Damage identification based on improved Kalman filter with the precise integration method

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Abstract

Damage identification for large size structures using extended Kalman filter (EKF) is difficult due to ill-conditioning and computation-convergence problems. To resolve these problems, the prior damage information, that sparse distribution of structural damages, is introduced to EKF through l1-norm regularization. Then the unconstrained optimization problem in classic EKF algorithm becomes the optimization problem with l1-norm constraint, and the solution for the constrained minimization problem is derived by endowing EKF with a pseudo-measurement equation. In addition, to improve the accuracy of the prediction update for EKF state vectors, the precise integration method is employed. A numerical example of a frame indicates that the proposed method can be used to identify large structure damage effectively.

1. Introduction

Civil infrastructure systems may suffer different levels of damage due to various factors, like earthquakes, overweight traffic loads, high winds, etc. However, most of the damages at early stages are minor, and it is difficult to detect them by visual inspection. If these damages go undetected for an extensive period of time, catastrophic failure of structures may occur. Therefore, it is very important to develop efficient methods for damage detection and damage growth monitoring. For the aforementioned purposes, many vibration-based techniques have been proposed, and they can be mainly classified into two categories of frequency-domain and time-domain methods [1]. Since time-domain methods directly identify structural damage using structural responses which retain all damage information, they have received considerable attention in recent years [2-6].

Extended Kalman filter, as a recursive time-domain algorithm with the advantage of small computation cost, has been an efficient tool to identify structural parameters or damage. Up to now, researchers have used EKF to identify the time-invariant parameters of different structures, like shear structure, beam, truss and plane frame [7-10]. Furthermore, the time-varying structural parameters can also be on-line identified by introducing a forgetting factor into EKF [11-12]. Moreover, EKF is applied to simultaneous identification of unknown excitation and structural parameters based on incomplete measurements [13-15].
However, the identification of local damage is a typical dynamic inverse problem, the large number of EKF state vectors make EKF difficult to convergence due to the ill-condition of dynamic inverse problems [8]. Consequently, some methods are introduced into EKF to decrease the dimension of the state vector, such as modal transformation [7], substructure method [8], static-dynamic condensation method [16]. In addition, the Tikhonov regularization, a widely used $l_2$-norm regularization-based technique for stabilizing the inverse problem, is introduced to Kalman filter algorithm to improve the identification accuracy [17-18]. However, since the $l_2$-norm-based regularization method tends to produce an over-smooth solution by blurring the singularity of the real solution in the original problem [19], the identification effectiveness usually will be decreased due to the over-smoothness effect.

Recently, sparse recovery theory has received increasing attention and it shows that the $l_1$ regularization technique performs better than $l_2$ norm regularization [20-21] in retaining the edge or abrupt characteristics. Consequently, sparse recovery theory has been widely used in wireless sensing, signal and image reconstruction [22-24]. For civil structures, damage usually occurs at a small part of elements, and most structural elements are not undamaged. Thus the damage parameters can be considered as a sparse vector because most of the damage parameters are zeros. In recent years, the sparse characteristic of damage parameters has begun to be applied in structural damage identification to improve identification accuracy [25-28]. In this paper, the $l_1$-norm regularization is also introduced to EKF to enhance the identification accuracy.

Moreover, in the EKF algorithm, the accuracy of numerical integration in the prediction update step has a great influence on the identification result. The numerical time step integration methods can be classified into two categories: explicit and implicit integration methods [29]. The explicit integration method, such as central difference method, is very efficient for one time step computation. However, to ensure stability of integration, the time step size must be very small. The implicit integration method, such as the Newmark and Wilson-θ methods [30-31], can enable a larger time step size by choosing proper integration parameters. However, the vibration components with higher frequencies will be distorted due to the large time step. Fortunately, it is validated that the precise integration method is superior to previous approaches with a more precise integration [32], which is almost independent of the time step size for a wide range of step sizes. In addition, the precise integration equation is derived within state space framework, which makes it very suitable for state updating in EKF algorithm.

In this paper, an improved EKF method, combining $l_1$-norm regularization and the precise integration, is proposed for damage identification. The sparse characteristic of local damage is introduced into EKF algorithm as $l_1$-norm constrain, then a pseudo-measurement (PM) technique is applied to solve the constrained optimization problem in the framework of EKF. Meanwhile, the precise integration method is used in the prediction update step to ensure the accuracy of numerical integration. A numerical example of a frame indicates that the proposed method can identify structural local damage effectively.

2. Conventional extended Kalman filter for damage identification
2.1 Dynamic equation of the system

Based on the finite element model, the motion equation of a linear structure can be written as

\[ M \ddot{x}(t) + C \dot{x}(t) + K x(t) = B F(t) \]  

(1)

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices, respectively. \( x, \dot{x}, \) and \( \ddot{x} \) are the displacement, velocity and acceleration response vectors, respectively. \( F \) is the external excitation vector, and \( B \) is the influence matrix associated with \( F \). To reduce the order of linear system, modal transformation technique is usually used to replace the response \( x(t) \) with general modal coordinate \( p(t) \), that is

\[ x(t) = \Phi p(t) \]  

(2)

where \( \Phi = [\phi_1 \ \phi_2 \ \cdots \ \phi_n] \) denotes the modal matrix which satisfies the mass normalization. Therefore, Eq. (1) is changed to

\[ \ddot{p}(t) + \Gamma \dot{p}(t) + \Lambda p(t) = \Phi^T B F(t) \]  

(3)

where \( \Gamma = \Phi^T C \Phi \), consider the modal damping, thus \( \Gamma = diag(2\xi_1 \omega_1 \cdots 2\xi_n \omega_n) \) and \( \xi_i \) is the \( i \)th mode damping ratio. \( \Lambda = diag(\omega_1^2 \cdots \omega_n^2) \), and \( \omega_i \) is the \( i \)th eigenfrequency of the undamped system.

2.2 Extended Kalman filter for damage identification

To simultaneously estimate structural state variables and damage based on EKF, the extended state vector is defined as

\[ \theta(t) = [X(t)^T \ \alpha^T]^T; \ X(t) = [p(t) \ \dot{p}(t)]^T; \ \alpha = [\alpha_1, \cdots, \alpha_m]^T \]  

(4)

where \( X(t) \) includes the structural state variables \( p(t) \) and \( \dot{p}(t) \), \( \alpha_m \) is the damage parameter of the element \( m \). Assume the structural parameters are constant, the state equation corresponding to Eq. (3) becomes

\[ \dot{\theta}(t) = \begin{bmatrix} \dot{p}(t) \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -\Gamma \dot{p}(t) - \Lambda p(t) + \Phi^T B F(t) \\ 0 \end{bmatrix} = f(\theta(t), F(t), t) + w(t) \]  

(5)

The discretized observation vector (measured responses) can be written as:

\[ y_{k+1} = h(\theta_{k+1}, F_{k+1}, t_{k+1}) + \epsilon_{k+1} \]  

(6)

where \( w \) represents a zero-mean process noise with covariance matrix \( Q \), \( y \) is the observation vector, \( \epsilon \) is the observation noise, which is a zero-mean white noise with covariance matrix \( R \).
Based on the conventional EKF algorithm, the recursive solution of the extended state vector can be obtained by the following equations:

\[
\dot{\theta}_{k+1} = \dot{\theta}_k + \int_{k\Delta t}^{(k+1)\Delta t} f(\theta(t), F(t), t) dt \tag{7}
\]

\[
\tilde{P}_{k+1} = A_{k+1} \tilde{P}_k A_{k+1}^T + Q_k \tag{8}
\]

\[
G_{k+1} = \tilde{P}_{k+1} H_{k+1}^T (H_{k+1} \tilde{P}_{k+1} H_{k+1}^T + R_{k+1})^{-1} \tag{9}
\]

\[
\dot{\theta}_{k+1} = \dot{\theta}_{k+1} + G_{k+1} (Y_{k+1} - h(\dot{\theta}_{k+1})) \tag{10}
\]

\[
\tilde{P}_{k+1} = (I - G_{k+1} H_{k+1}) \tilde{P}_{k+1} \tag{11}
\]

where \( \Delta t \) is the sampling time step, \( \dot{\theta}_k \) is the state estimate vector of \( \theta_k \) at time \( k\Delta t \), \( \tilde{\theta}_{k+1} \) is the prediction of state vector \( \theta_{k+1} \) at time \((k+1)\Delta t\), \( \tilde{P}_k \), \( \tilde{P}_{k+1} \) are the covariance matrix of the estimation error associated with \( \dot{\theta}_k \) and \( \tilde{\theta}_{k+1} \), respectively. \( G_{k+1} \) is the Kalman gain matrix. \( H_{k+1}, A_{k+1} \) are expressed as

\[
H_{k+1} = \partial h / \partial \theta_{k+1} \tag{12}
\]

\[
A_{k+1} \approx I + A_{k+1} \Delta t, \quad A_{k+1}^f = \partial f / \partial \theta_{k+1} \tag{13}
\]

3. Improved extended Kalman filter for damage identification

3.1 Extended Kalman filter based on the precise integration method

For the prediction equation (7) in the EKF framework, the accuracy of numerical integration in the prediction update has a great influence on the estimation. To obtain a high accuracy for the state prediction, the precise integration method is introduced into conventional EKF in this study.

Since the structural damage parameters \( \alpha \) are constant, only the state variable \( X(t) \) needs to be predicted in Eq. (7). Based on the precise integration method [32], the prediction of the state variable \( X(t) \) can be obtained as

\[
X_{k+1} = T_{\Delta t} [X_k + D_k^{-1}(r_1 + D_k^{-1}r_2)] - D_k^{-1}(r_1 + D_k^{-1}r_2 + \Delta tr_2) \tag{14}
\]

where \( T_{\Delta t} = \exp(D_k\Delta t) \), the precise computation for \( T_{\Delta t} \) can refer to [32].

\[
D_k = \begin{bmatrix} 0 & I \\ -A_k & -R_k \end{bmatrix} \tag{15}
\]

\[
r_1 = \begin{bmatrix} \Phi_k^T B F_k \\ \Phi_{k+1}^T B F_{k+1} \end{bmatrix} \tag{16}
\]

\[
r_2 = \begin{bmatrix} 0 \\ \Phi_{k+1}^T B F_{k+1} \end{bmatrix} / \Delta t \tag{17}
\]

3.2 Extended Kalman filter embedded with L1-norm regularization

In practical engineering, since only a small number of structural elements have damage whereas most of elements are undamaged, damage parameters \( \alpha_i \) are zeros with a small amount of non-zeros. Thus the distribution of local damage can be considered to be sparse,
the sparse property can be introduced into the conventional EKF as a constraint condition by l1-norm regulation method. The optimization problem with a constraint condition can be written as

$$\min_{\theta_k} E_{\theta_k|y_k} \left[ \|\theta_k - \hat{\theta}_k\|^2 + \gamma \|\alpha\|_1 \right]$$  \hspace{1cm} (18)

where \(\|\alpha\|_1 = \sum_i |\alpha_i|\), \(\gamma\) is the l1-norm regulation parameter. To obtain a solution within the framework of EKF algorithm, the l1 minimization problem (18) can be replaced with the following constrained optimization according to [33]

$$\min_{\theta_k} E_{\theta_k|y_k} \left[ \|\theta_k - \hat{\theta}_k\|^2 \right], \text{ s.t. } \|\alpha\|_1 < \varepsilon$$  \hspace{1cm} (19)

where \(\varepsilon\) is a sufficiently small value. Then a so-called pseudo-measurement (PM) technique [34] can be applied to incorporate the l1-norm condition into the EKF framework. The basic idea of PM is that the inequality constraint \(\|\alpha\|_1 < \varepsilon\) is replaced by a fictitious measurement process \(0 = \|\alpha\|_1 - \varepsilon\), and the PM is rewritten as

$$0 = \bar{H} \cdot \theta_k - \varepsilon$$  \hspace{1cm} (20)

where \(\bar{H} = [0_{1 \times N}, 0_{1 \times N}, \text{sign}(\alpha_1), \cdots, \text{sign}(\alpha_m)]\). \(N\) is the number of selected modal coordinates, \(\text{sign}(\alpha_i)\) denotes the sign function of damage parameter \(\alpha_i\).

Finally, the improved EKF combined with precise integration and l1-norm regularization method, can be summarized as follows:

1: Prediction Update

$$\tilde{X}_{k+1} = T_{\Delta t} \left[ \tilde{X}_k + D_k^{-1} \left( r_1 + D_k^{-1} r_2 \right) \right] - D_k^{-1} \left( r_1 + D_k^{-1} r_2 + \Delta t r_2 \right)$$  \hspace{1cm} (21)

$$\tilde{\theta}_{k+1} = \tilde{\alpha}_k$$  \hspace{1cm} (22)

$$\bar{\theta}_{k+1} = \left\{ \tilde{\theta}_{k+1} \right\}$$  \hspace{1cm} (23)

$$\bar{P}_{k+1} = A_{k+1} \bar{P}_k A_{k+1}^T + Q_k$$  \hspace{1cm} (24)

2: Measurement Update

$$G_{k+1} = \bar{P}_{k+1} H_{k+1}^T (H_{k+1} \bar{P}_{k+1} H_{k+1}^T + R_{k+1})^{-1}$$  \hspace{1cm} (25)

$$\bar{\theta}_{k+1} = \tilde{\theta}_{k+1} + G_{k+1} (y_{k+1} - h(\bar{\theta}_{k+1}))$$  \hspace{1cm} (26)

$$\bar{P}_{k+1} = (I - G_{k+1} H_{k+1}) \bar{P}_{k+1}$$  \hspace{1cm} (27)

3: Pseudo Measurement: Let \(P^1 = \bar{P}_{k+1}\) and \(\bar{\theta}^1 = \bar{\theta}_{k+1}\)

4: for \(i = 1, 2, \cdots, N_i - 1\) iterations do

5: \hspace{1cm} \bar{H}_i = \bar{H} (\bar{\theta}^i)$$  \hspace{1cm} (28)

$$\bar{G}^i = P^i H^T (\bar{H} P^i H^T + R_e)^{-1}$$  \hspace{1cm} (29)

$$\bar{\theta}^{i+1} = (I - \bar{G}^i \bar{H}_i) \bar{\theta}^i$$  \hspace{1cm} (30)

$$P^{i+1} = (I - \bar{G}^i \bar{H}_i) P^i$$  \hspace{1cm} (31)

6: end for

7: let \(\bar{P}_{k+1} = P^{N_i}\), \(\bar{\theta}_{k+1} = \bar{\theta}^{N_i}\)
4. Numerical simulation

4.1 Description of the four-story plane frame

To examine the effectiveness of the proposed algorithm, a numerical simulation of a four-story plane frame is investigated, as shown in Fig. 1. The plane frame is similar to the structure used in the reference [35] with two extra upper stories added to construct a larger example in this paper, and the related parameters of the frame are given in Table 1. In this study, the frame is discretized into 12 beam elements and 16 column elements, damage is assumed to be the reduction of Young’s modulus of element. The damage parameter \( \alpha_i \) represents that damage takes place if its value is greater than 0, \( \alpha_i = 0 \) denoting no damage and \( \alpha_i = 1 \) denoting complete damage. A multiple-damage case is considered in the paper as shown in Table 2.

![Fig. 1. Sketch of the 4-story plane frame](image)

<table>
<thead>
<tr>
<th>Area ( A(\text{m}^2) )</th>
<th>Density ( \rho(\text{Kg/m}^3) )</th>
<th>Young’s modulus ( E(\text{N/m}^3) )</th>
<th>Moment of inertia I (m(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 0.4 ( \times ) 0.4</td>
<td>2.5 ( \times ) 10(^3)</td>
<td>2.5 ( \times ) 10(^{10})</td>
<td>2.1 ( \times ) 10(^{-3})</td>
</tr>
<tr>
<td>Beam 0.25 ( \times ) 0.5</td>
<td>2.5 ( \times ) 10(^3)</td>
<td>2.5 ( \times ) 10(^{10})</td>
<td>2.6 ( \times ) 10(^{-3})</td>
</tr>
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</table>

In the numerical example, a white noise excitation is applied on the node 5 and the responses of all nodes are theoretically computed based on mode superposition method with a sampling time step 0.001s. Since in practice it is difficult to measure responses of
all the DOFs, particularly for rotational DOFs, only translational displacements from the 16 nodes with arrows in Fig. 1 are measured. Moreover, measurement noise is inevitable in reality, and noise may have a direct effect on damage identification. Therefore, Gaussian white noise $\epsilon(t)$ is added to simulate actual response $y(t)$.

$$y(t) = \tilde{y}(t) + \epsilon(t)$$  \hspace{1cm} (32)

where $\tilde{y}(t)$ is the theoretical response and $\epsilon(t)$ is the Gaussian white noise represented by ratio of signal power to noise power (SNR), where $SNR = 10 \times \log \left( \frac{SP}{NP} \right)$, $SP$ is the signal power and $NP$ is the noise power. Herein two noise levels of 25, 30 dB are investigated. In addition, to quantify the accuracy of estimated response, the relative percentage error (RPE) is calculated as

$$RPE = \frac{\|\hat{y} - y\|}{\|y\|}$$  \hspace{1cm} (33)

where $\|\cdot\|$ is the norm of vector. $\hat{y}$ is the estimated response.

4.2 Damage identification using displacement responses

In this section, displacement responses are used as observations to identify structural damage. Fig. 2 compares the structural damage estimated by the EKF with different types of regularizations. The black bars indicate exact damage degrees, whereas the white bars, the grey bars and slash bars indicate estimated damage degrees with the $l_1$-norm regularization, Tikhonov regularization and without regularization, respectively. It is observed that accurate damage estimation can be obtained based on the EKF with 1-norm regularization method, however many misidentifications appear when using Tikhonov regularization, and damage can hardly be identified based on the conventional EKF without regularization.

Damage identification using different integration methods in EKF is compared in Fig. 3. The grey bars indicate damage degrees estimated using EKF with the precise integration method whereas the white bars with the Euler integration method. It can be seen that the precise integration method provides an effective identification while most damages cannot be identified by Euler integration method. Moreover, Fig. 4 compares the estimated responses of different integration methods at node 8, in which the blue solid curve indicates the actual displacement whereas the red dashed curve indicates the estimated displacement. The RPE of estimated response equals to 0.07% when using precise integration method, however, it reaches up to 10.1% when using Euler integration method. Thus, the large error accumulation of estimated response in the Euler integration method contributes to inaccurate damage identification.
Fig. 2. Damage identification using EKF with different regularizations or without regularization.

Fig. 3. Damage identification using EKF with different integration methods.

Fig. 4. Comparison of the estimated and actual displacement at node 8 using different integration methods (a) the precise integration method (b) the Euler integration method.

Fig. 5 compares the identification of damage parameter at different noise levels. The black bars indicate the exact damage, the white bars, the grey bars and slash bars indicate the damage degrees estimated without noise, at 30dB and 25dB, respectively. It can be seen that the identification error slightly increases with the noise level increases, however the maximum error is very small (with an error of 1.93% at element 13) even if the noise level reaches 25dB. Fig. 6 shows the converge process of identified damage parameters, in which the dashed curve indicates the converge process of $\alpha_3$ whereas the solid curve and the dotted curve indicate the converge process of $\alpha_{13}$ and $\alpha_{17}$, respectively. It is seen that the damage parameters can quickly converge to the true values in one second even if the noise level reaches 25dB. The above estimation may be sufficient to support the conclusion that the proposed method can accurately identify the structural damage, and the method is insensitive to noise.

Fig. 5. Damage identification at different noise levels.
Comparisons between actual response $\mathbf{y}(t)$ and estimated response $\hat{\mathbf{y}}(t)$ of node 8 in different noise levels are shown in Fig. 7, in which the blue solid curve indicates actual displacement whereas the red dashed curve indicates the estimated displacement. It is noted that the estimated structural response almost coincide with the actual signal even at the noise level of 25dB, which proves excellent capability of the proposed method on filtering noise and tracing signal.

4.3 Damage identification using velocity or acceleration responses

In the proposed method, damage can also be identified with measured velocity or acceleration responses, not limited to displacement responses. Fig. 8 show the identification results when velocity or acceleration responses are used as the observation, where the black bars indicate the exact damage, the white bars, the grey bars and slash bars indicate the damage degrees estimated without noise, at 30dB and 25dB, respectively. It can be seen that the identification error of damage elements increases slightly with noise, and almost no false alarm appeared in the undamaged elements. It indicates that any type of responses (the acceleration, velocity, and displacement) can be used to accurately identify structural damage, and the measurement noise has little effect on the identification result. Fig. 9 shows the comparison between the actual response and estimated response of node 21 when using different responses as the observation, in which the blue solid curve indicates actual response whereas the red dashed curve indicates the estimated response. It can be seen that the estimated response can trace the actual response well no matter what type of response is used.
5. Conclusions

An improved EKF method for damage identification has been derived and presented. In this approach, the prior information, that the distribution of local damage parameters is sparse, is introduced to classic EKF through 1-norm regularization process. A pseudo-measurement technique is then utilized to incorporate the 1-norm constraint into EKF framework. Moreover, the precise integration method is used to improve the prediction accuracy of prediction process in the EKF. A numerical simulation is used to illustrate the effectiveness, robustness and reliability of the proposed method. In the example, multiple damage for a four-story plane frame structure subjected to ambient excitation on the node 5 is simulated. It is shown that the damage identification is very accurate and is insensitive to observation noise. Moreover, the identified result indicated that the proposed damage identification method is theoretically flexible for arbitrary type of measure responses (displacement, velocity or acceleration, etc), which improves its implementation potential in damage identification of engineering structures.

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