CONTOURING GEODETICALLY ACCURATE ACOUSTIC EMISSION SOURCES VIA KERNEL DENSITY ESTIMATES

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Abstract:

We deal with numerical model of localization of acoustic emission (AE) sources on real complex solid bodies. Our approach is based on exact geodesic curves on 3D vessels composed of several parametrized surfaces. The numerical computations are provided via Finite difference, Newton–Raphson, and Fixed-point iteration methods applied to geodesic equations acquired from differential geometry theory. To speed up computations, some technical improvements and optimizations are proposed. The variable propagation velocity and also the case when the geodesic curve has to bypass a given obstacle there is also included into the model. These techniques are employed in the real experimental setup on bodies with higher geometrical complexity. The results (localization maps) of AE localization principle using length (ΔL) or time (ΔT) differences, obtained by means of geodesics, are then processed through the two-dimensional Kernel probability density estimates executed directly on the 3-D surfaces, which give us the most probable areas of the AE source positions on the main body. The placement of piezo-ceramic AE sensors is outside the central part of the vessel because it can be inaccessible due to possible high temperature or radioactivity, such as in the case of nuclear power station health monitoring. This outward position of all AE sensors can result in a dispersed AE wave detected, or attenuated because of welded intersections of different surfaces. Thus, the Change-point analysis of AE signals is also discussed in order to obtain the most precise arrival times of AE events, which is crucial for ΔT / ΔL localization.

1. Introduction on geodesics

In our mathematical model of geodesic-based localization of acoustic emission sources we employ a posteriori evaluation of acquired localization maps. This is carried out by using the statistical kernel density estimates in 2D, which are applied either in parametric domain of the surface or directly on the 2D surface embedded in 3D. The resulting geodetic-kernel localization is applied to two experiments – the ionized iron watering can and steam reservoir. The method of geodesic localization itself is based on the well-known time differences localization principle. Measured time differences are compared (via certain functional F) with length differences that are computed using geodesic curves. These geodesic curves (geodesics) possess the property of the shortest length curves between two points and they are obtained from the following system of two non-linear ordinary differential equations [3] of the second order

\[
\ddot{u} + \Gamma^1_{11} \dot{u}^2 + 2 \Gamma^1_{12} \dot{u} \dot{v} + \Gamma^1_{22} \dot{v}^2 = 0, \\
\ddot{v} + \Gamma^2_{11} \dot{u}^2 + 2 \Gamma^2_{12} \dot{u} \dot{v} + \Gamma^2_{22} \dot{v}^2 = 0,
\]

with a boundary conditions \( u(a) = \alpha_1, u(b) = \alpha_2 \) and \( v(a) = \beta_1, v(b) = \beta_2 \), where \( u = u(t) \) and \( v = v(t), t \in (a, b) \), are coordinate functions of the geodesic in parametric domain of the surface with parametrization \( x = x(u,v), y(u,v), z(u,v) \). The symbols \( \Gamma^i_{jk} \) are called Christoffel symbols and they describe a metric on the surface. We go through the each point...
of a surface and numerically compute the geodesics to each sensor. Since many real bodies cannot be, in general, parametrized by a single parametrization, the algorithms obtaining a geodesic between two arbitrary points on the body are required [1, 2].

2. Numerical computation of geodesic curves

The above system of geodesic equations is solved via finite difference method, which leads to the system of algebraic non-linear equations [2], which is solved through the Newton-Raphson iterative method, given by the following principle

$$dF(\omega^k) \Delta \omega^k = -F(\omega^k), \quad \omega^{k+1} = \omega^k + \Delta \omega^k,$$

with initial root guess $\omega^0$ as a straight line in parametric domain. To be able to compute geodesics on real bodies we need to decompose them to elementary surfaces so that we can describe each of them parametrically and easily obtain its Christoffel symbols. Then we compute the geodesics on the every sectional surface between start/end points by using a connection point lying at the intersection of two surface sections (Fig. 1 left). Then we iterate through each point of intersection and minimize the sum of lengths of partial geodesics. The computational cost can be very high, mostly when we have to iterate through many intersections.

![Figure 1: Computing geodesics through intersection on compound body.](image)

To fasten the computation through intersections, the Sequential Algorithm (SA) is applied [1]. It is based on bisectional method and iterates through the points of intersection with minimal sum of length of geodesics (Fig. 1 right). Let $(a_0, b_0) \subset \mathcal{J}$ be one part of parametric domain of the intersection curve. Set $y_0 = (a_0 + b_0)/2$ and the step $\sigma_0 = |b_0 - a_0|$. Now we compute geodesics through the points $a_k, b_k, y_k$, $k \geq 0$,

$$a_{k+1} = y_k + \sigma_{k+1}, \quad b_{k+1} = y_k - \sigma_{k+1}, \quad \sigma_{k+1} = \frac{\sigma_k}{2},$$

$$y_{k+1} = \text{argmin}\{L_{a_{k+1}}, L_{b_{k+1}}, L_{y_k}\},$$

where $L_i$ means the length of geodesic through the point $i$. This SA algorithm is very fast but there may be possible disadvantage that the points, through which the SA iterates, are not restricted to the interval $(a_0, b_0)$. Then a simple modification of the algorithm is needed.

There occurs one serious problem when solving geodesic system of equations. In short, the curve obtained need not be the shortest possible path between two points (Fig. 2). It happens in practical applications while working with solid vessels whose elementary parts form closed rotary surfaces. It means that at least one of coordinates $u, v$ of its
parametrization domain takes values in $(0, 2\pi)$ or $(-\pi, +\pi)$. These intervals are mathematically equivalent apart from the discontinuity at $2\pi \leftrightarrow 0$ or $\pi \leftrightarrow -\pi$, which has an influence to the geodesic curve solution. Therefore, if $|\theta_A - \theta_B| > \pi$, we a priori change the computational parametric range of the angular coordinate, where $\theta_A$, $\theta_B$ are angular coordinates of selected two surface points. The idea of this criterion comes from the observation that greater angle between two points results in longer arc and consequently greater length of geodesic curve.

![Figure 2: Conditional a priori reparametrization of $(0, 2\pi)$ to $(-\pi, +\pi)$.](image)

We also have to deal with computations of geodesic curves around obstructions, e.g. see Figure 4. It is clear that the exact geodesic computations around obstacles can give more realistic signal paths and more exact AE source localization [2]. Thus, we have to find out sets of points for each obstruction through which we minimize the sum of length of all parts of geodesic (this may be tricky if we are dealing with multiple obstructions and advanced graphical methods are needed [4]). This algorithm is considerably time consuming because we are forced to compute very large number of geodesics in both reparametrizations.

![Figure 3: Geodesic curves in cases of path obstructions.](image)

3. **Localization of acoustic emission sources**

In this part we present results from experiments with watering can and steam reservoir using the exact geodetic method mentioned above. First, SIC algorithm [5] is used to determine time arrivals of acoustic signals (produced by pentests) and then the common localization of
AE sources is carried out by means of either length differences: \( F_L = \sum_{i<j} |c_{ij}\Delta t_{ij} - \Delta l_{ij}| \), or time differences: \( F_T = \sum_{i<j} |\Delta t_{ij} - \Delta T_{ij}| \), where \( c_{ij} \) denotes the specific velocity between i-th and j-th component of the surface. There may occur the differences between resulting localization maps because the velocities \( c_{ij} \) are in general different from the velocities on each component itself (Figure 4 (right)).

Figure 4: Sensor placement on the watering can (left) and an example of localization map using length (black+green) or time differences (blue+yellow), the source is red (right).

In the sequel, we use the time differences for our pentest experiments on steam reservoir, where the localization results are shown in Figure 5. As we can see, the precision of localization is quite high. This precision was mainly maintained due to incorporation of the variable velocity (non-constant on the whole surface) and also due to computing geodesics around obstacles (pouring ladle, welded pipes). We point out here that SIC criterion for obtaining the AE signal arrivals played a very important role in localization accuracy.

Figure 5: Localization maps on steam reservoir. Black dots with circles denotes pentest areas.

4. Kernel density estimates in 2D

The Kernel density estimate [6] of some unknown density \( f \) is defined by
\[ \hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K_H(x - X_i), \]

where \( X_i = [u_i, v_i] \in \mathbb{R}^2 \) represents the \( i \)-th localized point in the parametric domain of the surface, \( x \in \mathbb{R}^2, \ H = (h_{ij})_{ij=1}^2 \in \mathbb{R}^{2 \times 2} \) is a symmetric and positive definite smoothing matrix and \( K_H \) is determined through the kernel function \( K \) by the expression

\[ K_H(x) = |H|^{-1/2} K(H^{-1/2} x). \]

\( H \) plays a similar role as the window width in one dimensional case. The essential thing about kernel density estimates is that it weakly depends on \( K \), however it substantially depends on the smoothing matrix \( H \). We briefly present two algorithms which determine the matrix \( H \) automatically.

**Reference density method (RD):** If we suppose that the unknown density \( f \) belongs to the family of normal densities, then the estimate of \( H \) is \( \hat{H}_{RD} = n^{-1/3} \hat{\Sigma} \), where \( \hat{\Sigma} = (s_{ij})_{ij=1}^2 \) stand for the estimate of covariance matrix obtained from measured data set.

**Iterative method (IT):** IT estimate of \( H \), based on the family of normal densities again, is

\[ 2h_{11} \sqrt{S(s_{ij})} \sum_{i,j=1}^{n} \Lambda_H(X_i - X_j) = n V(K), \]

with \( S(s_{ij}) = (s_{11}s_{22} - s_{12}^2)/s_{11}^2, \ V(K) = \int_{\mathbb{R}^2} K^2(x) \, dx, \) and \( \Lambda_H(z) = (K \ast K \ast K \ast K - 2K \ast K \ast K + K \ast K)(z) \), where \( \ast \) denotes the convolution. The equation can be solved for example with Newton-Raphson method. The remaining \( h_{12}, h_{21}, h_{22} \) are then obtained from \( h_{11} \) as \( h_{ij} = h_{11}s_{ij}/s_{11} \). Note that \( H \) is symmetric and thus \( h_{12} = h_{21} \).

5. **Localization results using kernel density estimates**

The localization map from Figure 5 is further processed through kernel density estimate which tells us the most probable location of AE sources. This kernel density estimate can be carried out either in parametric domain of the surface and then transformed onto the surface in 3D, or preferably, can be performed directly on the curved surface 2D object embedded in 3D.

![Figure 6: Kernel density estimates on the steam reservoir applying RD procedure.](image)
The former resulting kernel density estimates can be seen in Figures 6 and 7, where the significant difference between IT and RD methods can be observed. The outstretched RD kernel estimate is caused by its application on quite dispersed pentest data with big variance, consequently leading to unusable estimates. However, if we treat the data separately for each localization cluster then the RD method provides practicable estimates. Further, we found out that although the IT algorithm also depends on the covariance matrix $\Sigma$, this IT method does not suffer from the dispersed estimates.

The second approach based on direct kernel computations on the curved surface in 3D is presented in Figure 8. It was realized by means of generalized definition of the kernel estimator using geodesic metric $\rho$ at the surface and restricted to the window radius $h$. This method offers wider variability of adjusting magnitude of the kernel window, unlike the methods for automatic estimate of smoothing matrices $H$. For comparison purposes, the direct surface kernel estimates were also transformed back into the parametric domains of the surface, see Figure 8 (left).

6. Conclusions

Mathematical model of sources of AE based on time differences principle expanded from plane to curved surfaces make use of geodesic curves obtained from geodesic equations. These equations are solved numerically via finite difference method and Newton-Raphson method. Both the variable propagating velocity of acoustic signal and the geodesics bypassing obstacles were incorporated into the model. This led, with other support techniques, to high accuracy of AE localization model. Computation is parallelized for the possibility of online localization. Resulting model was applied to laboratory experiments on ionized watering can and steam reservoir. The watering can represents higher geometric complexity, whereas the steam reservoir brings computations of geodesics bypassing multi-obstacles. Resulting localization maps were processed with 2D kernel density estimates.
which provided most probable locations of AE sources. Based on our comparison we point out the possibility of computing the kernel estimates directly on the 2D surface in 3D, which can be very useful and more accurate. All the results can be found in [2] in details.

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**References:**


