Numerical calibration of direct current potential drop measuring: A comparison of FEM- and Bayesian filtering-based approaches

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Abstract

The direct current potential drop method is a widespread non-destructive testing and evaluation technique utilised especially in shipbuilding and aerospace industry. Even without feasibility of optical assessment it allows for the accurate health monitoring of electrically conductive components and structures. At anticipated damage locations, the drop of the electric potential of an injected electrical current is measured and subsequently used to determine the respective damage extent on the basis of a specific calibration curve. Though knowledge on this specimen-bound relation between potential drop and damage extent is crucial, the experimental calibration approaches either entail considerable efforts and limitations in the test setup or ensue only afterwards. As opposed to this, FEM-based numerical calibration allows for a real-time damage extent monitoring, however only in a generalised form since no actual measurement data are incorporated into the process. Recent studies propose a new calibration methodology by means of Bayesian filtering where the damage extent is inferred probabilistically from potential drop measurements for fatigue loading conditions. The present work aims to provide a comparison of the aforementioned numerical approaches and their benefits and drawbacks.

Keywords: Potential drop measuring, Calibration curve, Finite element analysis, Bayesian filtering

1. Introduction

Measuring the difference of the electrical potential of two points with an intermediate defect to determine its extent within the direct current potential drop (DCPD) method is a widely accepted approach to testing and evaluation in various engineering domains like petrochemical and power generation industries, civil engineering, shipbuilding and aerospace industry [1]. At anticipated damage locations it can be conducted non-destructively and without optical assessment within the scope of structural health monitoring (SHM) whereas the linkage of measured potential drop and desired knowledge of the damage extent - a calibration curve - is crucial.
In crack growth applications, the various ways to determine the calibration curve comprise experimental [2–5], analytical [6, 7], analytic-numerical [8–10] and recently proposed numeric machine learning-based [11–13] approaches. The experimental calibration entails either considerable efforts and limitations in the test setup or ensues only afterwards whereas the analytical calibration is not applicable to more complex geometries. Consequently, the analytic-numerical calibration by means of finite element (FE) analysis and the machine learning perspective in the form of Bayesian filtering are to be considered and compared hereinafter.

In Section 2., the problem is stated by presenting the experimental setup, contemplating the basics of the electric potential formulation in the FE analysis as well as introducing the Bayesian approach and the utilised probabilistic state space model. Section 3. first emphasises the peculiarities of the FEM- and Bayesian filtering-based calibration results. Subsequently, the respective outcomes are compared and contrasted with a focus on accuracy and practicability. Section 4. then concludes the paper.

2. Problem statement

In DCPD measuring, a direct current injected into a specimen is utilised to obtain the electrical potential drop over a damage location. Since the damage extent directly influences the corresponding electrical resistance, a formal relation between potential drop and damage extent can be established. The objective of the DCPD method is hence illustrated in Fig. 1: Potential drop measurements over cycles (I) are to be utilised to determine the damage growth over cycles (III). This necessitates a linkage of the quantities of potential drop and damage size, the calibration curve (II). The specific form of the calibration curve however depends on the specimen as well as on the experimental setup and loading conditions, i.e. on the specimen geometry, measuring positions and the form of the crack which in turn depends on the applied loads. In the following, the experimental setup as well as the basic ideas regarding the approaches of finite element analysis of the electric...
potential and state and parameter inference by means of Bayesian filtering are revisited.

2.1 Experimental setup

![Figure 2: Applied stress, geometries, crack propagation orientation as well as electrode and probe positioning of the experimental setup](image)

The experimental testing of the fatigue crack growth is performed as shown schematically in Fig. 2. Pre-notched Udimet 720Li superalloy specimens with a quadratic cross section are subjected to cyclic uniaxial tensile stress $\sigma(t)$ with double amplitude $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$ to allow for Mode-I crack initiation and crack growth. Due to the loading conditions and the specimen geometry, the crack surface is assumed to evolve in a quarter-circular shape of dimension $a \leq 0.6W$ with $W = 10\text{mm}$ over cycles $N$. The electric direct current is injected through electrodes at positions 3 and 4 in sufficient distance to the damage location as to allow for initial electric field uniformity in the vicinity of the inspection area [1]. The corresponding potential drop is measured at positions 1 and 2 periodically at $\sigma(t) = \sigma_{\text{max}}$.

2.2 Finite element analysis

Under the assumption of stationary electric fields and ohmic charge transport, the reduced continuity equation and Ohm’s law

$$\text{div} \, j = 0$$  
$$j = \kappa E = -\kappa \text{grad} \Phi,$$

(1)  
(2)

can be rewritten as homogenous Poisson’s equation of an electric potential

$$\text{div} (\kappa \text{grad} \Phi) = 0,$$

(3)

where $j$ denotes the current density, $\kappa$ the electrical conductivity, $E$ the electric field and $\Phi$ the electric potential [7]. In a problem domain $\Gamma \subset \mathbb{R}^3$ with Dirichlet boundary conditions
The modelling and solving of the problem in Eq. (4) is herein accomplished by means of Comsol Multiphysics using the Electric Currents interface of the AC/DC module (see Fig. 3). The respective geometric dimensions of the positions of potential probes \((\delta_0^\text{x}, \delta_0^\text{z})\) and especially of the crack size \(a\) are modified utilising a coupling to Matlab.

### 2.3 Bayesian approach

In the application of Bayesian methods, the lack of knowledge of dynamic systems and unknown quantities is expressed by the utilisation of random variables. Thereby, various sources of uncertainty can be modelled and consequently be accounted for \([15–22]\). The following exposition is motivated by Särkkä \([23]\), additional details may be found for example in the works of Chen \([24]\) and Arulampalam et al. \([25]\).

Considering a dynamic system at time step \(k\) of hidden states \(x_k \in \mathbb{R}^{d_x}\) (e.g. unknown crack extents), of observations \(y_k \in \mathbb{R}^{d_y}\) (e.g potential drop measurements) and of unknown parameters \(\theta \in \mathbb{R}^{d_\theta}\) (e.g material-dependent constants of DCPD measuring and crack evolution laws) the dynamic and measurement model functions \(f\) respectively \(h\) can
be written as

\[ x_k = f(x_{k-1}, \theta) + q_{k-1}, \]
\[ y_k = h(x_k, \theta) + r_k. \]  

These model functions describe how the states of the previous time step \( x_{k-1} \) evolve in time towards the current state \( x_k \) and subsequently what actual observation \( y_k \) is to be perceived then. The terms \( q_{k-1} \) and \( r_k \) denote additive process and measurements noises whereas the dependency on inputs \( u_{k-1} \) is omitted for simplification. Since the states \( x_k \), observations \( y_k \) and unknown parameters \( \theta \) are treated as random variables, Eq. (5) and (6) are rewritten probabilistically as

\[ x_k \sim p(x_k|x_{k-1}, \theta), \]
\[ y_k \sim p(y_k|x_k, \theta), \]

where \( p(\cdot|\cdot) \) denotes a conditional probability density function (PDF). Starting from prior PDFs of the hidden states \( p(x_0|\theta) \) and unknown parameters \( p(\theta) \) which express an initial belief on the distribution of these random variables, Bayesian filtering is applied in a recursive scheme of prediction and updating. The prediction step

\[ p(x_k|y_{1:k-1}, \theta) = \int p(x_k|x_{k-1}, \theta)p(x_{k-1}|y_{1:k-1}, \theta)dx_{k-1}, \]  

and update step

\[ p(x_k|y_{1:k}, \theta) = \frac{p(y_k|x_k, \theta)p(x_k|y_{1:k-1}, \theta)}{p(y_k|y_{1:k-1})}, \]

allow the calculation of conditional states whereas the conditional PDF of the unknown parameters is given as

\[ p(\theta|y_{1:k}) = p(\theta|y_{1:k-1}) \int \frac{p(y_k|x_k, \theta)p(x_k|y_{1:k-1}, \theta)}{p(y_k|y_{1:k-1})}dx_k. \]

Generally, the integrals in Eq. (9) to (11) are intractable which necessitates the utilisation of approximation methods. A short overview of such approaches and technical details for the herein applied hybrid of unscented Kalman filtering (UKF) [26–28] and approximate grid-based methods [24, 25] can be found in Berg et al. [12, 13].

The fatigue crack growth illustrated in Fig. 2 is modelled by means of a Paris’ law [29] modification according to Forman/Mettu [30, 31] as

\[ \frac{da}{dN} = C \cdot F \cdot \Delta K(a)^n \left( 1 - \frac{\Delta K_{th}}{\Delta K(a)} \right)^p \left( 1 - \frac{1}{1-R_{\sigma}} \frac{\Delta K(a)}{K_c} \right)^q, \]

where \( a \) denotes the crack size, \( K(a) \) the stress intensity factor (SIF) range, \( N \) the number of cycles, \( F \) the crack velocity factor, \( R_{\sigma} = \sigma_{\min}/\sigma_{\max} \) the stress ratio, \( \Delta K_{th} \) the threshold SIF range, \( K_c \) the fracture toughness and \( C, n, p, q \) empirical constants. Setting \( F = 1, q = 1 \) and \( p = n \) as simplification, collecting the remaining empirical constants as well as the
threshold SIF range and fracture toughness in the parameter vector $\theta^{\text{dyn}} = \{C, n, \Delta K_{\text{th}}, K_c\}$ and associating the crack size $a$ with the one-dimensional state $x$, Eq. (12) is recasted as

$$\frac{dx}{dN} = \theta_1^{\text{dyn}} \Delta K(x) \left( 1 - \frac{\theta_3^{\text{dyn}}}{\Delta K(x)} \right) \frac{\theta_2^{\text{dyn}}}{1 - \frac{1}{R_{\sigma}} \frac{\theta_4^{\text{dyn}}}{\Delta K(x)}}. \tag{13}$$

The dynamic model function $f(x_{k-1}, \theta^{\text{dyn}})$ is then obtained as Runge-Kutta solution to the initial value problem in Eq. (13) with $x(N_0) = x_0$ and step size $N_k$. Additionally, the one-dimensional observation $y_k$ is interpreted as measured potential drop and assumed to be linearly dependent on the state $x_k$ [5, 32], yielding the measurement model function

$$h(x_k, \theta^{\text{ms}}) = \theta_1^{\text{ms}} x_k + \theta_2^{\text{ms}}. \tag{14}$$

The additive process and measurement noises in Eq. (5) and (6) are assumed to be normally distributed according to $q_{k-1} \sim \mathcal{N}(0, Q_{k-1})$ and $r_k \sim \mathcal{N}(0, R_k)$, whereas only the parameter $Q_{k-1} = Q = \theta^{\text{ns}}$ is not known. To complement the probabilistic state space model, the prior knowledge is surmised as follows: Due to the pre-notchting of the specimen, the initial state $x_0$ is considered normally distributed with mean $m_0$ and variance $\sigma^2$ whereas the unknown parameters $\theta = \{\theta^{\text{dyn}}, \theta^{\text{ms}}, \theta^{\text{ns}}\}$ are assumed to be distributed uniformly on $[\theta_1, \bar{\theta}_1] \times \ldots \times [\theta_d, \bar{\theta}_d]$. For further explanations and clarifications regarding the definition of the probabilistic state space model and a discussion of the impact of choices of methodology, model functions and boundaries, the reader is once again referred to Berg et al. [12, 13].

3. Results and discussion

This section aims at providing a comparison of FEM- and Bayesian filtering-based calibration of DCPD measuring. The information displayed encompass the experimental, FEM-based and Bayesian filtering-based calibration curves $\Delta \Phi$ over $a$ for eight different data sets, whereas the former serves as reference.

3.1 FEM-based calibration and probe position dependency

In addition to the aforementioned depiction of the simulated potential $\Phi$ and the thereof resulting equipotential surfaces in Fig. 3, the corresponding calibration curve $\Delta \Phi_0^c(a)$ is shown in Fig. 4 (black line). The linearity of the calibration curve for point-symmetrically aligned probe positions with dimensions $\delta_x^0$ and $\delta_z^0$ is apparent (emphasised by the straight dashed line) as assumed for the measurement model function in Eq. (14) and likewise reported for example by Cláudio et al. [9]. Nevertheless, the influence of the probe positioning is non-negligible as can be seen in the FE analysis results for relatively small lateral and axial deviations $\delta_x$ respectively $\delta_z$. Pure lateral displacements of the probe positions $\delta_x$ (blue lines), i.e. a translation along the edge of the cross section, result in either a shift
above (negative $\delta_x$) or below (positive $\delta_x$) the linear calibration curve $\Delta \Phi^{fe}_0(a)$ whereas the latter induce a buckling for smaller crack sizes. In contrast to that, pure positive axial displacements $\delta_z$ (red lines) lead to a shift above $\Delta \Phi^{fe}_0(a)$ as well as to a parabolic form that amplifies for higher values of $\delta_z$. Ultimately, the combination of lateral and axial displacements (yellow lines) entails a varying intensity of the parabolic form and a shifting of the respective intercept.

### 3.2 Bayesian filtering-based calibration

The Bayesian filtering-based calibration curves $\Delta \Phi^{bay}_i(a)$ for $i = 1, \ldots, 8$ of the eight experimental data sets are obtained at the last time step $k = T$ by approximating the conditional posterior density of the parameters in Eq. (11) for four different scenarios of prior knowledge regarding the dynamic model parameters $\theta^{dyn}$:

(a) Specimen-specific parameters $\theta^{dyn, true}$ are available leading to the conditional PDF $p(\theta^{ms}|y_1:T; \theta^{dyn, true})$ (the semicolon indicates that $\theta^{ms}$ is not conditioned on this random variable but in this case dependent on the deterministic vector $\theta^{dyn, true}$).

(b) Non-specific averaged parameters $\theta^{dyn, avg}$ are known, the conditional PDF takes the form $p(\theta^{ms}|y_1:T; \theta^{dyn, avg})$.

(c) The vector of known quantities is reduced to $\theta^{dyn, avg}_{red} = \{\theta^{dy, avg}_3, \theta^{dy, avg}_4\}$, i.e. the empirical constants $C$ and $n$ in the Forman/Mettu modification in Eq. (12) have to be inferred as well. This yields the marginalised conditional PDF $p(\theta^{ms}|y_1:T; \theta^{dyn, avg}_{red})$, where the unknown parameters $\theta^{dy}$ are integrated out.

(d) All dynamic model parameters are unknown, resulting in the marginalised conditional PDF $p(\theta^{ms}|y_1:T)$, where $\theta^{dy}$ is integrated out. The uncertainty introduced into the probabilistic state space model is largest in this scenario.

Additionally, the unknown parameter of the dynamic model noise $\theta^{ns}$ is integrated out in all four studies. The approximation of the respective conditional PDF is utilised to obtain maximum a posteriori (MAP) estimates of the inferred measurement model parameters.
\( \theta^\text{ms} \). MAP estimates of the form
\[
\hat{\theta} = \arg \max_{\theta} p(\theta | y_{1:k}) ,
\]
confine the posterior knowledge to a Dirac delta function \( p(\theta | y_{1:T}) \approx \delta(\theta - \hat{\theta}) \) which is exploited hereinafter for the sake of simplicity. The consideration of time step \( T \) in the comparison follows from the fact that the maximal amount of measurement data is to be harnessed to obtain the respective calibration curves. An earlier time step \( k < T \) is of course conceivable, however contradicts the findings of especially accurate results in the latter part of maximal possible cycles as shown in Berg et al. [13]. The MAP estimates \( \hat{\theta}_{i}^\text{ms} \) yield the respective calibration curve \( \Delta \Phi_{i}^\text{bay}(a) \) by employing the measurement model function in Eq. (14).

### 3.3 Comparison

The results of FEM- and Bayesian filtering-based calibration of DCPD measuring in corner crack specimens is depicted in conjunction with the experimentally obtained data for eight test series in Fig. 5. The numbering of the individual sub-figures is in accordance to the different scenario enumeration of the preceding Section. Each plot displays eight pairs composed of experimental calibration curves \( \Delta \Phi_{i}^\text{exp}(a) \) and a corresponding filtering outcome \( \Delta \Phi_{i}^\text{bay}(a) \). Additionally, one FEM-based calibration curve \( \Delta \Phi_{i}^\text{fe}(a) \) is shown. As mentioned in Section 3.1, the probe welding is intended at locations \( (\delta^0_x, \delta^0_z) \), however deviations \( \delta_x \) and \( \delta_z \) of the positions occurring for the respective specimens are not captured. A single calibration curve \( \Delta \Phi_{0}^\text{fe}(a) \) at \( (\delta^0_x, \delta^0_z) \) is therefore provided for the eight test series in the different scenarios of available prior knowledge (black line). In contrast to that, the experimental data \( \Delta \Phi_{i}^\text{exp}(a) \) (solid colored line) are plotted with the calibration curves \( \Delta \Phi_{i}^\text{bay}(a) \) (dashed colored lines) for each specimen \( i \). Additionally, the arithmetic means of the relative deviation of the respective maximal crack extents

\[
\begin{align*}
 r^\text{bay} &= \frac{1}{8} \sum_{i=1}^{8} \frac{\left| a_{i}^\text{exp}(T) - a_{i}^\text{bay}(T) \right|}{a_{i}^\text{exp}(T)}, \\
 r^\text{fem} &= \frac{1}{8} \sum_{i=1}^{8} \frac{\left| a_{i}^\text{exp}(T) - a_{i}^\text{fe}(T) \right|}{a_{i}^\text{exp}(T)},
\end{align*}
\]

are given in the captions.

As anticipated, the FEM-based calibration curve does not fit the experimental data of each individual specimen since deviations \( (\delta^0_x, \delta^0_z) \) in the probe positioning are not accounted for and variabilities for example in the electrical conductivity \( \kappa \) occur naturally. Nevertheless, the calibration curve coincides with the actual data quite well in the case of five test series with a mean relative crack extent deviation as defined in Eq. (16) adding up to mere \( r^\text{fe} = 0.0371 \). As opposed to that, the Bayesian filter-based calibration curves display a specimen-specific compliance with the experimental data which - self-evidently - follows from the utilisation of test series-bound observations \( y_{1:T} \). The quality of the respective compliance of the calibration curves \( \Delta \Phi_{i}^\text{bay}(a) \) and \( \Delta \Phi_{i}^\text{exp}(a) \) varies especially in light of the different scenarios of prior knowledge on the dynamic model parameters.
Figure 5: FEM-based (black line), experimental (solid colored lines) and Bayesian filtering-based calibration curves \( \Delta \Phi \) over \( a \) for different scenarios of available prior knowledge with mean relative deviation of the crack extent \( r_{fe} = 0.0371 \) for eight specimens.

\( \theta^{\text{dyn}} \). For given specimen-specific parameters \( \theta^{\text{dyn, true}} \) (which is of course unrealistic but included nonetheless to demonstrate the capability of the Bayesian filtering) and known average parameters \( \theta^{\text{dyn, avg}} \), the results are markedly superior compared to the FEM-based calibration. The mean of the relative deviations are calculated as \( r_{a}^{\text{bay}} = 0.0156 \) and \( r_{b}^{\text{bay}} = 0.0259 \). In case of increased uncertainty due to reduced knowledge on the dynamic model parameters, i.e. given \( \theta^{\text{red}} \) in (c) and completely unknown parameters \( \theta^{\text{dyn}} \) in (d), the calibration curves \( \Delta \Phi^{\text{bay}, i}(a) \) and \( \Delta \Phi^{\text{exp}, i}(a) \) partially differ noticeably with mean relative deviations \( r_{c}^{\text{bay}} = 0.0831 \) and \( r_{d}^{\text{bay}} = 0.0703 \) greater than \( r_{fe} \). When looking more closely at the outcome in for example Fig. 5 (d) however, the outlier \( \Delta \Phi^{\text{bay}, i}(a) \) that contributes to \( r_{d}^{\text{bay}} \) the most becomes apparent (without data set \( i = 2 \) the mean reduces to \( r_{d}^{\text{bay}} = 0.0541 \)). The inferior results for case (c) imply that the parameters \( \theta^{\text{dyn}} \) should better entirely be treated as unknown instead of utilising partly given values \( \theta^{\text{red}} \) in
this kind of application.

The FEM-based calibration is mainly affected by the positioning \( (\delta_0^x + \delta_x, \delta_0^z + \delta_z) \) of the electric probes and by a varying electric conductivity \( \kappa \) as the only material-dependent quantity. The calibration curve \( \Delta \Phi_{fe}^c(a) \) provides general compliance with the experimental data \( \Delta \Phi_{exp}^i(a) \), yet specimen-specific variations cannot be captured. In addition, it must be kept in mind that \( \Delta \Phi_{fe}^c(a) \) is computable a priori without the necessity of online or post-processing.

In contrast to that, the Bayesian filtering-based calibration can be applied individually with accurate results without specimen-specific knowledge on any parameters except basic boundaries by means of an understanding of the physical processes involved and the ability to model them. The outcome \( \Delta \Phi_{bay}^i(a) \) may be improved by adding information on the material-dependent constants of the dynamic model which results in superior calibration curves. Bayesian filtering facilitates the need for online processing where the accuracy of the outcomes obtained improve when new observations become available. Nonetheless, an assessment of the damage extent is possible for time steps \( k < T \).

4. Conclusion

In the present work, the numerical calibration of DCPD measuring in crack growth applications by means of FE analysis and Bayesian filtering has been conducted. The distinct aspects and characteristics as well as the impact on the obtained results have been discussed and emphasised for both approaches.

On the one hand, the FEM-based calibration offers an a priori calibration technique primarily dependent on the geometric dimensions of the probe positioning and on the respective electrical conductivity. Lacking knowledge especially of the former, the obtained calibration curves apply only generally without the capability of adjusting to specimen-specific crack growth behaviour. On the other hand, Bayesian filtering provides specimen-bound calibration curves a posteriori (in the sense that the most accurate results are available at the last time step). They are conditioned on actual measurement data and employ functional relations of the underlying physics (without knowledge of the corresponding parameters) that then account for the natural variability of the utilised material and processes.

Depending on the prior knowledge provided in the Bayesian filtering approach, different levels of accuracy can be achieved. Consequently, a choice of the presented calibration techniques has to be made with regard to the available information at hand. A worthwhile further step could be found in the combination of both approaches where the FE analysis provides prior distributions of the measurement model parameters of less entropy.
References


