Adaptation and implementation of Probability of Detection (POD)-based fault diagnosis in elastic structures through vibration-based SHM approach

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Abstract

The demand for effective, efficient, and safe operations leads to the development of improved Structural Health Monitoring (SHM) systems. Probability of Detection (POD) serves as a performance measure for quantifying the reliability of conventional Nondestructive Testing (NDT) procedures taking into account statistical variability of sensor-based measurements. For Fault Detection and Isolation (FDI) methods, similar approach does not exist. This results mainly from the complexity of the dynamical behavior of systems monitored in relation to faults and sensors position (observability).

In this contribution, POD evaluation of vibration-based fault detection in elastic mechanical structures is developed. As example using different sensor types in combination with mechanical modifications of an elastic beam, the $a_{90/95}$ criteria representing probability of 90% at a confidence level of 95% is examined to the measurements. Based on the analysis of a suitably chosen feature (like eigenfrequency) and dependent on the modes considered, the efficiency and deficiency of this novel approach is shown. To improve the detection quality, suitable assumptions in combination with sensor/information fusion are applied to feature-based analysis as detection task using vibration measurements. Based on experimental evaluation from the results it can be concluded that a suitable fault-feature-POD-analysis can be successfully implemented as a new reliable measure for vibration-based FDI approaches. Applying decision fusion, the combination of different measurements allows the improvement of results.

Keywords: Vibration-based SHM, Fault Detection and Isolation (FDI), Probability of Detection (POD), Sensor/information fusion

1. Introduction

Structural Health Monitoring (SHM) is the process of implementing a damage detection and characterization strategy in monitoring engineering structures (1, 2). Damage in this context describes changes that adversely affects systems performance. The field of SHM increasingly has become an essential aspect of industrial practice to ensure the quality of products, safe operations, and to save cost. Many engineering structures are approaching or exceeding their initial design life, making SHM relevant (2). In SHM, monitoring is mainly applied on-line for large structures. Non-destructive testing in contrast are usually applied off-line after damage localization though it is used for in situ monitoring of structures like pressure vessels and rails. Safety-critical and high capital expenditure elastic structures have a wide application in the mechanical, civil, and aerospace industries. These include structures like turbomachinery, blades and tower of wind turbines, aircraft fuselage, bridges, skyscrapers among others. Vibration monitoring of
these elastic structures is common in the industrial practice. Vibration-based SHM operates on the principle that structural defects result in changes in the dynamical behavior. Fault identification is mainly based on displacement, velocity, and acceleration measurements at a single point (3). However, faults are usually local and may not significantly affect the measured output. From this point of view useful assessments may include observable effects and multipoint measurements. The evolution of the damage and the changes in the dynamics of the structure happen on different time scales (3). The evolution of the damage is slower compared to the vibration of the structure except for impact damage.

Despite the advances in SHM, questions remain. These include the transition from theory to practical implementation, early detection of faults, and reliability assessment of diagnostic statements (1, 4). The dynamics of vibrating systems can be altered by changes in the mass, stiffness, and damping properties. Making a reliable decision is of importance. Current FDI approaches utilize classification-related performance measures. These performance measures are subject to uncertainties hence the reliability is not quantified. Conversely, conventional NDT approaches use the Probability of Detection (POD) as a reliability measure. The POD quantifies a sensor/filtering approach with respect to mostly static measurements. Adapting and implementing this measure in the field of vibration-based SHM is strenuous. The difficulty results from the complexity of dynamical behavior in relation to faults, sensors position (observability), and related data analysis procedures. The POD description is usually based on the POD curve.

2. Brief introduction to the POD reliability measure

The POD measure specifies flaw size that can be detected/missed with a specific NDT method (consisting of sensor and postprocessing), considering statistical variability. Data used in producing POD curves are categorized by:

I. Hit/miss: produce binary/qualitative information as to the presence or absence of a flaw.

II. Flaw size vs. response (a vs.  ): data which provide quantitative information about the related target size to be detected.

Images must be pre-processed to provide response  or hit/miss as input for analysis. Flaw size detectability is based on the  criteria which represents 90% probability of detection with 95% confidence level. In the derivation of the POD (a) the first task is to undertake a regression analysis of the data gathered (5, 6).

Let \( x = f(a) \) and \( y = f(\hat{a}) \). The regression equation for a line of best fit to a data set is given by

\[
y = b + mx,
\]

where m is the slope and b the intercept.

Here the 95% Wald confidence bounds on y is constructed by

\[
y_{(a=0.95)} = y + 1.645 \tau_y,
\]

where 1.645 is z value of 0.95 and \( \tau_y \) the standard deviation of the regression line.

The Delta method, a statistical technique for deriving the variance of a function of asymptotically normal random variables with known variance, is an effective method to
transition from regression line to POD curve. It is useful to construct the confidence bounds of the POD(a) curve. This is done by computing the covariance matrix for the mean and standard deviation POD parameters $\mu$ and $\sigma$ respectively (5). To estimate the entries, covariance matrix for the parameters and distribution around the regression line need to be determined. This is done using the Fisher's information matrix $I$. The information matrix is derived by computing the maximum likelihood function $f$ of the standardized deviation $z$ of the regression line values. The entries within the Fisher's matrix are calculated by the partial differential of the logarithm of the function $f$ using the parameters of $\theta (m, b, \tau)$ of the regression line (5, 7).

From

$$z = \frac{(y-(b+mx))}{\tau}$$  

$$f = \prod_{i=1}^{n} \frac{1}{2\pi} e^{-\frac{1}{2}(\frac{y-(b+mx)}{\tau})^2}$$

the information matrix $I$ can be computed as

$$I_{ij} = -E \left( \frac{\partial}{\partial \theta_i \partial \theta_j} \log(f) \right).$$

The inverse of the information matrix yields $\varphi$ as

$$\varphi = I^{-1} = \begin{bmatrix} \sigma_b^2 & \sigma_b \sigma_m & \sigma_b \sigma_\tau \\ \sigma_m \sigma_b & \sigma_m^2 & \sigma_m \sigma_\tau \\ \sigma_\tau \sigma_b & \sigma_\tau \sigma_m & \sigma_\tau^2 \end{bmatrix}. \quad (6)$$

Next the mean $\mu$ and standard deviation $\sigma$ of the POD $(a)$ curve has to be computed. The parameters are related by $\mu = \frac{c-b}{m}$, where $c$ is the decision threshold and $\sigma = \frac{\tau}{m}$.

The POD is derived utilizing the cumulative density function $\Phi$ as

$$POD(a) = \Phi \left[ \frac{a - \mu}{\sigma} \right]. \quad (7)$$

The POD method qualifies a sensor/filtering approach with respect to mostly static measurements of effects (like cracks within material). In the context of this contribution the idea is transferred to the observation of vibrating systems. The challenge is how to implement this reliability measure for SHM-systems applied to dynamic environments.

3. Experimental results

To illustrate the proposed new approach experimental data (taken from the vibrations of an elastic beam) are used for analysis. Displacement, strain, and acceleration
measurements are taken. As features, eigenfrequency analysis are carried out on the first and second modes. The obtained results and the analysis are presented. The experiment is carried out on an elastic mechanical beam using the test rig in Fig. 1.

![Test rig (Chair SRS, UDuE) consisting of a one side-clamped beam with (1) bonded strain gauges (2) laser sensors, and (3) accelerometers](image)

**Figure 1**: Test rig (Chair SRS, UDuE) consisting of a one side-clamped beam with (1) bonded strain gauges (2) laser sensors, and (3) accelerometers

An elastic steel beam of dimensions 545 x 30 x 5 mm is clamped on one side and the other end is free. For suitable sensor placement the length of the beam is divided into five equal parts (Fig. 2). Piezoelectric accelerometers are attached at three positions (1, 2, and 3) on the beam. Two strain gauges are bonded onto the beam at positions 1 and 3. Two displacement measurements are taken at the two positions (2, 4) using non-contact laser sensors. The beam can be excited manually or by modal hammer.

In this contribution, injected changes (by adding masses) are used to apply faults within the elastic structure to modify the existing system. Two cases of point mass placement are examined. Case A involves placing the point mass at midpoint of position P2 and P3. Case B involves the placement of point mass at the midpoint of position P3 and P4. These masses are added to the specified locations in increment of 15 g. For every incrementally placed mass the beam is excited, and the corresponding data recorded.

The analysis is carried out for the first and second mode for each case (A and B) of mass placement. The regression analysis, confidence bounds, prediction bounds, and the POD characterization for each sensor (eight in all) is carried out. From the results, it becomes evident that the POD curve distribution is different depending on the sensor type and mode considered.
A summary of the results is given in Tab. 1. Based on the fault position, sensor location relative to fault, sensor type, and the mode considered, different results are obtained. The numbers indicate the maximum mass that can be missed with a 90% POD at a 95% confidence level. The lowest masses (denoted in blue) represent best results, in that the
sensor detects least mass change whilst the worst POD sensor characterization (denoted in red) requires high mass values to be detected with $a_{90/95}$ reliability.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Case A $a_{90/95}$ POD (g)</th>
<th>Mode 2 $a_{90/95}$ POD (g)</th>
<th>Case B $a_{90/95}$ POD (g)</th>
<th>Mode 2 $a_{90/95}$ POD (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC 1 at P1</td>
<td>74.04</td>
<td>48.15</td>
<td>52.21</td>
<td>9.915</td>
</tr>
<tr>
<td>ACC 2 at P2</td>
<td>74.04</td>
<td>55.78</td>
<td>52.15</td>
<td>9.293</td>
</tr>
<tr>
<td>ACC 3 at P3</td>
<td>74.04</td>
<td>72.59</td>
<td>52.15</td>
<td>13.36</td>
</tr>
<tr>
<td>SG 1 at P1</td>
<td>85.19</td>
<td>72.38</td>
<td>52.15</td>
<td>11.07</td>
</tr>
<tr>
<td>SG 1 at P3</td>
<td>126.7</td>
<td>62.23</td>
<td>54.08</td>
<td>9.394</td>
</tr>
<tr>
<td>Laser 1 at P1</td>
<td>67.3</td>
<td>61.03</td>
<td>52.1</td>
<td>43.49</td>
</tr>
<tr>
<td>Laser 2 at P4</td>
<td>74.04</td>
<td>–</td>
<td>52.15</td>
<td>–</td>
</tr>
</tbody>
</table>

Legend: - ACC: Accelerometer, SG: Strain gauge blue: best POD result, red: worst POD result

From the results (Tab. 1) it can be concluded that the POD of vibration-based analysis of elastic structures strongly depend on sensor type (dynamic range differs for each sensor), sensor position (same sensor at different positions produce different results), fault position (case A or B), and attribute selected.

4. POD-oriented view to vibration-based SHM

Structural Health Monitoring systems applied to vibrating elastic structures usually involves monitoring dynamical systems. The question that arises is how reliable measurements and related statements with respect to FDI-oriented statements in this field are and how to qualify/quantify vibration-based measurements. Here a new adaption of POD measure is proposed and implemented with respect to the integration of the vibration analysis results. Conventional NDT approaches apply statistical analysis and use the relationship between signal strength $\hat{a}$ and the size $a$ of the target initially causing the measurement. The variation of this can be discussed from a SHM-oriented view. In SHM, additional data analysis is required to convey information about the signal's attributes. The response is a feature-based response. Experiments have to be carried out including variations of faults (denoted by $a$). The output sensor values for varied fault size has to be recorded. Through signal processing, suitable features (eigenfrequencies) has to be extracted and used as response. Four possible graphs \{ $a$ vs $\hat{a}$, $a$ vs $\log(\hat{a})$, $\log(a)$ vs $\hat{a}$, and $\log(a)$ vs $\log(\hat{a})$ \} have to be plotted. The graph with best linearity, uniform variance, and uncorrelated observations is selected.
The detection task corresponds to the response threshold value (ε). This can be obtained from the regression analysis. From the general equation of the confidence bounds Eqn. 2, the values of slope, gradient, and standard deviation can be measured directly from the regression line. However, the size α generating \( a_{90/95} \) can be obtained from the regression line in combination with the POD curve as indicated in Fig. 6. The threshold defines the response detection value above which the fault can be detected with \( a_{90/95} \) reliability. The \( a_{90/95} \) value quantifies the maximum fault size that can be missed with a 90% POD at 95% confidence level.

5. **New approach to fusion of vibration-based POD values**

As concluded from previous section it can be stated that the use of a single sensor may not lead to satisfying fault detection accuracy. To improve this a novel approach to fuse POD values is developed based on sensor/information fusion. To fuse the detection results of the sensors related to their POD and \( a_{90/95} \) value, the Bayesian combination rule can be applied. For classification-related fault detection, the precision value is usually used as a performance measure, which is considered in the fusion process. Here the precision value is not available, but the POD values for specific fault sizes (here: masses) are measurable. Therefore, the POD for specific changes of each sensor (as shown in Tab. 2) can be used to calculate the belief value of the specific change (Tab. 3). Based on different combinations and as an example, the POD results for all three acceleration sensors and mode 2 features are examined for case B.
Table 2: POD values of specific masses

<table>
<thead>
<tr>
<th>Sensor / Mode</th>
<th>POD (9.29 g) [%]</th>
<th>POD (9.92 g) [%]</th>
<th>POD (13.36 g) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC 1 Mode 2</td>
<td>88.13</td>
<td>90.00</td>
<td>98.52</td>
</tr>
<tr>
<td>ACC 2 Mode 2</td>
<td>90.00</td>
<td>92.07</td>
<td>96.90</td>
</tr>
<tr>
<td>ACC 3 Mode 2</td>
<td>76.19</td>
<td>78.89</td>
<td>90.00</td>
</tr>
</tbody>
</table>

The proposed approach gives consistent and reasonable results. For case 1 where no sensor detects the fault, the belief values range from 0.01% to 0.47%. Case 8 on the contrary, using all three sensors to detect the fault will give belief values from 99.53% to 99.9%.

Table 3: Fusion results

<table>
<thead>
<tr>
<th>Case</th>
<th>ACC 1 Mode 2</th>
<th>ACC 2 Mode 2</th>
<th>ACC 3 Mode 2</th>
<th>bel (9.293 g) [%]</th>
<th>bel (9.915 g) [%]</th>
<th>bel (13.36 g) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.47</td>
<td>0.26</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4.57</td>
<td>3.45</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>27.49</td>
<td>25.66</td>
<td>4.96</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>20.49</td>
<td>17.18</td>
<td>19.14</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>79.51</td>
<td>82.82</td>
<td>80.86</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>72.51</td>
<td>74.34</td>
<td>95.04</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>95.43</td>
<td>96.55</td>
<td>99.57</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>99.53</td>
<td>99.74</td>
<td>99.99</td>
</tr>
</tbody>
</table>

6. Summary and conclusion

This contribution presents a new reliability assessment for vibration-based SHM-systems. The POD measure is adapted and applied as a reliability metric for Fault diagnosis. The measurement paths used are displacement (laser sensors), strain, and acceleration. The results indicate that the POD-values depends on the sensor type, sensor position, fault position, and the feature selected. This allows a new (POD-based) insight to the use of vibration-based measurements and FDI-related approaches. The $a_{90/95}$ criteria representing probability of 90% at a confidence level of 95% is successfully implemented in vibration-based FDI as a reliability measure. A
novel approach to fusion of $a_{90/95}$ POD through sensor-information fusion values is presented. The results are compatible with the detection task.

REFERENCES