Guidelines for Using the Finite Element Method for Modeling Guided Lamb Wave Propagation in SHM Processes

M. GRESIL, V. GIURGIUTIU, Y. SHEN and B. PODDAR

ABSTRACT

The aim of the work presented in this paper is to provide guidelines for extending the modeling capacities and improve quality and reliability of 2-D guided wave propagation models using commercially available finite element method (FEM) packages. Predictive simulation of ultrasonic nondestructive evaluation (NDE) and structural health monitoring (SHM) in realistic structures is challenging. Analytical methods can perform efficiently modeling of wave propagation are limited to simple geometries. Realistic structures with complicated geometries are usually modeled with the finite element method (FEM). Commercial FEM codes offer convenient built-in resources for automated meshing, frequency analysis, as well as time integration of dynamic events. We propose to develop FEM guidelines for 2-D Lamb wave propagation with a high level of accuracy. The proposed 2-D guided wave problem will be the pitch-catch arrangement in a full 3-D geometry plate involving guided waves between a transmitter piezoelectric wafer active sensor (PWAS) and receiver PWAS. In addition, corrosion damage is added to this problem to simulate the detection of damage, and assess the detectability threshold. The general approach is to run a series of FEM models. These FEM models will be compared with the experimental data and with our 1-D analytical homemade software.

INTRODUCTION

The goal of SHM research is to develop a monitoring methodology that is capable of detecting and identifying, with minimal human intervention, various damage types during the service life of the structure. Active structural health monitoring (SHM) systems using interrogative Lamb waves are able to cover large areas from a single location making such systems cost effective and efficient.

Matthieu Gresil, Department of Mechanical Engineering, University of South Carolina, Columbia, 29208, SC, USA. matthieu@engr.sc.edu
Another advantage is that Lamb waves provide through-the-thickness interrogation which allows detection of internal defects in materials. Piezoelectric wafer active sensors (PWAS) are used for active SHM technique. However, Lamb waves present some difficulties: they are dispersed, at a given frequency, and thus several modes can propagate at different speeds. Work has been done to establish analytically the dispersion curves [1-6], to validate experimentally [7] and to study the effect of dispersion over long distances [8].

This paper presents the use of guided wave and the capabilities of embedded PWAS to perform in situ nondestructive evaluation (NDE) are explored. Finite element method (FEM) codes are used to simulate Lamb wave 3D structure and compared with analytical and experimental results.

PIEZOELECTRIC WAFER ACTIVE SENSORS (PWAS)

PWAS are the enabling technology for active and passive SHM systems. PWAS couples the electrical and mechanical effects through the tensorial piezoelectric constitutive equations

\[
S_{ij} = s_{ijkl} T_{kl} + d_{ij} E_k
\]

(1)

\[
D_j = d_{jk} T_{ik} + \varepsilon_{jk}^T E_k
\]

(2)

where, \(S_{ij}\) is the mechanical strain; \(T_{ij}\) is the mechanical stress; \(E_k\) is the electrical field; \(D_j\) is the electrical displacement; \(s_{ijkl}\) is the mechanical compliance of the material measured at zero electric field (\(E = 0\)), \(\varepsilon_{jk}^T\) is the dielectric permittivity measures at zero mechanical stress (\(T = 0\)), and \(d_{ij}\) represents the piezoelectric coupling effect.

In our preliminary study, the analytical modeling of the pitch-catch process between two PWAS transducers separated by a distance \(x\) was developed [9-11].

EXPERIMENTAL SET-UP

The test specimen were constructed to develop and calibrate the damage-detection methodology using a simple geometry specimens, and also to validate the analytical, and the FEM results. Thin aluminum plate specimens were constructed from 3.2-mm-thick 2024-alloy stock in the form of a square plate \((1118 \text{mm} \times 1118 \text{mm} \times 3.2 \text{mm})\). The specimens were instrumented with arrays of 7-mm square and 7-mm circular PWAS (Figure 1).
Figure 1: Arrangement of the PWAS bonded on the plate.

**FEM MODELING**

The effectiveness of conventional finite element modeling of elastic waves propagating in structural components has been shown in the past. The case of Lamb waves in free plates is a classic example [12, 13]. The package used in the present study, ABAQUS/Explicit, uses an explicit integration based on a central difference method [14]. The stability of the numerical solution is dependent upon the temporal and the spatial resolution of the analysis. To avoid numerical instability ABAQUS/Explicit recommends a stability limit for the integration time step is [13]

\[
\Delta t = \frac{1}{20 f_{\text{max}}} \tag{3}
\]

The maximum frequency of the dynamic problem, \( f_{\text{max}} \) limits both the integration time step and the element size. The size of the finite element, \( L_e \) is typically derived from the smallest wavelength to be analyzed, \( \lambda_{\text{min}} \). For a good spatial resolution 20 nodes per wavelength are required [13]. This condition can be written as

\[
L_e = \frac{\lambda_{\text{min}}}{20} \tag{4}
\]

In non-destructive evaluation (NDE), a common understanding is that the ultrasonic scanning technique can usually detect damage. Furthermore the fundamental anti-symmetrical mode (A0) is preferable and more sensitive to damage because its wavelength is shorter than that of the S0 mode at the same frequency. However, the A0 mode exhibits more dispersion at low frequencies. The FEM
simulation of the A0 mode requires fine spatial discretization with substantial computational cost for the sake of the short wavelength. In contrast, the mode shapes of the S0 mode are simpler and the stresses are almost uniform throughout the thickness of the plate at low values to the frequency and plate thickness product. For these reasons, the two modes S0 and A0 were selected in this study to compare with the analytical and the experimental results.

We used the ABAQUS/explicit solver because it gives a better trade-off between accuracy and computation time. The piezoelectric element C3D8E doesn’t exist in ABAQUS/explicit; hence we applied 12 and 8 self-equilibrating forces as shown in Figure 2a and Figure 2b to simulate the wave excitation for the square and round PWAS respectively. The plate was discretized with C3D8R brick elements of size 0.5mm. A 3-count smoothed tone burst with a central frequency of 150 kHz was used to modulate the excitation.

![Figure 2: Self equilibrating force excitation; (a) for the square PWAS; (b) for the round PWAS.](image)

RESULTS AND DISCUSSIONS

The analytical, finite element and experimental results for a 3.2-mm thick aluminum plate with 200-mm PWAS distance for a frequency of 150 kHz are shown in Figure 3. S0 and A0 mode wave packages could be observed. The wave speed of S0 mode is higher than the A0 mode, so the S0 wave packet is picked up earlier than the A0 wave packet. Furthermore, a perfect matching on the magnitude is observed between the FEM and the experimental results for the S0 and also the A0 mode. However, as observed on the Figure 3, a magnitude difference and a time shift are observed on the first wave packet (S0 mode) between the analytical and the other results. In addition, a very slight time shift is observed at the end of the A0 packet between the three different results. It may be due to the approximation of the excitation in the analytical and the FEM model compared with the real case in the experimental results.

Giurgiutiu and Bottai-Santoni [15] developed a shear lag solution for the stress/strain transfer between a structurally attached PWAS and the support structure. This generic solution takes into account the exact thickness distribution of displacements and stresses corresponding to the Lamb wave modes existing at particular frequency-thickness values. They showed that the shear stress distribution varies with PWAS length and thickness of the bond layer.
The analytical model using the shear lag effect is compared with the finite element and the experimental results for a 3.2-mm thick aluminum plate with 200-mm PWAS distance for a frequency of 150 kHz (Figure 4). When the size PWAS decrease to 4-mm a perfect matching on the magnitude of the S0 packet is observed between the analytical and the other results. Another interesting thing observed that the analytical model is developed with the 1-D wave propagation and match perfectly with the 3-D FEM model and also with the experimental results.

FEM CORROSION DETECTION

A series of FEM model are performed to detect corrosion in 3.2-mm thick aluminum plate. Metal structures exhibit a wide range of corrosions types including uniform, pitting, galvanic, crevice, concentration cell, and graphite corrosion [16]. When the guiding structure has changes due to corrosion in the geometry, materials properties, supports, or attachments, the guided waves that propagate through will be...
modified accordingly. Hence, loss of material due to corrosion presents geometrical changes which will cause the guided waves scattering and can be used for inspection of corrosion.

In this study, we simulated uniform corrosion on an aluminum plate. The depth was increased gradually in order to simulate corrosion progression. This thickness loss produced a change in the waveguide impedance and thus caused (i) scattering and reflection and (ii) modification of the wave speed of the Lamb waves crossing the corrosion area. In practice, corrosion defects are geometrically complex and require multiple parameters to describe them and their scattering behaviors. We used simplified shapes for propagating Lamb wave paths to reduce number of parameter in order to better understand the changes caused by material loss. In the mode, simulated corrosion was made on 3.2-mm thick. The location of the corrosion was halfway between the two PWAS transducers. The distance between the R-PWAS and the T-PWAS is 260 mm. A 50 x 38 mm$^2$ area is used to simulate the corrosion. This simulation set-up is in agreement with our previous experimental set-up in order to compare this FEM simulation with our previous experimental study [17].

In order to have a system able to evaluate in real time, in situ, the health of the structure in an automatic way, it is necessary to define a damage index (DI). In this paper, we choose the DI based on the root mean square deviation (RMSD). The RMSD-DI is a scalar quantity that results from a statistical comparison between the signal in the present state and the signal in the reference state (baseline). Such a scalar reveals the difference between pristine data and measurement caused by the presence of damage and provides an overall change of the structure between sensors. This feature would be ideal for hole or corrosion detection since it carries information of both the amplitude and the phase changes from the growth of the corrosion. The RMSD DI is defined as the relative ratio of the difference between each measurement and baseline signals as follows:

$$\text{RMSD DI} = \sqrt{\frac{\sum_{j=0}^{N-1} [s_i(j) - s_0(j)]^2}{\sum_{j=0}^{N} s_0^2(j)}}$$

In Equation (5), $s_i$ is the $i^{th}$ measurement and, $s_0$ is the baseline signal, and $N$ is the length of the data set.

The curves obtained from the RMSD DI are given in Figure 5. First, the A0 mode for both experimental and simulate is more sensitive that the S0 mode. Second, The DI curves changes from 0 to 2.5, indicating significant corrosion development occurring along the wave propagation path. However, the change of DI with corrosion development is not monotonic. The DI curve increases first for the corrosion depth from 0 to 1.2 mm for the simulation results, and from 0 to 0.94 mm for the experimental results. Then the RMS DI decrease. The reason for this phenomenon is that the RMSD DI shows the changes of both amplitude and phase while is it more sensitive to magnitude changes.
From these simulations it is found that, with 99% of confidence, the minimum detectable corrosion depth’s size is 0.1 mm., and from our previous experimental results [17], the minimum detectable corrosion depth’s size is 0.38 mm. Nevertheless, in our experimental results, 0.38 mm was the first measurement after the pristine case.

![Figure 5: Comparison of the damage index based on the RMSD between our preliminary experimental results [17].](image)

CONCLUSION

This paper addressed the need for developing a predictive modeling methodology for structural sensing in active structural health monitoring (SHM) with piezoelectric wafer active sensor (PWAS). A few examples of preliminary work done towards such a predictive methodology have been presented. The analytical modeling was used to simulate the traveling of guided waves from a transmitter PWAS to a receiver PWAS and was compared with the FEM model and the experimental results. We have developed guidelines to simulate the 2D pitch-catch signal between a transmitter and a receiver piezoelectric wafer active sensor.

The corrosion detection method used in these simulations was pitch-catch method. It was found that, with 99% confidence, the minimum detectable depth of the corrosion size was 0.1mm. Moreover, it was found that the A0 wave mode was much more effective in detecting corrosion damage than S0 mode.

However, the results presented here are just preliminary. Future work should attempt to combine the efficiency of analytical methods with the detailing capability of the FEM approach such as to develop a hybrid global local method for modeling realistic structures with sufficient computational efficiency as to permit parameter studies. In this way, we will be able to advance from an empirical approach into an analytical rational development of structural health monitoring systems and maintenance strategies.

ACKNOWLEDGEMENT

Support of Office of Naval Research # N00014-11-1-0271, Dr. Ignacio Perez, Technical Representative; Air Force Office of Scientific Research #FA9550-11-1-0133, Dr. David Stargel, Program Manager; are thankfully acknowledged.
REFERENCES

7. Grondel, S; Bastien; Assaad, J; Delebarre, C; Blanquet, P; Moulin, E (1999) "The propagation of Lamb waves in multilayered plates: phase-velocity measurement", Measurement Science and Technology, 10(5), 348-353, 1999