Analysis of the uncertainty of measurement in pulsed eddy current signal evaluation

Yuriii KUTS, Anatoliy PROTASOV, Iuliia LYSenko, Oleksandr DUGIN

Department of Non-Destructive Testing Instruments and Systems, National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"; Kyiv, Ukraine
Phone: +38044 2049547, Fax: +38044 2048501; e-mail: j.lysenko@kpi.ua

Abstract
The probability of the pulsed eddy current NDT results depends on the measurement accuracy of the signal parameters of eddy current transducers. In particular, among such parameters are the frequency and attenuation ratio of harmonic oscillations, which arise in the pulse mode of the transducer. The process of evaluation of these parameters is accompanied by the emergence of a error random component, which is due to the noise effect of different nature, as well as the methodical component, which is determined by the features of the applied methods of digital signal processing and modes of obtaining experimental data. The standard uncertainty of measuring the frequency and attenuation ratio of the transducer signals obtained from signal's amplitude and phase characteristics is analyzed in the article. The characteristics are determined by the discrete Hilbert transform of the transducer signals. The performed analysis and the obtained formulas can be used for engineering calculations of numerical values of measurement uncertainties of attenuation and internal oscillations frequency and recommendation justification for choosing the time length of observation intervals and signal analysis. The efficiency of the obtained results is confirmed by an example of the signal processing of the multi-differential eddy current transducer in the pulsed excitation mode for the crack detection tasks. The method of determination and analysis of transducer signals information parameters is given. The experimental results of the influence estimation of the electrical conductivity value of the sample material and the cracks depth on the informative parameters of the signal are given. The general character of the dependence of the attenuation and the frequency of the transducer signal oscillations from the crack depth in the testing object is established. The sensitivity of the transducer signal informative parameters to the change in the crack depth is determined.

Keywords: pulsed eddy current, frequency and attenuation of a signal, uncertainty of measurement, Hilbert transform, signal evaluation

1. Introduction

The concept of uncertainty of measurements is actively used in metrological practice for quantifying the accuracy of the measurement results of physical values [1]. To date, its application in a number of technical terms has not been properly disseminated. This also applies to the field of eddy-current non-destructive testing (ECNDT) of materials and products. One of the reasons is the lack of techniques and examples of calculating uncertainties for various practical tasks.

The accuracy of the parameters measurement of the eddy current transducer (ECT) signals has a significant effect on the testing results [2, 3, 4]. In the case of pulsed excitation, the ECT signal under certain conditions is a harmonic dampening oscillation with proper parameters for it - the frequency of its own oscillations and the attenuation coefficient. The methods of digital signal processing allow determining the parameters of the ECT signals by estimating the amplitude and phase characteristics of these signals obtained by using Hilbert's discrete transform. The process of evaluating the informational parameters of the ECT signals is accompanied by the emergence of both the random component of the error, and the methodical component. The random component of the error is due to the effect of noise of different nature and the methodical component is determined by the peculiarities of the applied digital methods of signal analysis and modes of obtaining experimental data [5].

The purpose of this work is to obtain and analyze analytical expressions for estimating the standard uncertainty of the determination of the decrement and frequency of natural
oscillations of the ECT signal in pulsed mode in the tasks of control of conductive materials and products. The task was solved by model and field experiments, based on:

1) formation of signals of multidifferential overlay ECT (MECT) at pulse excitation mode;
2) determination and substantiation of the informational parameters of the ECT signals;
3) analysis of the standard uncertainty of the measurement characteristics of the eddy current transducer signals;
4) an example of the application of standard uncertainties at the analysis of model and experimental data of ECNDT.

2. Carrying out the experiment

2.1 Formation of multidifferential ECT signals

ECT signals were obtained using the developed ECNDT system. The Fig. 1 shows the structure of this system. It consists of an overhead multidifferential ECT (mdECT), a pulse signal generator (G), a digital oscilloscope (DO), a digital interface (DI) and a personal computer (PC) with original software (SW).

The formation of signals occurred during the scanning of the standard samples VSO-1 and VSO-2 by the MECT worked in pulsed mode. The samples were plates 5 mm thick, 100 mm long, 30 mm wide (Fig. 2). VSO-1 was made of steel grade St.20, and VSO-2 was aluminum alloy D16. The surfaces of both samples had three artificial defects imitating surface cracks 0.2 mm in width and δ = {0.2, 0.5, 1.0} mm in depth. The interval between cracks was equal. The roughness of the working surface did not exceed 1.6 μm.

The analysis of the received signals was carried out using the developed software, which is based on the algorithm of processing the ECT signals in the time domain using the discrete Hilbert transformation to obtain and further analysis of the amplitude and phase characteristics of the transducer signals [6].

The multidifferential ECT received an exciting pulse signal from the generator (U = 5V, the repetition period T = 0.1ms, duration τ = 50μs). The graphs of the ECT signals implementation in the testing object (TO) are in the form of damped harmonic oscillations [7]. ECT was a component of the measuring circuit, which formed an information signal in the form of voltage at the output. This signal was the ECT reaction to the front of the exciting pulse. The signal model is represented as an additive mixture of damped harmonic oscillations and Gaussian noise:

\[
s(t, \delta) = S_m(\delta) \cdot e^{-\alpha(\delta) t} \cdot \cos(2\pi f(\delta) \cdot t) + s_m(t), \ t \in (t_1, t_2)\]

Where: \(S_m(\delta)\) is the amplitude value of the information component of the ECT signal, \(\alpha(\delta)\) is the signal attenuation coefficient, \(f(\delta)\) is the frequency of the natural oscillations of the signal, \(t\) is the current time, \((t_1, t_2)\) is analysis time interval of the ECT signal, \(t \in (t_1, t_2)\).
\( s_{\text{ns}}(t) \) is the noise component of the signal, which was considered as the realization of a Gaussian random process with zero mathematical expectation and dispersion \( \delta^2 \).

It was investigated the influence of the crack depth on such informative parameters of the ECT signal as the frequency and attenuation of this signal [4, 7].

### 2.2 The algorithm of signal processing

The initial data for determining the standard uncertainty of the attenuation and the natural frequency of the ECT signals is the measurement equation that connects these parameters with the measurement results functionally. The ECT signal (1) is represented by its discrete analog \( s[j, \delta] \), obtained by signal sampling of frequency \( f_0 \gg f \). The processing and analysis of the ECT signal characteristics consisted of:

1) Definition of the Hilbert - image of the ECT signal:

\[
\hat{s}[j, \delta] = H[s[j, \delta]], \ j \in \mathbb{N}, \ N = \text{int}[(t_2 - t_1) \cdot f_0],
\]

where: \( H \) is the operator of discrete Hilbert transform, \( \text{int} \) is an integer part of the number;

2) Determination of phase and amplitude characteristics (PCS and ACS) of the ECT information signal:

\[
\Phi[j, \delta] = \arctg \left[ \frac{S[j, \delta]}{s[j, \delta]} \right] + L(s[j, \delta], s[j, \delta]),
\]

\[
S[j, \delta] = \sqrt{s^2[j, \delta] + \hat{s}^2[j, \delta]},
\]

where: \( L \) is the operator of the PCS deployment out-of-range of uniqueness of the \( \arctg \) function;

3) Smoothing of the function (3) by the method of determining the linear regression of Bartlett-Kenya;

4) Determination of the frequency of the ECT signal with the linear trend of the function (3):

\[
f_1(\delta) = \frac{\Delta \Phi[\delta]}{2 \pi \Delta T},
\]

where: \( \Delta \Phi[\delta] \) is accumulated phase of the signal ECT during \( \Delta T \in (t_1, t_2) \) obtained by the function of linear regression;

5) Application of the exponential approximation of function (4) to increase the accuracy of determining the decrement of the information signal. It was taken into account that the ACS region corresponded to the first periods of the ECT signal to increase the accuracy of the estimation of the exponential approximation coefficients;

6) Determining the decrement of ECT signals by the formula:

\[
\alpha(\delta) = \frac{1}{\Delta T} \ln \left( \frac{\hat{S}(t_1', \delta)}{\hat{S}(t_2', \delta)} \right),
\]

where: \( \hat{S}(t_1', \delta) = \hat{S}_1 \) and \( \hat{S}(t_2', \delta) = \hat{S}_2 \) are the value of the approximation curves of the ACS at moments of time \( t_1', t_2' \in \Delta T \).
2.3. Analysis of standard uncertainty of measurement of ECT signals characteristics

First, let us consider the phase characteristic of this signal (2) to analyze the standard measuring uncertainty of the ECT signal natural frequency. Taking into account the non-correlation of the components $s$ and $\hat{s}$ at the coinciding moments of time, the total standard uncertainty of the estimation of the PCS is:

$$u_\Phi = u \sqrt{\frac{\left(\frac{\partial \Phi}{\partial s}\right)^2}{2}} + \frac{u}{\sqrt{s^2(t) + \hat{s}^2(t)}} = \frac{u}{S(t)}, \quad (7)$$

where: $\frac{\partial \Phi}{\partial s}$ and $\frac{\partial \Phi}{\partial \hat{s}}$ are coefficients of influence, $u$ is standard uncertainty of estimation of the ECT signal values, $u = u_s = u_\hat{s}, u_s$ and $u_\hat{s}$ are standard uncertainties in the measurement of the ECT signal and estimation of the values of the Hilbert-image of the ECT signal respectively, $S(t)$ is signal envelope.

The standard uncertainty of estimating the natural oscillations frequency of the ECT signal is determined by the values of the PCS as:

$$f = \frac{\Phi_2 - \Phi_1}{2\pi \Delta T} = \frac{\Phi_2 - \Phi_1}{2\pi \Delta T} + \frac{L[s_2, \hat{s}_2] - L[s_1, \hat{s}_1]}{2\pi \Delta T}, \quad (8)$$

where: $\Phi_{l(2)} = \Phi_{l(2)} + L[s_{l(2)}, \hat{s}_{l(2)}]$.

Assume that $L[\ ]$ is obtained without a gross error, then the total standard uncertainty of the estimation of the signal natural oscillations frequency is determined by:

$$u_f = \sqrt{\left(\frac{\partial f}{\partial \phi_1}\right)^2 u_2 \frac{2}{\phi_1} + \left(\frac{\partial f}{\partial \phi_2}\right)^2 u_2 \frac{2}{\phi_2}}, \quad (9)$$

where: $\frac{\partial f}{\partial \phi_1} = -\frac{1}{2\pi \Delta T}$ and $\frac{\partial f}{\partial \phi_2} = \frac{1}{2\pi \Delta T}$ are coefficients of influence, $u_\phi_1$ and $u_\phi_2$ are standard uncertainties for evaluating the values of the PCS at times $t_1$ and $t_2$ respectively, which are determined by the formula (7):

$$u_\phi = \frac{u}{2\pi \Delta T} \sqrt{\frac{1}{s_1^2(t) + \hat{s}_1^2(t)} + \frac{1}{s_2^2(t) + \hat{s}_2^2(t)}}. \quad (10)$$

Taking into account that $s_1^2 + \hat{s}_1^2 = S_1^2$ and $s_2^2 + \hat{s}_2^2 = S_2^2$ are signal envelopes and $S_2 = S_1 e^{-\alpha \Delta T}$ we have:

$$u_\phi = \frac{u}{2\pi S_1} \cdot \frac{1}{\Delta T} \cdot \sqrt{1 + e^{2\alpha \Delta T}} \cdot \frac{1}{\Delta T}. \quad (11)$$

From the analysis of the obtained expression (11) it follows that the minimum error of frequency determination will take place for $\Delta T \approx 1.1089/\alpha$, and the minimum total standard uncertainty of estimation of the signal natural oscillations frequency is equal to:

$$u_{f_{\min}} = \frac{u}{2\pi \Delta T S_1} \sqrt{1 + e^{2.1089}} \approx \frac{u}{2\Delta T S_1}. \quad (12)$$
Considering, that the dispersion noise $\sigma_{ns}^2$ and quantization noise with the dispersion $\sigma_q^2$ render the greatest influence on the magnitude $u$, it can be determined by the formula:

$$u \approx \sqrt{\sigma_{ns}^2 + \sigma_q^2} = \sqrt{\frac{\sigma_{ns}^2}{2^{2n+2}} + \frac{S_n^2}{3}},$$

(13)

where: $n$ is bit ADC, $S_n$ is nominal value of ADC voltage.

The standard uncertainty in the estimation of the attenuation of the ECT signal (6) is equal to:

$$u_\alpha = \sqrt{\left(\frac{\partial \alpha}{\partial \Delta T}\right)^2 u_\Delta T^2 + \left(\frac{\partial \alpha}{\partial S_1}\right)^2 u_{S_1}^2 + \left(\frac{\partial \alpha}{\partial S_2}\right)^2 u_{S_2}^2},$$

(14)

where: $\partial \alpha/\partial \Delta T$, $\partial \alpha/\partial S_1$ and $\partial \alpha/\partial S_2$ are coefficients of influence, $u_\Delta T$ is standard uncertainty of time interval estimation (is determined by the stability of the clock pulse generator and can be reduced to a negligible small value), $u_{S_1}$ and $u_{S_2}$ are standard uncertainty of estimation of the ECT signal envelope.

Total standard uncertainties for estimating the values of signal envelopes $u_{S_{1,2}}$ can be determined as the result of indirect measurement taking into account the coefficients of influence:

$$\hat{\partial S}_1 = \frac{s_1}{\sqrt{s_1^2 + \hat{s}_1^2}}, \quad \hat{\partial S}_1 = \frac{\hat{s}_1}{\sqrt{s_1^2 + \hat{s}_1^2}}.$$

(15)

Thus, the total standard uncertainty of the estimation of the signal envelope $S_1$ is determined by:

$$u_{S_1} = \sqrt{\left(\frac{\hat{\partial S}_1}{\partial S_1}\right)^2 u_{S_1}^2 + \left(\frac{\hat{\partial S}_1}{\partial S_2}\right)^2 u_{S_2}^2},$$

(16)

Taking into account, that $u_s = u_{\hat{S}_1} = u$ and the expressions (15) we have:

$$u_{S_1} = \sqrt{\frac{s_1^2}{s_1^2 + \hat{s}_1^2} u^2 + \frac{\hat{s}_1^2}{s_1^2 + \hat{s}_1^2} u^2} = u.$$

(17)

The total standard uncertainty of the estimation of the signal envelope $u_{S_2}$ is determined similarly to (17). Then expression (14) can be represented as:

$$u_\alpha = \frac{u}{\Delta T} \sqrt{\frac{1}{S_1^2} + \frac{1}{S_2^2}} = \frac{u}{\Delta T} \sqrt{\frac{1}{S_1^2 e^{-2\alpha \Delta T}} + \frac{1}{S_2^2 e^{-2\alpha \Delta T}}} = \frac{u}{S_1 \Delta T} \sqrt{1 + e^{-2\alpha \Delta T}}.$$

(18)

The structure of the resulting equation (18) is similar to equation (11). So, the minimum error of determining the attenuation of the ECT signal will occur on condition of $\Delta T \approx 1.1089/\alpha$. The minimum total standard uncertainty of the signal attenuation is equal to:
\[ u_{\alpha, \text{min}} = \frac{u}{\Delta TS_1} \sqrt{1 + e^{2\cdot1.089}} \approx \frac{\pi u}{2\Delta TS_1}. \] (19)

The analysis of the errors of determining the frequency of the natural oscillation and the attenuation coefficient of the ECT signal can determine the optimal time for the analysis of this signal, which can significantly affect the probability of the testing results in the conditions of noise.

3. Simulation and Experimental Research and Results Discussion

3.1 Simulation Research

The signal-to-noise ratio varied from 0.1% (corresponding to 60 dB) to 10% at the time interval of analysis for simulated signal types (1). Figure 3a shows graphs of confidence regression regions for the values of calculated coefficients of ECT signals attenuation using the exponential trend of ACS (curve 1 is confidence regression region for individual calculated values \( \alpha' \), curve 2 - for average values \( \alpha' \), sample size \( n = 5 \)). It shows also graphs of confidence regression regions for the attenuation coefficients calculated without using the exponential trend of ACS (curve 3 is a confidence regression region for the individual values of the signal attenuation coefficient, curve 4 - for mean values).

Fig. 3b shows the graphs of confidence regression regions for the values of the calculated frequencies of the ECT signal natural oscillations using the linear trend of the PCS (curve 1 is the boundary of the confidence regression region for individual values of \( F \), curve 2 - for the mean values of \( F \)). Fig. 3b shows also the graphs of confidence regression regions for the frequencies of the ECT natural oscillations calculated without using the linear trend of the PCS (curve 3 is the boundary of the confidence regression region for individual values \( F' \), curve 4- for the mean values \( F' \)).

Fig. 3. Fragments of confidence regression areas for values \( \alpha' \pm u_{\alpha} \) (a) and values \( F \pm u_{F} \) (b)

According to the results of the simulation, the maximum relative standard uncertainty of the determination of the ECT signal attenuation coefficient for ACS and its exponential trend are 9% and 3%, respectively, and the frequency of the ECT signal natural oscillations for the PCS and its linear trend are 0.5% and 0.06%, respectively.
3.2 Experimental Results

Fig. 4a and 4b show the analysis results of the attenuation and frequency of the ECT signal for the two samples. The comparative analysis of these graphs shows that for the TO from different materials, a different dependence of the attenuation coefficient on the crack depth is observed. It can be used to evaluate the physical and mechanical parameters of the TO material [8]. The $f(\delta)$ dependence is close to the linear one and tends to decrease because of an increase in the crack depth $\delta$.

Fig. 4. Dependence of the attenuation (a) and frequency (b) of the ECT signal for the sample VSO-1 (curve 1) and VCO-2 (curve 2) and the limits of their confidence intervals (curve 3 and 4).

In the process of analyzing experimental results, it was used the condition for obtaining the minimum error of the determination of the ECT signal informational parameters.

4. Conclusions

It is shown that under the condition of pulsed excitation, the ECT generates a signal in the form of dampening harmonic oscillations. Informative parameters of such signals are the decrement and signal frequency. In this paper, it is obtained analytical expressions for standard uncertainties of these parameters derived from the amplitude and phase characteristics of the ECT signal.

From the analysis of the standard uncertainties of the attenuation factor and the natural oscillations frequency of the ECT signal in terms of their amplitude and phase characteristics, it was obtained and substantiated the conditions for choosing the time interval for the analysis of the ECT signals to minimize these uncertainties. It has been established that in order to obtain the minimum error of determining the attenuation coefficient $\alpha$ and the frequency of the signal natural oscillations, the time required for analyzing this signal is $\Delta T \approx 1.1089/\alpha$. 

\[ \alpha (s^{1} \cdot 10^5) \]

\[ f (Hz \cdot 10^5) \]

\[ \text{Crack depth, } \delta (\text{mm}) \]

\[ 0.2, 0.4, 0.6, 0.8, 1 \]

\[ 3.5, 4, 4.5, 5, 5.5, 8.05 \]

\[ 7.25, 7.35, 7.45, 7.55, 7.65, 7.75, 7.85, 7.95, 8.05 \]

\[ 0.2, 0.4, 0.6, 0.8, 1 \]

\[ 3.5, 4, 4.5, 5, 5.5, 8.05 \]
The results of the simulation showed that the accuracy of determining the attenuation coefficient and the natural oscillations of the ECT signal, taking into account the condition of obtaining the minimum error in determining the trends of the signal characteristics increases in ~ 3 times and ~ 8 times, respectively.

References