Analytical, Numerical and Experimental formulation of the sensor placement optimization problem for guided waves

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Abstract

Guided waves (GW) are one of the most promising tools for structural health monitoring (SHM). They allow fast inspection of a large area. Thus GW based SHM is finding applications in several fields like aerospace, automotive, wind energy, etc. The GW propagate along the surface of the sample and get reflected from the boundaries and damage. Through proper signal processing of the reflected waves based on their time of arrival, the damage can be detected and isolated. But in real applications the structures often have complex geometry and additional structural features like rivets, stiffeners, holes, etc. which make the use of GW a challenge. For complex structures a higher number of sensors may be required which increases the cost of the equipment as well as the mass. The mass increase is especially detrimental in aerospace applications where added mass leads to increase in the cost of operation. Thus there is an effort to reduce the number of sensors. In addition, for the safety and reliability of the aircraft it is of utmost importance that the entire structure can be investigated. Hence it is necessary to optimize the locations of the sensors in order to maximise the coverage while limiting the number of sensors used.

The optimization problem is very large and extremely non-linear hence use of trial and error based techniques is not useful. The use of only numerical tools too is computationally expensive as for accurate wave propagation, precise modelling and use of large number of elements is necessary. On the other hand the analytical tools make several simplifying assumptions which leads to deviation from the observations seen in real life. Thus there is a need to balance the three approaches to obtain relevant sensor placements which fulfill the goals of the optimization.

Thus, this research focuses on the formulation of the problem of optimization of sensor placement. The work starts with the definition of the cost function followed by the optimization based on analytical tools. The optimized placement is then simulated using the time domain spectral element method and then validated through experiments. The experimental observations are then back propagated to overcome the simplifying assumptions made in the analytical approach and update the numerical model which will allow application of the analytical approach for more complex optimization problems.

1. Introduction

Guided waves (GW) are one of the most promising tools for structural health monitoring (SHM) of large plate like structures. They allow fast inspection of large areas and may be used not only for detection but also for damage localization. They have been shown to be sensitive to small levels of damage as well as damage.
originating from several different mechanisms including impact, fatigue, as well as moisture and temperature induced damage scenarios (1-4). The use of GWs in metallic components is quite common, and due to their success, their use in composite structures is being investigated with promising results. Unfortunately, composite structures pose a complex problem due to their higher attenuation and the anisotropic nature. To counter the high attenuation shown by composites, a higher excitation energy as well as increase in the number of sensors is required. The increase in number of sensors is associated with an increase in cost and difficulties with the instrumentation of the system. Not only does the extra number of sensors increase the cost of implementation, but in applications like airplanes the added sensors lead to added mass of the instrumentation which leads to higher operating costs. Hence there is a need to reduce the number of sensors to lower the cost of implementation as well as the secondary costs without compromising on the quality of the structural health monitoring of the system. For this purpose optimization of the sensor placement is essential.

The problem of optimization is quite relevant and has started to attract attention from several researchers, unfortunately the subject has not been treated with enough rigor, and hence this study addresses the lack of literature in this area. The earliest work in the area of optimization of sensor placement made use of the analytical approach for determining the sensor placement (5-6). Some recent work has built on this work and have proposed optimization taking into consideration the different parameters to which the GW based SHM techniques are sensitive (7-9). But this approach is resource intensive and makes several simplifying assumptions. Thus there is a need for a different approach to tackle the optimization problem. The present study aims at providing a framework of methodology for the optimization of sensors placement through a combination of analytical, numerical and experimental approach. The combination of the three approaches promises to reduce the simplifying assumptions which lead to deviation between the computed optima and the reality, without allowing the effort required for optimization to increase. The methodology at the first stage is applied for a simple aluminium structure, but considering that the work is at the methodology level, it can be easily extended for complex composite structures at a later stage.

2. Methodology

The aim of the paper is to outline a sensor placement methodology based on analytical, numerical and experimental analysis. The aim is to streamline the entire process and make it less resource intensive both in terms of computational and material resources. Thus a combination of the three approaches is proposed to achieve accuracy at a reasonable cost. Each of the three approaches has their own set of advantages and disadvantages, for instance the analytical approach allows faster computation and is less intensive in terms of computations as well as requires low levels of instrumentation. But the simplifying assumptions made for the analytical approach make it too simplistic and it does not reflect the true nature of the problem. In the numerical approach, with the development of novel modelling approaches, and parallel computing the time requirement for the computations is lowered considerably, but this approach remains computationally demanding requiring several days for simulated studies. Also the numerical modelling uses some simplifying assumptions and the results obtained are often too idealistic as compared to real application. The experimental approach is the
most time consuming and resource intensive of all. It requires time for preparing the samples, performing the measurements and analysing the data. The experimental approach is not scalable or may not be customized easily for other application of the sensor placement, but it reflects all the facets of the optimization problem as well as the uncertainties in the measurements. It also takes into consideration all the physical phenomena occurring in the sample, some of which may not be possible to replicate in the numerical and analytical approach. Hence, by using a combination of the three approaches the individual shortcomings can be overcome while allowing synergy in the advantages offered by each of the approach.

![Figure 1. Synergy and data transfer between three approaches](image)

2.1 Optimization

The combination of the three approaches is the aimed at obtaining the optimization of the sensor placement. Irrespective of the approach used, the most important feature of the optimization problem is the definition of the application demands or in other words the cost function for the optimization. The aim is to design a sensor placement for damage detection and isolation using the GW based approach. In order to ensure the damage detection is possible on the majority of the structure, coverage of the structure with the sensors is a significant criterion. In addition to the detection, isolation of damage too is desirable. The isolation of the damage may be carried out by using triangulation. Thus it is desirable that any point on the structure be investigated by at least three sensors. Unfortunately the number of sensors employed should be as low as possible. This grid provides points for the assessment of the coverage. If the grid point in the structure lies within the sensing range of the sensor then the coverage is said to be achieved.

Hence in nutshell: we need to develop a metric which ensures coverage of maximum portion of the structure with at least 1 sensor as well as with at least 3 sensors while
using the minimum sensors possible. Thus keeping in mind these requirements the cost function was developed given by equation (1)

\[ cost = \alpha \frac{-cov^3}{s} + \beta \times pen0 \]  

(1)

where, cov3 is the number of points of the grid which lie within the sensing range of 3 or more sensors, pen0 is the number of points which do not lie in the sensing range of a single sensor. \( \alpha \) and \( \beta \) are weighting values to determine the relative merit for each of the parameters and \( s \) is the number of sensors.

The determination of the cov3 and the pen0 for each sensor placement was carried out analytically as the problem size is too big to be tackled through experiments alone or through use of numerical tools. The determination of the sensing range of the sensors for composite structures needs to be determined based on experiments but for the present case the study was restricted to aluminium for which the propagation wave-front is well studied.

It is envisaged that for an isotropic structure, the sensing area will be elliptical with the sensor and actuator at the foci. While for composite structures, the shape of the sensed area may take complex shapes based on the fibre orientation. The range and directionality of the wave propagation for composites can be determined from full-field measurements using the Laser Doppler Vibrometer (LDV).

For the optimization, the integer genetic algorithm was implemented with each gene corresponding to the sensor location. The GA is ideally suited for large problems and where the cost function is not linear or differentiable. The parameters of the GA were based on the experience of the authors as number of generations=5000, the number of chromosomes=64. The mutation rate =1% and elitism= 50%.

The work carried out using each of the three approaches is given below. Once the cost function has been established, the integer GA was chosen for optimization.

2.1 Analytical Approach

For the isotropic aluminium case, the wave velocity in all directions is considered constant. Also, aluminium is a well-studied material and the wave velocities have been well documented. Also the attenuation in aluminium is quite low, so, the range was considered large enough for the plate under investigation. So the scope of the tasks under the analytical approach was to determine the highest coverage obtained by a particular sensor-actuator pair. This can be achieved by determining the largest ellipse that will fit in the plate. This is done by solving the equation for tangency of ellipse with known foci (sensor-actuator pair locations). The use of the tangency condition is required as the ability of the basic signal processing algorithms for location and detection of damage is compromised for cases where the wall reflections occur. The maximum coverage achieved can be changed based on the fidelity of the signal processing tool and the ability of the algorithm to adjust for reflections from the edges. Once the largest ellipse is determined, the coverage for that sensor pair can be calculated. Similarly the coverage for all sensor-actuator pairs are combined to obtain the cumulative coverage of the sensor network. The coverage matrix obtained then may be used for calculating the cost function.
2.2 Numerical Approach

The numerical approach is necessary for giving insight into the complex wave interactions that occur. The key aspect for using signal processing tools for damage detection is the estimation of the first arrivals of the waves and ability to differentiate between the arrivals of the reflections from structural components and damage. The numerical model allows us to visualize the wave propagation in time and space, which is possible with the experimental approach only in few conditions. Also at a later stage when more structural features like stiffeners and rivets will be investigated, the knowledge of these physical interactions will allow to update the analytical approach for the complex cases.

2.3 Experimental Approach

The experimental approach will be required for determining the attenuation, directionality of wave propagation and studying the physics of the interaction between the waves and the different structural features like stiffeners, rivets etc. Also, the ultimate goal of the sensor placement optimization is the deployment of the sensors and the experimental validation of the sensor placement and the ability to detect damage. Hence, damage will be simulated at different locations and the ability of the sensor network to detect it will be investigated. The performance will be compared to other placement strategies to determine the performance of the sensor placement optimization exercise.

3. Results and Discussion

The optimization approach was applied on a simple aluminium plate of dimension $1\times 1\times 1$ mm as shown in Figure 2. The material properties of aluminium alloy were Young’s modulus $E = 72$ GPa, Poisson ratio $\nu = 0.33$, mass density $\rho = 2660$ kg/m$^3$. The plate was instrumented with 9 PZT sensors at locations which were determined based on the optimized results obtained from the analytical approach. The locations of the sensors are shown in Figure 3.

![Figure 2. Aluminium plate with PZT sensors](image)
The analytical approach was validated using the numerical and the experimental approach. The validation of the numerical model with experimental full wave field data has been carried out and has been reported in (10). The results for the optimization and validation are presented and its implications are discussed below.

### 3.1 Genetic Algorithm

The Table. 1 gives the performance of the GA as compared to the random placement. As can be seen the Cov0 (complement of pen0), Cov3 as well as the cost function for the optimized placement are better than the one for the random placement. This shows that there is merit in optimizing the sensor placement. It is worth noting that the implementation of the GA and the cost function have a great bearing on the optimization. For instance the Cov0 and Cov3 needs to be normalized or coded in terms of percentage coverage of the plate, otherwise the metric is sensitive to the discretization of the plate which is not desirable. Also the optimized values for the GA need to be determined based on sensitivity studies. The Figure 4 shows the convergence of the GA. Also, the Figure 5 shows the surface plot indicating the coverage for the two sensor placements.

<table>
<thead>
<tr>
<th>Run</th>
<th>Placement</th>
<th>Number</th>
<th>Cost</th>
<th>Cov0 %</th>
<th>Cov3 %</th>
</tr>
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<tr>
<td>1</td>
<td>1, 9, 10, 21, 36, 40, 44, 46, 73, 81</td>
<td>10</td>
<td>-15189</td>
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<td>90.7</td>
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<td>-13141</td>
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<td>96.10</td>
<td>88.66</td>
</tr>
<tr>
<td>4</td>
<td>1, 9, 21, 23, 67, 69, 72, 73</td>
<td>8</td>
<td>-17909</td>
<td>96.68</td>
<td>87.07</td>
</tr>
<tr>
<td>Rand</td>
<td>3, 14, 26, 38, 47, 52, 69, 75, 81</td>
<td>9</td>
<td>-462.44</td>
<td>90.61</td>
<td>86.43</td>
</tr>
</tbody>
</table>
3.2 Validation
To verify if the largest ellipse fitting inside the plate is computed correctly the results obtained from the time of flight analysis for the numerical model and the experimental analysis are compared with the analytical approach. The time signals using the numerical model and the experimental data are shown in Figure 6.
The numerical and the experimental results are in relatively good agreement, the differences in the measurements may be explained based on the low sampling frequency of the instrument. The Figure 7 shows the time of flight for actuation at position 1 and sensing at position 20. The arrivals of the $A_0$ wave and its reflection from edge at Y axis as well as $S_0$ were identified based on the full wave-field simulation from the numerical analysis shown in Figure 8. This proves the need of the numerical approach for the additional insight.

![Figure 7. Time signal for actuation at 1 and sensing at position 20](image-url)

![Figure 8. Still for full wavefield animations from numerical study](image-url)

The time of flight analysis was used to determine the coverage achieved for each sensor pair. The compared plots for the analytical, numerical and experimental approach are shown in Figure 9.

![Figure 9. Validation of Analytical approach](image-url)
The experimental results for the sensor pair 58, 65 were difficult to extract due to the complex interactions between the wave reflections. A clear reflection for the $A_0$ or $S_0$ could not be extracted, but the numerical results were available through the full field simulation and the results are in good agreement.

### 4. Conclusions

The paper outlines a methodology for optimization of sensor placement for damage detection using GW technique. The methodology is based on analytical, numerical as well as experimental approach where input for the optimization is secured using experimental data and the insights into the complexity of the problem are gained from the numerical simulations.

The methodology yields an improved coverage using the same number of sensors, faster computation speeds as compared to numerical only approach or only experimental approach as well as lesser simplifying assumptions than purely numerical or analytical approach. The better coverage ensures a more reliable SHM of the specimen.

The authors agree that at the current stage the simple structure of the sample, the isotropic material simplify the problem and application to complex shapes with additional structural features and anisotropic materials will make the problem challenging. In theory the approach proposed can be easily extended to any material through the use of beam forming approach or parametrization of the wave-front.

Also the numerical approach using similar element formulation to that in the study has been incorporated for composite materials as well, so even though problem for real applications is complex, it can be tackled with the above approach.

The future work identified is study of the optimization problem for more complex shapes, and in the presence of other structural features. Also study for the use of this technique for anisotropic materials will be undertaken. Also there is a need to quantify the performance of the optimized sensor placement for damage detection through experimental study.

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### References and footnotes


