STRUCTURAL DAMAGE CLASSIFICATION COMPARISON USING SUPPORT VECTOR MACHINE AND BAYESIAN MODEL SELECTION

Zhu Mao¹, Michael Todd¹

¹ University of California at San Diego, 9500 Gilman Dr, MC0085, La Jolla, CA, USA 92093-0085
mdtodd@ucsd.edu

ABSTRACT

Since all damage identification strategies inevitably involve uncertainties from various sources, a higher level of characterization is necessary to facilitate decision-making in a statistically confident sense. Machine learning plays an important role in the decision-making process of damage detection, classification, and prognosis, which employs training data (or a validated model) and extracts useful information from the high-dimensional observations. This paper classifies the type of damage via support vector machine (SVM) in a supervised learning fashion, and selects the most plausible model for data interpretation. Therefore the separation of damage type and failure trajectory is transformed into a group classification process, under the influence of uncertainty. Given data observation, SVM is obtained under a training process, which characterizes the best classification boundaries for any future feature set. A rotary machine test-bed is employed, and vibration-based damage features are evaluated to demonstrate the proposed classification process.

KEYWORDS: damage classification, machine learning, support vector machine, bearing failure.

1 INSTRUCTION

As the technology of structural damage detection and health monitoring is getting more and more mature, lots of considerations have been addressed in dealing with the uncertainties involved in making damage diagnosis/prognosis decisions. To make better decisions under uncertain scenarios such as noisy measurements and operational variability, quantifying the uncertainty will enhance the overall performance, and as a result, a quantified confidence in the decision will be available [1-3]. In our previous research, probabilistic uncertainty quantification (UQ) models of frequency response function (FRF) estimations are established, and the distributions of estimations from different algorithms are fully characterized by the analytically derived probability density functions [4, 5]. Bayesian statistics fuse collected evidence to update prior confidence and are powerful for making decision especially when there is ambiguity caused by all sorts of uncertainty. For damage classification applications, a Bayesian framework can be used to select the most plausible model to characterize the data observations and classify the structural condition with respect to maximum posterior probability [6].

Besides the uncertainty involved in damage detection and classification processes, extracting sensitive and specific features from large volume of data set is another burden. Oftentimes, there is great “fuzziness” and redundancy in the raw data coupled to the aforementioned uncertainty, and this fundamentally causes the damage detection and classification processes to become more complicated. Machine learning technologies have been employed widely in data processing and feature extraction, among which support vector machines (SVM) are particularly powerful for solving classification problems. This paper adopts an SVM to classify damage cases of the Machinery Fault Simulator (MFS) from SpectraQuest, Inc., where damages on the outer race and
balls of bearings are differentiated from undamaged baseline. In this work, features are selected based on FRF magnitude and phase, and rate of correct classification is defined as comparison metric for different case studies.

A brief overview of FRF and the estimations on the test-bed will be given in section-2, and SVM implementation on the data acquired from the test-bed, with a parametric study, is available in section-3. In the end, a summary of the result is given in section-4.

2 FREQUENCY RESPONSE FEATURES

Frequency response function is very widely used in system identification and is also a good feature for damage detection because of its easy accessibility and clear physical interpretation. Equation (1) illustrates the definition of FRF, also known as transfer function in the frequency domain:

\[ H(\omega) = \frac{V(\omega)}{U(\omega)}, \]  

in which \( U \) and \( V \) are the Fourier transforms of clean input and output \( u(t) \) and \( v(t) \). Under the realistic conditions, the measurements are always contaminated by noise (uncertainty), denoted as \( x(t) \) and \( y(t) \), and estimation of FRF is often calculated via estimators. Equation (2) is the H-1 estimator of FRF, which is the ratio between cross- and auto-power density functions of input and output measurements:

\[ \hat{H}(\omega) = \frac{\hat{G}_{xy}(\omega)}{\hat{G}_{xx}(\omega)}, \]

where the ^ denotes the average of power spectra according to Welch’s algorithm [7]. If the number of averages is sufficient, the distributions of FRF magnitude and phase estimations, as random variables, can be characterized by the probability density functions \( p_m \) and \( p_\theta \) in Equation (3) and (4), under Gaussian assumption [8].

\[ p_m(z | \mathcal{M}_j) = \frac{1}{\sigma_m \sqrt{2\pi}} e^{-\frac{(z-\mu_m)^2}{2\sigma_m^2}}, \]  

where in the context of classification, \( \mathcal{M}_j \) is the \( j \)th condition of structure, and \( \mu_m \) and \( \sigma_m \) are the mean and standard deviation of magnitude estimation respectively.

\[ p_\theta(z | \mathcal{M}_j) = \frac{1}{2\pi} e^{-\frac{(\mu_\theta + \mu_\theta^*)^2}{2\sigma_\theta^2}} + \frac{\eta_j}{2\sqrt{2\pi}\sigma_\theta} e^{-\frac{(\mu_\theta \sin(z) - \mu_\theta \cos(z))^2}{2\sigma_\theta^2}} \left(1 + \text{erf} \left( \frac{\eta_j}{\sqrt{2}\sigma_\theta} \right) \right), \]

in which \( \eta_j = \mu_\theta \cos(z) + \mu_\theta \sin(z) \), and erfc(.) is error function. In Equation (4), \( \mu_\theta \) and \( \mu_\theta^* \) represent the mean of real and imaginary parts of FRF estimation, while \( \sigma_\theta \) is the standard deviation of both parts.

FRF magnitude and phase characterize the system’s input-output transformation dynamics, and their estimations fall into different distributions for different system damage conditions. Probability density function in Equation (3) and (4) quantify the measurement confidence and facilitate detection with statistical significance, especially when the feature clusters have large overlap. However, when there is large overlap in the feature space, UQ models only find out the best decision boundary, but do not improve the capability of detection and classification. SVM partitions the feature sets by means of machine learning, and applies kernel transformation to separate the complex data space.
3 SVM AND IMPLEMENTATION

3.1 SVM and kernelization
SVM employs training data to form a hyperplane as the decision boundary, in order to discriminate different sets of data, and all the data points determining the hyperplane are called support vectors. Equation (5) describes the decision maker $h(\mathbf{x})$, which maps feature vector $\mathbf{x}$ into a binary space:

$$
    h(\mathbf{x}) = \begin{cases} 
    0 & \text{if } g(\mathbf{x}) > 0 \\
    1 & \text{if } g(\mathbf{x}) < 0 
    \end{cases}
$$

(5)

The function $g(\mathbf{x})$ is a hyperplane in the feature space:

$$
    g(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + b.
$$

(6)

When the data from the classification model are non-separable, slack variables are introduced to solve a soft margin problem, but this may not always practical for highly overlapped/complicated feature spaces. Kernel functions are employed if necessary, so that the feature dimension is increased, and all clusters are being better distinguished in a higher dimensional state, as Equation (7) shows:

$$
    h(\mathbf{x}) = \text{sign} \left[ \sum_{i \in SV} y_i \alpha_i K(\mathbf{x}_i, \mathbf{x}) + b^* \right],
$$

(7)

in which $\mathbf{x}_i$ are all the support vectors, $K(\mathbf{x}, \mathbf{x})$ is the selected kernel function, $\alpha_i$ is the Lagrange multiplier for constrained optimization, and $y_i$ is the classification label of feature $\mathbf{x}_i$.

3.2 Testbed
A rotary machine is adopted as the testbed to implement the SVM classification, and two different bearing failures are designed to be distinguished from the baseline, namely damaged balls and damaged outer race. Figure 1 shows the testbed, and on the right-hand-side support of the shaft, the aforementioned two damage conditions are tested on the bearing here. Acceleration time series are acquired in the directions $y$ and $z$, and transfer function is calculated from the acceleration spectra in the two directions via estimator in Equation (2).

![Rotary machine test-bed](image)

Figure 1: Rotary machine test-bed

For various bearing conditions, the FRF feature estimations will be different, as Figure 2 demonstrates in different colors. However, as the uncertainty exists, the realizations of FRF features are highly overlapped as expected. Figure 3 shows the state space of FRF feature at a single frequency line as example, and the overlap causes difficulty in damage classification.
3.3 SVM implementation

At first, a simplified condition is considered for SVM analysis, in which the feature vector is defined as:

$$\mathbf{x} = \begin{bmatrix} H(\omega) \\ R H(\omega) \end{bmatrix} \in \mathbb{R}^2. \quad (8)$$

In Equation (8), $\omega$ is the same sample frequency as in Figure 3. Obviously, the feature space is 2-D, which can be visualized easily. Before doing a full damage classification, Figure 4 and Figure 5 illustrate the SVM implementation on a binary case. Only cases with damaged ball are included to be differentiated from undamaged baseline. For about 250 testing cases, the true labels and the classification result for each case are also plotted on the right hand side. For each class, the percentage of correct labelling from SVM is calculated for comparison. As abovementioned, kernelization increases the decision space dimension, and the decision boundaries plotted in Figure 4 and 5 are only projection of the real decision boundaries to the 2-D plane. In Figure 4, linear kernel function is adopted, while in Figure 5, a more flexible radial bases function kernel (a.k.a. Gaussian kernel) is employed. Comparing the rate of correct classification, the Gaussian kernel does better than the linear hyperplane separation. One thing need to be clarified is, the performance of
correct classification rate needs to be compared considering all classes. For the binary case as shown in Figure 4 and Figure 5, the rate for labelling ball damages via linear kernel is higher than 98%, but this does not necessarily indicate good performance, because of tremendous false labelling of baseline data. The Gaussian kernel does slightly more accurate classification, which gives about 85% of correct decisions with only 2-D feature, while the linear kernel SVM only gives a little less than 80% of correct decisions. As a reference, 0% means totally flipped (wrong) classification, and 50% indicates a random guess for this binary testing.

For the original trinary group classification problem on the rotating machine, data from damaged ball and outer race both need to be discriminated from the undamaged baseline. The multi-class problem is reduced into three binary classification problems, in which SVM models are built to distinguish one of the classes from all the rest. Therefore the one-versus-rest idea partitions the trinary classification into (1 vs 2,3), (2 vs 3,1) and (3 vs 1,2), all three sub-problems.

Figure 6 presents the support vectors and decision boundaries for each sub-problem, using Gaussian kernel. In each sub-problem, the data to be distinguished mostly fall into the boundary given by SVM, and the other two classes are grouped as “ELSE”. Again, although the dimension of feature vector is only 2, the SVM maps the feature space to a higher dimensional domain, and what plotted is only the projection of decision boundary back to the 2-D feature space.
Figure 6: SVM implementation for trinary classification, Gaussian kernel

To evaluate the performance of the SVM for trinary case, the true and predicted label for all testing cases are plotted in Figure 7, and the rate of correct classification is also calculated. Generally speaking, the SVM classifiers obtained from both kernel functions tag the data less accurately than the performance of binary case. This is mainly caused by the greater complexity and ambiguity. The average rate of correct classification for each case is about 57% and 71%, which is lower than the binary classification, given the same feature dimension. Compare to linear kernel, Gaussian function outperforms linear function due to its flexibility.

Figure 7: Correct classification rate for SVM implementation for trinary classification
left: linear kernel; right: Gaussian kernel

However, all results shown in Figure 4 to Figure 7 are the SVM implementation using 2-dimension feature sets described in Equation (8), and the rate for correct classification is 85% and 71% respectively for binary and trinary cases. Actually classifying damage types is a very complicated process for the possible damage modes. Using only magnitude and phase at a single
frequency line is not appropriate to address such a complicated classification problem, but only for visualizing the SVM flow conceptually. Instead of eliminating most of the FRF information, Equation (9) defines the state space feature by using more frequency lines:

\[
x = \left[ H(\omega_1), H(\omega_2), \ldots, H(\omega_n), RH(\omega_1), RH(\omega_2), \ldots, RH(\omega_n) \right]^T \in \mathbb{R}^{2n}.
\]

in which \( n \) frequencies are considered to build up the feature vector \( x \). The new feature state has a lot greater dimension. For example, in this paper, all the frequency lines are included in the vector \( x \), and this leads to a \( \sim 1000 \)-D state space. Under this circumstance, the state space is not able to be visualized, and Figure 8 only plots the projection to three arbitrary dimensions.

Given feature space with such a high dimension, Gaussian kernelization becomes burdensome and could not return a reliable classifier easily, especially due to the relative lack of training data. Instead, linear kernel function is adopted, and classifications for both binary and trinary cases are deployed. By using the high-dimensional feature, the rates of correct classification for both binary and trinary cases are 100%, as shown in Figure 9, which means all the test cases are correctly labeled.

![Figure 8: High-dimension feature state space projected to arbitrary three dimensions](image)

![Figure 9: Rate of correct classification via SVM classifiers, linear kernel](image)
4 SUMMARY

This paper focuses on the frequency response function features of a rotary machine, and detects/classifies the bearing damages. There are three system statuses being distinguished, namely the undamaged baseline, bearing with damaged balls, and damaged outer race. FRF estimations are contaminated by operational variability and environmental noise, and the separation of features is ambiguous.

A support vector machine is adopted to learn the optimal decision boundaries, so that the noisy feature state can be partitioned and good classification is achievable thereby. In this paper, both binary and trinary classifications are implemented, with parametric study presented. Generally, the performance of binary classification is better than trinary cases because of the lower complexity, and higher-dimensional features will lead to a more specific classification. On the other hand, higher dimensional feature spaces will cause more computational burden in the SVM training procedure. Two types of kernel functions are adopted, namely linear kernel and radial bases function (Gaussian) kernel. SVM with linear kernel function separates data with a hyperplane and has the advantage of fast training, while Gaussian kernel has better flexibility to handle more complicated data sets. In this paper, the average rate of making correct classification is used as the comparison metric. SVM with Gaussian kernel outperforms linear kernel, for the lower dimension feature space, where the number of training set is about 100 times of the feature dimension. For high-dimensional feature space, the same amount of trainings cannot provide a reliable learning, therefore, a hyperplane decision boundary is a better solution.

REFERENCES