchhoff approximation, the echo amplitude of a reflector is proportional to its illuminated area. However, it is a good approximation only if the reflector curvature is small compared to the wave length. Obviously, this is the case for the inner area of a disc or FBH but not for its edge. This means that the approximation can be used only for larger reflectors, where the inside dominates over its edge. Consequently, the use is possible only for defects not significantly smaller than the wave length (6, 7, 8).

Both KSRs and FBHs are models for indications and are alike, except that FBHs extend in one direction all the way to the surface. This means that FBHs can also be seen as a model for KSRs with the benefit that they can be machined. Meaning the abstraction chain goes from real indications over KSRs to FBHs. Usually this difference between KSRs and FBHs is insignificant. However, for the indication sizes discussed in this paper they get important as section 3.2 shows.

With the enhancements in ultrasonic testing (9) and metallurgy the sizing of small defects becomes increasingly important. Therefore, the questions arise:

- How far can DGS, DAC and equation 1 be used?
- How to extend sizing for small reflectors?

Standard ultrasonic simulation tools use a point source approach and are restricted, like equation 1, by the Kirchhoff approximation. Experiments with small flat bottom holes are challenging and it needs to be questioned whether small FBHs are a good model for small KSRs.

2. Methodology

Goal of this work is to determine the reflection amplitude of defects (including defects smaller than a wave length) by means of a physically accurate simulation. By physically accurate it is meant that the simulation covers all physically allowed phenomena (and is not restricted by the Kirchhoff approximation). 3D-EFIT was chosen, as it simulates all physical wave modes possible in linear elastodynamics and allows to quantitatively assess numerical dispersion (10). Hence the only constraint is linearity, which is generally well satisfied in NDT.

2.1 Elastodynamic Finite Integration Technique (EFIT)

The basic equations of linear elastodynamics are the conservation of momentum (Newton-Cauchy equation), the strain rate equation, and the material equations. Putting these together the wave equations in their integral form specialized for isotropic, homogenous media can be written as follows (10, 11):

\[
\int \int \int_{V} \rho \ddot{v}(R,t) dV = \int \int_{S} \{\nabla \times \{\nabla \times \vec{v}(R,t)\}\} dS + \int \int \int_{V} h(R,t) dV \tag{2}
\]

\[
\int \int \int_{V} \rho \dddot{v}(R,t) dV = \int \int_{S} \dot{T}(R,t) \cdot dS + \int \int \int_{V} f(R,t) dV . \tag{3}
\]

Hereby \(T(R,t)\) denotes the symmetrical stress tensor, \(v(R,t)\) the particle velocity vector, \(h(R,t)\) the induced deformation rate tensor, \(f(R,t)\) the induced force density vector, \(s\) the compliance tensor, and \(\rho\) the mass density at rest.
EFIT discretizes the integral form of the wave equations in space and time so that they can be calculated numerically (The differential form of the equations could be solved with methods like the FDTD (Finite Difference Time Domain)). For the spatial discretization a dual orthogonal grid system is used as shown in figure 2. The time discretization of the velocity vector and the stress tensor are shifted by half a time step against each other. To model reflectors boundary conditions are introduced. For a KSR or FBH reflector stress free boundaries are placed on the surface of the reflector.

2.2 Accuracy and computability

Not only the physics of the wave propagation has to be simulated, but high amplitude accuracy has to be reached at the same time. And the simulation effort – in terms of number of operations and memory usage – must be manageable. First, the discretization must be fine enough to account for the smallest possible wave length (shear waves) at the highest relevant frequency (center frequency plus upper part of bandwidth). Secondly, the reflector discretization must be fine enough to accurately model its area as the interest lies in the relation between defect area and echo amplitude. And thirdly, the numeric dispersion must be small enough to ensure that the simulated signal shapes are not distorted by numerical effects.

Considering a typical inspection scenario (steel, 2 MHz, band width 100%, sound path 100 mm), the 3D-EFIT requirements on the grid are:

\[
\Delta x \leq \Delta x_{\text{min}} = \frac{\lambda_{\text{min}}}{G} = c_{\text{min}} \left( G \cdot f_{\text{max}} \right)
\]

\[
\Delta t \leq \Delta t_{\text{min}} = \Delta x \left( \sqrt{3} \cdot c_{\text{max}} \right)
\]

where \( \Delta x \) and \( \Delta t \) define the grid in space and time.

With an accuracy factor \( G \) of 10, a maximum frequency of 3 MHz (taking into the bandwidth), and the shear and long wave sound speed of standard steel, the grid must be 100 \( \mu \)m spatially and 9 ns temporal or finer. The required relative accuracy \( p \) of the reflector discretization is calculated from the ratio of defect area \( A \) over circumference \( U \):

\[
\Delta x \leq p \cdot \left( \frac{A}{U} \right) = p \cdot \left( \frac{D_f}{4} \right)
\]

For a KSR with \( D_f = 1 \) mm and a relative accuracy \( p \) of 2% the maximum grid size calculates to 5 \( \mu \)m / 0.45 ns, which is quite an ambitious requirement. And finally, the maximum simulated time is limited by the condition for avoiding numerical dispersion (10).
2.3 Utilization of reciprocity

For a sound path length of 100 mm and a limitation of the simulated volume to a small tube with 10 mm width the simulated volume would comprise $20 \cdot 2k \cdot 2k = 80G$ cells. With at least 9 float values per grid cell this would impose a memory requirement of 2,9 TByte of RAM. Without even considering the number of computations it is obvious, that a more economical way of calculation is needed.

As the interest lies in the far-field reflection coefficient of the defect only, the simulation of the sound propagation over the long distance can be omitted. Instead a plane wave is coupled into the EFIT simulation. This not only simplifies the simulation, it automatically normalizes the result to the incident wave amplitude.

In analogy to the method presented in (12, 13) Auld’s reciprocity theorem (ART) is used instead of simulating the propagation back to the probe. The ART allows calculating the reflection coefficient change between two scenarios based on the fields on a surface $S$ enclosing the defect. The reflection coefficient refers to the electrical signal of the transducer. Hence, if (1) a scenario with defect and (2) another one without defect is considered, the echo signal $E(t)$ of the defect can be calculated as (11, 12, 13):

$$E(t) = \text{FFT}_t^{-1} \left\{ \frac{1}{4 f(\omega)} \iint_S \left[ \frac{1}{2} T^{(2)} - T^{(1)} \right] \, ndS \right\}. \tag{7}$$

The layout of the simulated volume is illustrated in figure 3. The defect is in the center which imposes the stress-free boundary conditions. The defect is enclosed by the ART box, which serves to capture the input data for equation (7). Outside the ART box there is more simulated volume surrounded by a convolutional perfectly matched layer (CPML), which is optimized to absorb incoming energy with almost no reflections. The coupling plane, where the incident wave is introduced is just left of the CPML on the right side.

![Figure 3. Layout of the simulation volume (left) and example for a simulated reflection (right).](image)

The incident P-wave travels from right to left.

3. Results

For the investigation how far DGS, DAC with FBHs, and equation 1 can be used different sizes of FBHs and KSRs were simulated for an ultrasonic longitudinal wave with a wavelength of 2,95 mm (2 MHz wavelength in steel). The largest KSRs and FBHs had a diameter of 9 mm, the smallest 0,1 mm. Meaning the largest artificial
reflectors are clearly bigger than the wavelength. The smallest on the other hand are clearly smaller and should be a good indicator how to extend the theory.

3.1 KSRs

Figure 4 shows the simulation results for KSRs as red points together with the quadratic dependency on the diameter $D_f$ as a black dashed line (the expected dependency according to equation 1). In addition, figure 4 shows a cubic dependency on the diameter $D_f$ as a red dashed line.

In the results three regions can be identified:
- Between 1.5 ($\lambda/2$) and 9 mm the simulation results follow the known DGS theory. This is the region of geometrical scattering, where the Kirchhoff approximation holds.
- Around 0.7 mm ($\lambda/4$) the simulation results are higher. Those results are created by resonance effects and depend on the shape of the reflector.
- For defects smaller than 0.5 mm ($\lambda/6$) the simulation results follow the cubic dependency. The cubic dependency results of Rayleigh scattering (14) (not to be confused with the well-known fourth-power dependency of the frequency for Rayleigh scattering processes).

The following formula was found to match the complete simulation results well and is shown in Figure 4 besides the simulation results (the basis for this equation is a heuristic approach based on Bode diagrams):

$$V = 20 \log_{10} \left( \frac{\left( \frac{i D_f}{\lambda/4} \right)^3}{\left[ 1 + \frac{1}{Q} \left( \frac{i D_f}{\lambda/4} \right)^2 + \left( \frac{i D_f}{\lambda/4} \right)^2 \right] \frac{1}{16} D^2 \left( \frac{\pi}{4 \cdot S} \right)^2} \right).$$

(8)
Hereby $Q$ denotes the quality factor of the resonance effect. As the shape of a real reflector is unknown a conservative solution needs to be chosen by ignoring any resonance effects ($Q = 1$):

$$V = 20 \log_{10} \left( \frac{D_f^2}{\lambda^2} \frac{4D_f}{\sqrt{-16 \cdot D_f^2 + 4iD_f \lambda + \lambda^2}} D^2 \left( \frac{\pi}{4 \cdot s} \right)^2 \right).$$

(9)

The difference in reflectivity of large ($D_f^2$) and small ($D_f^3$) KSRs is caused by the relative size of an KSR in comparison with the wavelength. Figure 5 shows three snapshots (after 0.25, 0.5 and 1.1 $\mu$s) of the displacement at the front and at the back of the KSR for both a 3.0 mm $\varnothing$ and a 0.1 mm $\varnothing$. KSRs larger than $\lambda/2$ see in the beginning mainly a displacement of the front surface. KSRs smaller than $\lambda/6$ on the other hand see a displacement of both the front and back surface. This is caused by the wave enclosing the small KSR and moving it completely – instead of exciting only the top surface.

This is also the reason why large KSRs cause longer return signals than small KSRs. Small KSRs just move with the exciting wave where the excitation of large KSRs causes waves from the rim to the middle of the KSR.

![Figure 5. All images show in blue the displacement at the front of an KSR and in red at the back (the 4 or respectively 14 stationary dots left and right of the reflector show the starting point)](image)

- Top row: displacement of a “big” reflector with 3.0 mm $\varnothing$ (after 0.25, 0.5 and 1.1 $\mu$s)
- Bottom row: displacement of a “small” reflector with 0.1 mm $\varnothing$ (after 0.25, 0.5 and 1.1 $\mu$s)

For KSRs between $\lambda/6$ and $\lambda/2$ those different wave modes can interfere on the surface, shifting the maximum height of the echo signal.
3.2 FBHs

Often the DAC calibration based on FBHs is used instead of DGS or to measure DGS diagrams. As detailed in section 1 FBHs are like KSRs – however they extend in one dimension to the surface.

Figure 6 shows the simulation results for FBHs as red points together with the quadratic dependency on the diameter $D_f$ as a black dashed line (the expected dependency according to equation 1). In addition, figure 6 shows the cubic dependency on the diameter $D_f$ as a red dashed line.

![Figure 6. Quadratic (black dashed line) and cubic dependency on the diameter $D_f$ (red dashed line); simulation results for FBHs (red points)](image)

Like KSRs, FBHs follow the known theory between 1 and 9 mm. However, no resonance effects occur, and below $\lambda/4$ the simulation results are between the quadratic and the cubic dependency (close to the quadratic dependency). This is caused by the bore. Due to the bore extending in one direction the wave cannot enclose the complete FBH. Real defects have no bore extending in one direction and real small defects will get excited more like the small KSRs. Therefore, in the case FBHs are used for calibration or as a source for DGS diagrams a compensation is necessary for small reflectors.

4. Ultrasonic calibration for small defects

4.1 Conventional Methods

Figure 7 shows the undersizing of both conventional methods, DGS and DAC with FBHs. Both methods work up to a lower limit of $\sim \lambda/4$. Below that limit both methods show serious undersizing.
4.2 Extended DGS

By using equation (8) or (9) DGS can be extended for any indication size. For indications larger than $\lambda/4$ this results in the well-known 12 dB spacing, for indications smaller than $\lambda/4$ an 18 dB spacing has to be used. Figure 8 shows such an extended DGS diagram. It also shows the good agreement with the conventional DGS (up to $\lambda/4$) by including the conventional DGS diagram as dashed lines.
5. Conclusions and Outlook

The simulation results show that both conventional DGS and DAC (using FBHs) can be used up to a lower limit of a quarter of the wavelength. Below that limit both lead to serious undersizing. However, as shown in this paper, DGS can be extended to smaller wavelengths by changing from a quadratic dependency from the indication diameter to a cubic dependency. This extension is given both by an extended DGS diagram and an extended DGS formula. For calibration using flat bottom holes smaller than a quarter of the wavelength the reflectivity difference to disc shaped reflectors must be considered. For shear waves and dual element pitch catch probes it is expected that those results can be transferred.

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