Evaluation of Electrical Conductivity of Metals via Monotonicity of Time Constants

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Abstract. This manuscript addresses the “classical” problem of estimating electrical conductivity of metals. The novel contribution of this work is the use of a new set of features, namely time constants of the source-free response in a pulsed eddy current testing (PECT) experiment. Time constants characterize the source-free response and increase monotonically with the electrical conductivity of the specimen. Time constants form a particularly important set of features because they do not depend upon the probing system and, hence, they are not sensitive to probe lift-off and tilting which are responsible for significant experimental errors.

Keywords. Electrical conductivity evaluation, pulsed eddy current testing, Nondestructive testing, monotonicity of time constants

1. Introduction

Nondestructive testing (NDT) measurements of the electrical conductivity of conductors is extremely common and useful in many applications. Two distinct families of methods for evaluating the electrical conductivity are available: eddy current and contact (two/three/four points) based methods.

The conductivity of a material can be obtained by measuring the bulk resistance and the physical dimensions of the material, usually of canonical shape [1]. The two-point methods require to apply two terminal/wires to both ends of the material. A voltage source applies a voltage across the material and an ammeter, which is connected in series with the material and the source, measures the current passing through the material. The bulk resistance is then the ratio of the applied voltage and the measured current. Three and four-point techniques are an improvement of this simpler two-points method. For instance, for the four-point probe, a current source forces a current through the ends of the material. A voltage meter simultaneously measures the voltage across the inner part of the material, thus providing the electrical resistance and, hence, the electrical resistivity. This approach avoids the contact resistances affecting the two-point technique and, hence, is widely used for measuring the resistivity of soil, semiconductor and metals. The four-point method is able to measure the electrical

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conductivity of metals with very low uncertainty [2], however, there are several restraining factors for its practical application [3], for instance, it requires good electrical contact with specimen. In addition, a large excitation current is required to produce a sufficiently high potential drop signal for highly conductive materials.

The conductivity of a material can also be determined by measuring the eddy current induced when the material is exposed to an external alternating magnetic field. In practice, a primary external field is usually produced by a coil carrying an AC current. The magnetic flux density due to the induced eddy current can be measured via the voltage induced in a pick-up coil or by using a field sensor (Fluxgate, Hall effect sensor, magnetoresistance sensors, etc.). Compared to the four-point technique, the eddy current based method requires no electrical contacts and, hence, is suitable for high speed operation. The conductivity of the specimen is determined via calibrated standards or by solving an inversion problem. Also, eddy current is a rather localized phenomenon and may reveal the local property.

Conventional eddy current methods use single-frequency magnetic field as excitation. For non-ferromagnetic materials, a well-designed eddy current system can measure the electrical conductivity with very small uncertainty [2, 4, 5]. However, conventional eddy current measurement is a function of many operation parameters, e.g. lift-off, frequency, sensor tilting, and requires extra effort to reduce these effects [6]. Pulsed eddy current method has recently emerged as a promising modality since it provides rich information in harmonic context. Current studies mostly investigate the time-domain signatures [3, 7] and the measured signal suffers from the effect of operation parameters. In [8], Ye et al. proposed using the decay time, which is insensitive to sensor lift-off, as a proper feature to evaluate minor changes in electrical conductivity of materials. This paper introduces a new set of features, the time constants, for estimating the electrical conductivity of a material. Time constants have been successfully introduced in Pulsed Eddy Current Imaging of defects in a conducting material ([9] - [12]). The present paper is the first contribution where time constants are exploited for estimating the electrical conductivity of a material. Time constants are attractive because they are monotonic in some sense w.r.t. the spatial distribution of the electrical conductivity [9]. In the specific problem addressed in this paper, in addition to being monotonic, time constants are also linear w.r.t. the electrical conductivity. This makes them very attractive for electrical conductivity measurement. Moreover, they depend only on the specimen geometry and the related conductivity. This set of features is completely independent of any operation parameters such as, probe lift-off, tilting, shape and nature, excitation waveforms, etc.

### 2. Monotonicity of time constants

In this paper, we consider an eddy current system comprising a planar coil and a controlled current source. The measurement is the voltage across the coil when a prescribed and known excitation current is injected into the same coil (Figure 1). Hereafter we assume linear and non-magnetic conductive materials.

The system is operated in the time domain. The source is periodically switched on and off. The free response of the system, i.e. the response of the system when the source is switched off, is characterized by a proper set of time constants:
where $\mathbf{J}$, $\mathbf{B}$ and $v$ represent the eddy current density, the magnetic flux density and the pick-up coil voltage, respectively, and the $c_i$'s are linear coefficients. It is also worth noting that, thanks to the Biot-Savart law and the Faraday’s law, all measured quantities in eddy current testing share same set of time constants. It can be proved that the $\tau_i$’s are real, positive, bounded and approach to zero when ordered in decreasing order, as usually done. Moreover, the time constants depend monotonically upon the electrical conductivity [9, 10, 11].

Figure 1. Schematic of an eddy current testing system.

2.1. Natural modes and time constants

Assuming that the conductive specimen is linear, spatially and temporally non-dispersive, isotropic and neglecting the displacement current, the eddy current density $\mathbf{J}(t)$ satisfies the following time dependent equation:

$$\int_D \mathbf{J}(\mathbf{r}, t) \cdot \sigma^{-1}(\mathbf{r}) \mathbf{J}(\mathbf{r}, t) dV = -\mathbf{\partial}_t \int_D \mathbf{J}(\mathbf{r}, t) \cdot \mathbf{A}_i(\mathbf{r}, t) dV - \mathbf{\partial}_t \int_D \mathbf{J}(\mathbf{r}, t) \cdot \mathbf{A}_j(\mathbf{r}, t) dV, \forall \mathbf{v} \in \hat{H}$$

where $D$ is the conducting domain, $\hat{H} = \{ \mathbf{v} \in H(div; D) | \mathbf{\nabla} \cdot \mathbf{v} = 0, \mathbf{v} \cdot \mathbf{n} = 0 \text{ on } \partial D \}$, $\mathbf{J} \in \hat{H}$ for any given $t$, $\sigma$ is the electrical conductivity of the conductor and $\mathbf{A}_i(\mathbf{r}, t)$ is the magnetic vector potential due to the driving currents and the operator $\mathbf{A}_j$, acting on the spatial coordinates only, is defined as:

$$\mathbf{A}_j[\mathbf{J}] = \frac{\mu_0}{4\pi} \int_D \frac{\mathbf{J}(\mathbf{r'}, t)}{\| \mathbf{r'} - \mathbf{r} \|} dV'.$$

The discrete equivalent of (1) is given by [13, 14]:

$$\mathbf{J}(t, \tau) = \sum_{i=1}^{\infty} c_i(t) \mathbf{J}_i(\mathbf{r}) e^{-\tau / \tau_i}$$

$$\mathbf{B}(t, \tau) = \sum_{i=1}^{\infty} c_i(t) \mathbf{B}_i(\mathbf{r}) e^{-\tau / \tau_i}$$

$$v(t, \tau) = \sum_{i=1}^{\infty} c_i(t) v_i e^{-\tau / \tau_i}$$
\[
\left( R + \frac{d}{dt} L \right) T(t) = s(t),
\]

where \( T(t) \) is the electric vector potential and

\[
R_{ij} = \int_D \nabla \times N_i \cdot \sigma \nabla \times N_j \, dV,
\]

\[
L_{ii} = \frac{\mu_0}{4\pi} \int_D \left( \nabla \times N_i(r) \cdot \nabla \times N_i(r') \right) \frac{dV'}{\|r-r'\|},
\]

\[
s_i(t) = -\frac{d}{dt} \int_D \nabla \times N_i(r) \cdot A_i(r,t) \, dV,
\]

and \( N_i \) is the \( i \)-th edge-element shape function.

The natural modes in pulsed eddy current testing are source-free solutions of (3) of the exponential type, i.e. \( T(t) = u e^{-\tau t} \). These solutions can be found by solving the following generalized eigenvalue problem [9]:

\[
Lu = \tau Ru.
\]

### 2.2. Monotonicity of time constants with respect to the electrical conductivity

It is worth noting that if we replace \( \sigma(r) \) with a scaled version \( \alpha \sigma(r) \), where \( \alpha \) is a positive constant, then (10) admits the same generalized eigenvectors, whereas the corresponding generalized eigenvalues (the time constants) are scaled by \( \alpha \), i.e.

\[
\sigma(r) \rightarrow \alpha \sigma(r)
\]

\[
u \rightarrow u
\]

\[
\tau \rightarrow \alpha \tau.
\]

This makes the estimation through time constants very attractive because, from the practical point of view, this requires calibrating the measurements system onto a single reference specimen only. This is not the case when a nonlinear relationship between the measured feature and the electrical conductivity is observed, as in “traditional” eddy current methods.

### 2.3. Estimation of time constants

Estimation of time constants \( \tau_i \)'s from the time domain waveform \( v(t) = \sum_{i=1}^{n} \beta_i e^{-\tau_i t} + \xi(t) \), where \( \xi(t) \) is the random noise, falls into the category of exponential analysis. In [15], Istratov gave a comprehensive review on available algorithms for extracting the exponential depending on how many exponential terms are subject to find (single exponential, multi exponential and exponential spectrum).
The problem of estimating time constants is naturally classified as multiexponential analysis. In this study, an algorithm modified from the Laplace-Padé approximant approach is exploited to estimate the time constants. The original Laplace-Padé approximant approach was presented in [16], which renders the benefit that it does not require any \textit{a priori} knowledge of the number of exponential terms. A modified approach tailored for estimation of time constants is presented in [12]. This approach takes into account multiple measurements sharing same set of time constants and introduces new stopping criteria accounting for the noise level and the presence of non-physical (negative or complex) time constants.

3. Validation

A numerical model was built to calculate the time constants of a square plate (figure 1). A circular coil above the center of the plate was used to produce the primary field and to measure the response produced by the induced eddy current density.

The width, length and height of the plate is 76.2 mm, 76.2 mm and 3 mm, respectively. The numerical model was discretized into a 30×30×5 mesh. The cross section of the circular coil is 1mm×1mm and its average radius is 5.5 mm. The lift-off between the coil and the plate is 0.5 mm.

The applied waveform was a train of square pulses of duration 1ms and duty cycle of 50%. We assumed the conductive plate to be homogeneous, and the baseline conductivity was $\sigma_0 = 5\times10^6$ S/m. The actual conductivity of the specimen varied from -10% to 10% of this baseline value. The source-free responses (pick-up coil voltages) corresponding to this range are given in figure 2(a). Time constants are numerically calculated according to (10) and are showed in figure 2(b). As expected from the theory, the time constants are linear with the electrical conductivity of the specimen.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figures.png}
\caption{Voltages at the pick-up coil (a) and corresponding time constants (b) as a function of the electrical conductivity. The 2\textsuperscript{nd} and 3\textsuperscript{rd} time constants are identical due to the geometrical symmetry of the plate.}
\end{figure}

The transient signals in figure 2(a) were sampled at discrete time $t = k\Delta t$ where $\Delta t = 2.5\times10^{-7}$s. A total of 2000 samples per waveform were collected (1ms×50% duty cycle/\Delta t). Assuming there is no noise on the measured waveforms, the first three time constants extracted from the signals by the modified Laplace-Padé approximant approach [12] are presented in figure 3.
Assuming an additive noise of about 1% of the peak of the signal amplitude, we performed the extraction of time constants on the signal obtained after averaging a total of 10,000 waveforms. Given 1ms for measuring an individual waveform, the total data collection time is 10s for 10,000 waveforms, thus making the average process feasible. The estimated time constants are presented in figure 4.

3.1. Sensitivity to lift-off

In principle, time constants are only a function of the electrical conductivity and geometry of the specimen. Time constants are independent upon probe lift-off and tilting, shape and nature of the probe and excitation waveforms. Despite of this, we found a weak dependency of the time constants upon the lift-off. There are two possible reasons for this: (i) when changing the lift-off, a different subsets of time constants is observable and/or (ii) the non-ideal performances of the algorithm (Laplace-Padé approximant) used to estimate the time constants.

The motivation underlying the first point is that with a single, or a limited number of measurement coils, only a subset of time constants can be observed. However, this
point can be easily investigated by considering the energy of the individual terms appearing in (3) as a function of the lift-off. The energy carried by each natural mode is given by:

$$E_k = \int_0^\infty \left( c_k e^{-\tau_k t} \right)^2 dt = \frac{1}{2} \tau_k c_k^2.$$  \hspace{1cm} (12)

Figure 5 illustrates the energy of the first 20 natural modes with different coil lift-offs. As aforementioned, when using a single pick-up coil, only a subset of time constants is observable, as appears from Figure 5. However, the subset of observable modes does not depend on the lift-offs, for the typical variations (from 0.1mm to 1mm) considered in ECT, as the peaks in the energy occur at the same positions. Therefore, the sensitivity of time constants w.r.t. lift-off is due to the extraction algorithm.

Eventually, we compared the performances in terms of sensitivity to lift-off of the proposed approach with a “traditional” eddy current methods based on time harmonic operations. Specifically, Figure 6 compares the changes in measured time constants and the changes in coil impedance with different lift-offs. Results indicate that extracted time constant is much less sensitive to lift-off if compared to the traditional approach.

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Figure 6. Changes in measured quantities with lift-off. (a) first time constant, relative difference = 8.2%. (b) second time constant, relative difference = 7.5%. (c) coil impedance, relative difference = 43%.
4. Conclusions and future work

In this paper, the monotonicity of time constants is exploited to measure the electrical conductivity of conductive materials. Moreover, we exploited a specific feature that time constants are homogeneous function of order one with the electrical conductivity of the specimen. This makes the estimation of the electrical conductivity through time constants very attractive because it requires a single reference conductive specimen to calibrate the measurement system.

Time constants depend on the electrical conductivity and geometry of the material only. They are completely independent of the type, shape, lift-off and tilting of the probe, as well are independent of the excitation waveforms. Despite of this, the intrinsic limit of the method to extract the time constants (Laplace-Padé approximant approach) makes the estimated time constants weakly depending on the lift-off of the probe. However, the method resulted to be much less sensitive to lift-off variation if compared to conventional frequency domain eddy current approaches.

References