Evaluation of elastic constants in composite laminates with guided wave tomography

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Abstract

Post-impact structural performance of a laminated composite is a concern for aerospace and other advanced industries. Low-velocity impact may cause barely visible but considerable internal damage in structures, which is difficult to detect. To identify and accurately characterize such localized damages, efficient non-destructive evaluation methods are required. In this study, the potential of guided wave tomography to quantify the changes in material properties of a quasi-isotropic composite with a stiffness defect is investigated. The method reconstructs the velocities of guided waves by the inversion of ultrasonic signals captured by a transducer array around the inspection area. The resulting velocity maps are then converted to specific elastic constant maps by the material dependent dispersion characteristics of selected guided modes. The reconstruction is based on a full-waveform inversion algorithm and was implemented on the data obtained from finite element simulations. The sensitivity analysis showed that the Lamb mode $A_0$ is suitable for the determination of effective in-plane Young’s modulus and out-of-plane shear modulus. The reconstruction of these parameters can be decoupled by considering varying sensitivity of the velocity depending on material properties at different frequencies. Numerical results on a localized defect with reduced elastic constants illustrate the interest of this strategy.

1. Introduction

One of the most commonly encountered types of damage in composites is caused by impact. The resulting damage is a localized stiffness loss which usually appears in the form of matrix cracking, fiber failure or delaminations and can severely degrade the structural performance of the composite [1]. In case of low-velocity impact the damage can arise in the subsurface and is difficult to detect and challenging to characterize. The aim of this paper is to investigate the potential of ultrasonic guided waves for tomographic mapping of localized damages in plate-like composite structures. Maps of damage would enable to assess the structure and damage in detail compared to approaches that only detect and locate the damage.

Ultrasonic guided wave tomography offers a way to image spatially localized wave propagation data, by inverting the ultrasonic signals captured by transducer array around the inspection area. Previously, the major focus has been directed on the evaluation of the thickness of the wave-guide to help assessing corrosion damage in plates and pipes [2-5]. In this approach, the velocity map is initially obtained by some inversion
algorithm, which is then converted to thickness by using the dispersion characteristics of the selected dispersive mode. A similar idea has been applied to map other parameters of the wave-guide such as Young’s modulus [6]. In composite materials, the stiffness damage can be viewed as the deterioration of material properties which can be also related to the changes in velocities of the guided modes [7]. However, it is more challenging to obtain specific elastic constants from the velocity as the material is characterized by a number of constants which all can affect the velocity. For specific composite layups and guided modes the number of dependent parameters can be reduced [8] and a simpler problem can be obtained. Specifically, in quasi-isotropic laminate, the wave propagation in the plane can be considered as isotropic and the velocity of the fundamental Lamb modes primarily depends on an effective in-plane Young’s modulus and out-of-plane shear modulus [9]. In such a case, a simple inversion model based on an isotropic wave propagation assumption can be used and elastic constants from velocity with orthotropic wave propagation model [10] can be assessed. This paper will demonstrate this method on simulated damages with reduced elastic engineering constants.

2. Method

2.1 Velocity mapping using full waveform inversion

Elastic constants in the imaging area are obtained from the phase velocity map of a selected guided mode by using full waveform inversion (FWI) algorithm [4]. This approach uses a forward solver to predict the scattering of the guided wave through defects in an acoustic model, and an iterative inverse model to reconstruct the needed velocity map. The forward modeling is performed in frequency domain by using the finite-difference method. The inversion is based on a linearized least-squares problem to minimize the misfit between the modeled and measured wave-fields. Compared with other inversion algorithms which are limited by linear scattering, FWI accounts for higher order scattering, thus providing more accurate inversion results [11].

It should be noted that the implemented acoustic model is isotropic and two-dimensional and has some limitations to describe the guided wave interaction problems in a three-dimensional composite plate. Acoustic approximation limits the inversion accuracy and resolvability as the scattering from defects is different in these models [12]. Secondly, the wave propagation in the damaged area can be anisotropic. At larger impact energies, the shape of the damage becomes more elliptic towards the zones of maximal bending and the elastic constants are more affected in this direction. Isotropic assumption can be used when the induced damage is not so intense and its progression is more homogeneous in respect of the impact centre [13]. Finally, the acoustic model does not depict mode conversions. Mode conversions emerge when the guided wave encounters abrupt changes in waveguides. The present approach is applicable to describe smoothly varying defects where the extent of mode conversions is limited and 2D acoustic model is sufficiently accurate.
2.2 Effect of elastic constants on phase velocity

The sensitivity of different elastic constants on the propagation characteristic of the fundamental anti-symmetric Lamb mode $A_0$ is examined. The composite plate used in the present study is a 16-ply carbon fiber/epoxy (CF/EP) laminate with a quasi-isotropic stacking sequence $[0/45/90/-45]_{2S}$ and the thickness of 2.4 mm. Its homogenized engineering constants are shown in Table 1. The reference coordinate system used to define the laminate is set so that the fibers lie along the $x_1$ axis and the layer interfaces are normal to the $x_3$ axis. Phase velocity dispersion curves are calculated with the help of the homogenized model for an anisotropic material [10] over the range of frequencies from 40 to 100 kHz.

Table 1. Homogenized material properties of quasi-isotropic composite plate

<table>
<thead>
<tr>
<th>$\rho$ (kg/m$^3$)</th>
<th>$E_1/E_2$ (GPa)</th>
<th>$E_3$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$G_{13}/G_{23}$ (GPa)</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13}/\nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1540</td>
<td>41.5</td>
<td>9.71</td>
<td>15.9</td>
<td>3.61</td>
<td>0.307</td>
<td>0.294</td>
</tr>
</tbody>
</table>

Fig. 1 shows the dependence of the phase velocity of the $A_0$ mode from elastic constants in polar coordinates at 40, 70 and 100 kHz. The resultant normalized velocity is the ratio of the velocity which has been obtained by reducing specific elastic constant by 20% to the nominal plate velocity which is the same for all propagation angles. It can be seen that the velocity of the mode reduces and is primarily sensitive to the in-plane Young’s $E_1/E_2$ and out-of-plane shear modules $G_{13}/G_{23}$. The velocity depending on the variation of $G_{13}/G_{23}$ is isotropic while the velocity depending on $E_1/E_2$ is sensitive to the propagation direction. At 40 kHz the velocity reduction from both of the elastic constant pairs is similar. However, with increasing frequency the velocity becomes less sensitive to $E_1/E_2$ and more sensitive to $G_{13}/G_{23}$.

2.3 Extracting elastic constants from velocity

In Fig.1 it could be seen that the velocity of the guided mode $A_0$ mainly depends on four parameters $E_1$, $E_2$, $G_{13}$ and $G_{23}$. In order to evaluate elastic constants from a single reconstructed parameter which is the phase velocity, the number of dependencies must be reduced. To achieve this the model is further simplified. Firstly, it is assumed that the wave propagation in the damaged area is isotropic and the material remains orthotropic.
As mentioned before, this assumption can be used if the defect is mild and with smoothly varying material parameters. In this case the in-plane Young’s and out-of-plane shear modulus can be described by single parameters as $E_{in-pl} = E_1 = E_2$ and $G_{out-pl} = G_{13} = G_{23}$. Secondly, in Fig. 1 it could be seen that the velocity depending on $E_1/E_2$ is not constant in all propagation directions. This relationship is simplified by averaging the velocity over all propagation directions. This can have an effect on the conversion accuracy and will be discussed later.

Fig. 2a) shows the contour curves for normalized phase velocity depending on the variation of $E_{in-pl}$ and $G_{out-pl}$ at 70 kHz. $E_{in-pl}$ and $G_{out-pl}$ are normalized by nominal value. It can be seen that different combinations of $E_{in-pl}$ and $G_{out-pl}$ give the same velocity value. In order to get the unique combination, the velocity needs to be reconstructed at different frequencies as the sensitivity to elastic constant is different. In Fig. 2b) the normalized velocity is presented at 40, 70 and 100 kHz and they intersect when normalized $E_{in-pl} = 0.8$ and $G_{out-pl} = 0.8$. For the determination of the constants the velocities only at two frequencies are required. The concept of the method can be summarized as follows:

1. Velocity maps are obtained at two different frequencies using FWI.
2. Theoretical velocities are calculated for a range of $E_{in-pl}$ and $G_{out-pl}$.
3. Each pixel value of the map is processed by finding an optimal $E_{in-pl}$ and $G_{out-pl}$ pair corresponding to correct velocity at both frequencies.

4. Tomographic reconstruction

4.1 Measured data from finite element modeling

The configuration of the imaging problem is presented in Fig. 3(a). In this model, transducers were placed around the circular stiffness defect to transmit and receive waves through the plate. The array was 200 mm in diameter. The plate was modeled with the material properties given in Table 1. The defect is shown in Fig. 3(b) and was
modeled as a simple axisymmetric Hann-shaped defect in the plate centre [6]. The material properties in the defect area were modeled in two ways: 1) all elastic constants of the intact plate were reduced by 20%, 2) only elastic constants affecting the velocity \((E_1, E_2, G_{13}, G_{23})\) were reduced by 20%. The aim was to investigate the contribution of different groups of elastic constants on the reconstruction accuracy.

Wave propagation simulations were performed using Abaqus Explicit. 40 transducers were used in the array, with a 3 cycle Hann-windowed 70kHz toneburst as the input signal, exciting the A0 mode. A full matrix of scattering data (i.e. one measurement for each combination of sending and receiving transducers) was taken. The signals were transformed into frequency domain and the required frequency components for the inversion were extracted.

![Figure 3. Configuration. a) Plate setup with a central stiffness defect and a circular transducer array surrounding it, b) cross-section of the meshed plate with a through-thickness defect](image)

### 4.2 Results

Velocity maps were obtained from FE data at 55 and 85 kHz by starting from the homogeneous velocity model and using 30 iterations for the inversion. The maps were processed and converted to elastic constants \(E_{\text{in-pl}}\) and \(G_{\text{out-pl}}\), shown in Fig. 4. These results were obtained for a defect with reduced \(E_{\text{in-pl}}\) and \(G_{\text{out-pl}}\) by 20%. The defect can be clearly identified on both maps. However, the extent of the reduction of the constants in the damaged area has been predicted differently.

![Figure 4. Reconstruction results. Normalized elastic constants \(E_{\text{in-pl}}\) and \(G_{\text{out-pl}}\) for a quasi-isotropic laminate with a 30 mm Hann-shaped stiffness defect](image)
Cross-sections through the reconstructions are presented in Fig. 5. Additionally, the case in which all elastic constants have been reduced by 20% is shown. It can be seen that $E_{\text{in-pl}}$ is underestimated and $G_{\text{out-pl}}$ is reconstructed quite accurately. The reconstructions are slightly different when the reduction has been considered in all elastic constants. The impact is small on $E_{\text{in-pl}}$ but is larger on $G_{\text{out-pl}}$, showing an overestimation in the drop.

Figure 5. Cross-section of the defect. Normalized elastic constants $E_{\text{in-pl}}$ and $G_{\text{out-pl}}$ of a quasi-isotropic laminate with a 30 mm Hann-shaped stiffness defect

5. Discussion and conclusions

The performance of the $A_0$ Lamb mode for guided wave tomography to reconstruct elastic constants in quasi-isotropic composite plate has been evaluated. The method is based on velocity mapping using tomographic inversion and frequency dependent relations between velocity and elastic constants. It has been shown that in the studied plate the velocity mainly depends on the in-plane Young’s modulus and out-of-plane shear modulus. The performance to reconstruct out-of-plane shear modulus was good while the in-plane Young’s modulus was strongly underestimated. It is believed that the main reason for this is the inability of the used isotropic model to properly consider the anisotropy of the velocity, which is caused by the variation of the Young’s modulus. Also, it should be noted that the used defect was described by a simple model. The stiffness loss through the plate thickness is averaged, which does not allow to capture the full complexity of real damages. The exact form of the damage is usually much more complicated since the damaged material may become more anisotropic and the delaminations can develop between the laminas. Further studies are planned to improve the reconstruction accuracy by considering the effect of material anisotropy. In addition, the influence of the intensity of the damage on the reconstruction performance will be investigated.

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References