Investigation of fracture processes using moment tensor inversion technique

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Abstract

The development and progress of fractures is accompanied by the radiation of acoustic waves due to microcracking. In the transient waveforms of acoustic emissions a huge number of information about fracture processes and the state of the material is inherent. Signal-based analysis of acoustic emissions allows, amongst other studies, for a fracture mechanics based investigation of failure.

From the spatial distribution of seismic energy, an inversion on a system of equivalent forces – the so-called seismic moment tensor – is performed. The inversion is simplified by the assumption of a point source and the neglection of near field effects, which means that the source volume is small compared to the distance between source and receiver. With the help of the moment tensor, arbitrary and complex source mechanisms can be described. For an interpretation of the results, the moment tensor is decomposed into simple fracture modes on the basis of an eigenvalue analysis. Furthermore, the appropriate radiation pattern of particle motion of the moment tensor is plotted on a sphere enclosing the source, revealing a 3D-visualization of the stress field. These results and additional data from further investigations can be combined in a comprehensive representation of the failure.

Basic principles of moment tensor inversion and some examples of experimental data and their analysis will be presented. The results will be discussed with regard to the influence of near field effects.

Keywords: moment tensor, decomposition of MT, near-field

Introduction

The investigation of failure mechanisms is an important task in the field of concrete technology. With the help of signal-based acoustic emission technique damages in concrete, such as concrete matrix cracking or debonding of reinforcement, can be registered and localized. Furthermore, an analysis of the wave field emitted during microcracking allows for a fracture mechanics based interpretation of failure. With this background, the concrete composition and the structural design can be optimized during the period of development and planning, and after completion, the quality and the state of the structure can be evaluated and monitored.

The concept of the moment tensor as a physical representation of an earthquake was invented in the early 1970's [e. g. GILBERT, 1970] for a detailed analysis of fracture processes and the role of non-double couple mechanisms. The moment tensor as a mathematical description of equivalent forces and moments in a point source, is very well suited to investigate source processes with a better resolution. Using an eigenvalue analysis, the source mechanism is decomposed into basic models of fracture mechanics [JOST & HERRMANN, 1989].

A complete inversion on the seismic moment tensor implies a broad knowledge of the source time function, the size of the source, the Green's functions of the medium and the displacement at the point of observation [AKI & RICHARDS, 2002; UDIAS, 1999]. Since the mathematical description of this inverse problem is extremely complex and the transfer functions of various components within the travel path of seismic energy is underdetermined, simplifications are performed to calculate moment tensors of seismic sources. The acoustic waves are assumed to be observed in the far-field of a time invariant point source. The onset of the analyzed wave phases must be well determined with a reasonable signal to noise ratio.

In these studies, a hybrid moment tensor inversion approach introduced by ANDERSEN [2001] and a relative inversion method by DAHM [1993] were used. Results of a splitting test of a concrete cube are shown. Tensile stress generated a mode I failure along a well defined damage zone. In another experiment – the pull out of a ribbed steel bar out of a concrete cube – failure is accompanied by mode II failure.

Theoretical Principles

In the following, the principles of the concept of the moment tensor are summarized briefly. A detailed derivation of the displacement field of a general elastodynamic source within a volume can be found in the literature (e. g. AKI & RICHARDS [2002], BEN-MENAHEM & SINGH [1981] and JOST & HERRMANN [1989]). To simplify the forward problem, the damage zone is assumed to be small compared to the distance between source and receiver and the source time function is approximated by a delta-pulse function. Such a point source can be represented by the physical model of the moment tensor **M**. In the common form the displacement **d** at sensor position **x** at time *t* can then be calculated from the convolution of the Green's functions **G** and the moment tensor **M**

$$\mathbf{d}(\mathbf{x},t) = \mathbf{G} * \mathbf{M} \quad (1)$$

The Green's functions are representing all propagation effects of the medium on elastic waves.

M is represented by the scalars M_{kj} with the unit of a moment of force. Additional external forces are neglected. This symmetric 3x3-tensor is containing six independent elements:

$$M_{kj} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \quad (2)$$

The physical meaning of the moment tensor elements is illustrated in figure 1. The components of the moment tensor can be thought of dipoles (see the joints) oriented in the three spatial dimensions (columns of M_{kj}) on which ends forces (see the arrows) act in the three spatial dimensions (rows of M_{kj}). On the diagonal, the axes of the joints are parallel to the forces. For the other elements, the forces lead to a torque around the axis perpendicular to the plane containing the force and the dipole [AKI & RICHARDS, 2002].

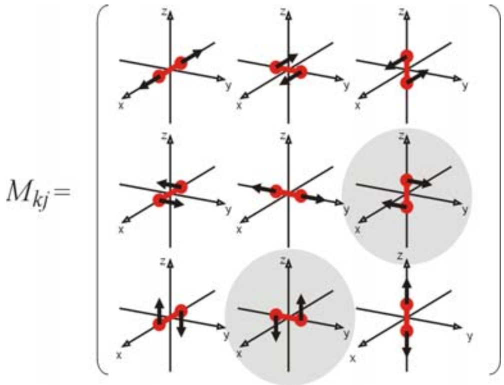

Figure 1: The elements of the moment tensor as dipoles in the spatial dimensions and the equivalent forces, which lead to moments of force.

The symmetry of the tensor yields a total zero moment of forces. This leads to the model of the double couple. In figure 2 the double couple concept is illustrated for the two highlighted elements M_{yz} and M_{zy}.

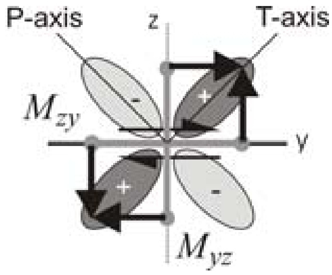

Figure 2: The p-wave radiation pattern of a pure shear event with the corresponding pair of a double couple. The symmetry of the tensor yields a zero total torsion.

The two elements M_{yz} and M_{zy} build a double couple, which leads to a pure shear failure when the stress in the material exceeds a critical mechanic threshold. Dependent on the global stress regime or the constitution of the material, a crack will occur in the *xy*-plane or the *xz*-plane. These two planes are called the nodal planes. From the radiation pattern alone, it can not be resolved if the stress was released by a relative slip of the upper part to the right or a relative slip of the right part upwards. To solve this ambiguity, more information about the stress regime or an analysis of other acoustic events is necessary. Along the nodal planes the amplitudes are zero, parallel to the T-axis (**T**ension) the amplitudes are positive (compressional) and along the P-axis (**P**ressure) the amplitudes are negative (dilatational). This nomenclature appears confusing at a first glance. It originates from different points of view, the source with its particle motion following the stress regime (P- and T-axis and vectors in figure 2), and the inverse corresponding amplitudes at a receiver.

Various methods for an inversion on the moment tensor are in use. Absolute inversion can be performed on single events, but require detailed knowledge of the Green's functions. Relative methods with or without a reference mechanism are applied on clusters of events where the Green's functions can be neglected by the assumption of common ray paths [DAHM, 1993]. Therefore, the size of a cluster has to be in the order of the investigated wavelength and has to be short enough to make sure, that the Green’s functions are constant within that period. The effects of alternating Green’s functions with the progress of damage are discussed in KURZ et al., [2004].

Problems occur when the signal to noise ratio is poor or the events within one cluster have similar mechanisms, which is often the case in acoustic emission analysis. ANDERSEN [2001] introduced a new approach, the hybrid moment tensor inversion, as a combination of the previous methods, where the effects of noise, low quality data, site effects, etc. are minimized and the solutions become more robust. The method is based on an iterative weighting scheme using the median of the distribution of residuals (for a particular geophone site, channel and wave phase), calculated using all events in the cluster.

With the introduction of the double couple we already began a mechanical interpretation of the moment tensor. In figure 3 some basic source models are introduced and correlated with the estimated radiation pattern of seismic p-waves. The upper part contains a collection of seismic sources and failure mechanisms which could play a major role in material sciences. The coordinate systems on the left help to connect these models with the radiation pattern images in the lower part of figure 3, where the respective columns belong to the same models above. The illustration of the radiation pattern as a projection into the *Schmidt net* (FOWLER, 1990) gives all important information of the 3D-spatial distribution of seismic energy on a fictive focal sphere around the source point in a view from above (indicated by a circle). The normalized amplitudes are plotted in a gray scale from black (+1) to white (-1). Black lines denote the zones with zero amplitudes (i. e. nodal planes for the double couple).

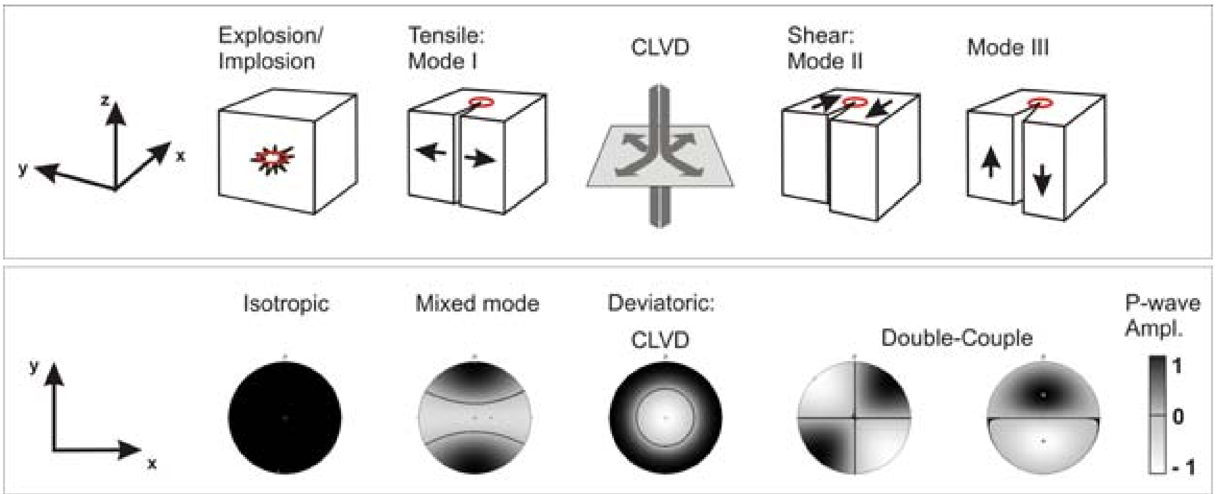

Figure 3: Correlation between models of failure mechanisms (top) and the estimated radiation patterns of these events in a spatial projection with view from above (same column, respectively) [FINCK et al., 2003].

The simplest model is a pure isotropic source with constant energy in all directions, comparable to an ideal explosion (positive amplitudes) or implosion (negative amplitudes). Uni-axial tension (mode 1), displayed in the second model from left leads to an opening crack. Here, the mean particle motion is parallel to the tension axis. In the plane perpendicular to this axis some small energy might be radiated with positive or negative amplitudes, depending on the elastic properties of the medium. A change of volume will be remaining. A similar mechanism is the so called *compensated linear vector dipole* (CLVD), shown in the middle. Here, the change of volume is compensated by the particle motion in the plane parallel to the largest stress. The CLVD was suggested as a possible deep earthquake mechanism due to mineralogical processes [KNOPOFF & RANDALL, 1970]. A second pure deviatoric mechanism can be formulated by a double couple. Shear is the corresponding failure mechanism. Mode 2 and mode 3 can de distinguished considering the direction of the crack growth relative to the particle motion. For the first, the directions are parallel, for the latter, perpendicular to each other. In reality, the stress field and homogeneities in the material lead to a superposition of these basic mechanisms.

The radiation of seismic energy can also be projected on a 3D focal sphere to be implemented in 3D visualizations of the specimen. This is performed by assuming isotropic and homogeneous Green's functions. The simplified Green's functions yield a constant factor, which is neglected, since the amplitudes are normalized to maximum amplitude equal to 1. Then the P-wave amplitudes can be calculated after PUJOL & HERRMANN [1990]:

$$d_{\mathrm{P}}(\mathbf{x}) = \frac{\mathbf{R}}{R}(\gamma_i M_{ij} \gamma_j) \quad . \tag{3}$$

Geometrical spreading is realized by the factor $1/R$. The second Term accounts for the convolution of the moment tensor and the direction cosines γ.

The basis of a physical interpretation of the real and symmetric moment tensor is a transformation into the system of its principal axes by an eigenvalue analysis [JOST & HERRMANN, 1989]. With an eigenvalue analysis, the moment tensor can be decomposed into an isotropic and a deviatoric component. Furthermore, the deviatoric portion can be separated into a double couple and a CLVD component:

$$\mathbf{M} = \mathbf{M}^{ISO} + \mathbf{M}^{DC} + \mathbf{M}^{CLVD} \quad . \tag{4}$$

To measure the magnitudes of the mechanisms involved, percentage decompositions can be calculated [e. g. ANDERSEN, 2001]

$$\begin{aligned} \%ISO &= \frac{100\,\mathrm{tr}(\mathbf{M})}{\left|\mathrm{tr}(\mathbf{M})\right| + \sum_{i=1}^{3} \left|m_i^*\right|} \\ \%DC &= \frac{m_3^*(1-2F)}{\left|m_3^*(1-2F)\right| + \left|2m_3^*F\right|}(100 - \%ISO) \\ \%CLVD &= \frac{2m_3^*F}{\left|m_3^*(1-2F)\right| + \left|2m_3^*F\right|}(100 - \%ISO) \quad , \end{aligned} \tag{5}$$

where the m_i are the eigenvalues of $\mathbf{M}$ with

$$\left|m_3^*\right| \geq \left|m_2^*\right| \geq \left|m_1^*\right| \tag{6}$$

and $F = - m_1^* / m_3^*$.

The model of the CLVD is not undisputable, since a Poisson’s ratio of 0,5 would be required for its realization, even under an idealized stress regime. In construction material sciences, the CLVD alone has no great significance as a mechanism of failure. In combination with a mayor positive isotropic component it can be seen as an opening crack (mode 1). One possibility to distinguish between various different focal mechanisms is simply to plot the isotropic component over the double couple component.

Mode I failure in a splitting test

To generate tensile stress within a well defined zone, compressive load was applied on a concrete cube (edge length 200 mm) over two parallel steel edges (see figure 4, left side) [FINCK et al., 2003]. The load in the test device was controlled over the crack opening, which was observed with two displacement sensors, one on the front, the other on the back side of the specimen. Eight acoustic emission sensors registered the microcracking, which occurred during the growth of a tensile crack. The localization of these events yielded the run of the crack through the specimen, starting on the back to the front side. This correlated very good with the visible damage progress, as well as with the crack opening data. Subsequent to the experiment, the specimen was ground in a stepwise manner, to digitize the run of the crack through the various sections. Thereby, the topography of the crack could be evaluated. The results of the localization of the acoustic emissions (black dots) and the inner crack surface (reticule) are plotted in figure 4, right side. Additionally, the radiation patterns of two selected events are visualized.

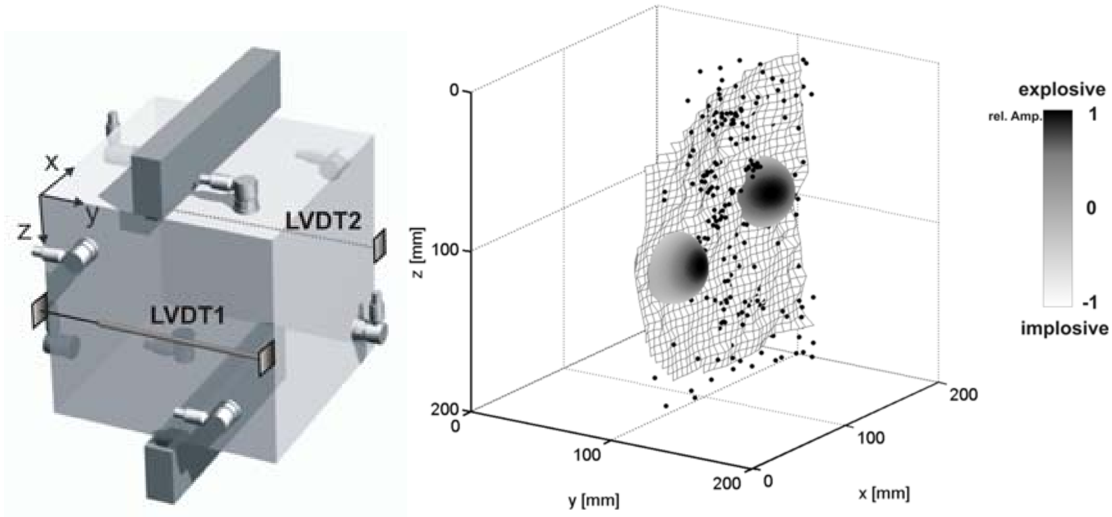

Figure 4: Left: Setup of the splitting test. Right: The topography of the crack surface and the radiation patterns of selected events from the two clusters. The radiation patterns were evaluated from the moment tensor solutions inverted with the hybrid method. Both radiation patterns reveal positive (dark) amplitudes parallel to the mean tensile stress and some negative (light gray) amplitudes parallel to the z-axis due to compressive stress. [FINCK et al., 2003].

The hybrid moment tensor inversion for these selected events revealed solutions, which very well approve the presumption of mode I failure. The principle tensile stress is oriented parallel to the y-axis, where the highest positive amplitudes occurred in the radiation patterns.

Mode II failure in a pull-out test

The bond behavior of steel reinforcement in a concrete matrix was investigated with signal-based acoustic emission technique in a pull-out test. A steel bar (Ø 15.6 mm) with five ribs (see figure 5, left) was pulled out of a concrete cube (edge length 200 mm). In the zone, where the ribs provided bond between the concrete matrix and the steel reinforcement, numerous acoustic emissions were localized with the POLAR^{AE} software [ROSENBUSCH, 2003] (dots in the view from above, figure 5, right side). Moment tensors were calculated for some selected events using the relative inversion approach by DAHM [1993]. The solutions of these inversions yielded stable results with nearly 100 % double couple mechanisms, and for each event one of the nodal planes was oriented tangential to the boundary the steel bar. The radiation patterns of the moment tensors indicate the relative downwards motion (blue, negative amplitudes) of the bar relative to the concrete block (red, positive amplitudes).

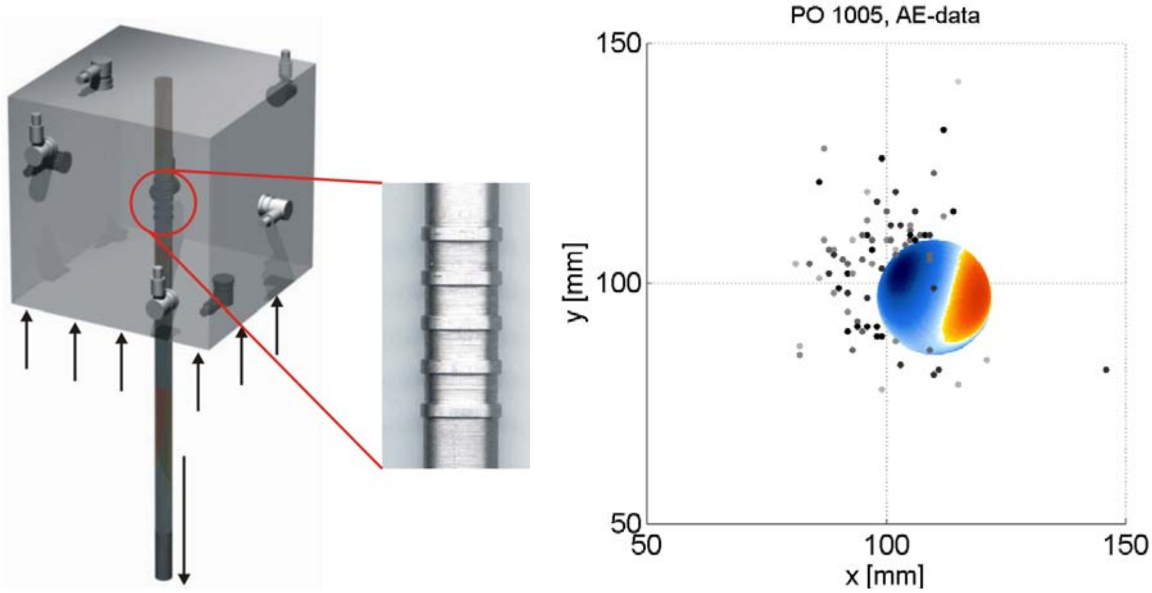

*Figure 5: Left: Setup of the pull-out test. Right: Localization of acoustic emissions, which occurred during the pull-out of a ribbed steel bar out of a concrete cube. The solution of the moment tensor inversion illustrates the pull-out of the bar, with a downwards motion relative to the concrete cube [*FINCK *et al., 2002].*

Conclusions

On the basis of a signal-based analysis of acoustic emissions, valuable information is obtained about failure processes. An accurate localization of microcracks yields the temporal and spatial distribution of damages. The concept of moment tensor inversion is well-suited for a fracture mechanics based interpretation of failure in engineering structures. Relative and hybrid moment tensor inversion methods were applied on different data sets and plausible solutions were obtained.

In a splitting test the evolution of a tensile crack was observed from the localization of the events and the moment tensors. In a pull-out test, the failure of bond and the slip of a steel bar in concrete was investigated in detail.

A 3-dimensional visualization of the moment tensors was developed to give a descriptive presentation of the results. This visualization method can easily be implemented into the illustration of the results of other investigations.

Understanding of the waveforms and development of standard routines during data processing allows for the automation of the evaluation of fracture processes. Some of these routines are implemented in the new POLAR^{AE} (*Program for Onset Detection, Localization and Amplitude Readout for Acoustic Emission*) software developed at the IWB.

The experiences gained in digital signal processing and signal-based AE analysis is the basis for future applications in the field of construction health monitoring for example.

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