Structural system Identification method using Extended Kalman filter With Optimized $P_0$, $Q$, $R$

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Abstract
Structural Health Monitoring System Identification research is actively being conducted as an evaluation index for the deterioration and damage of structures. There are many ways to identify the dynamic characteristics of a building. Identification technique Based on Extended Kalman Filter for Structural system Identification recently is proposed by the authors. Kalman Filter is a state estimation technique that estimates the optimal state through estimation of the predicted state and the next state. The Kalman filter is determined according to the initial value assumption whether the state converges or diverges. If the initial error covariance matrix is set too large or too small, the state will diverge or a biased estimate. To solve this problem, In this study, we propose the optimized state estimation method using genetic algorithms. The initial value of the Kalman filter converged optimally by the genetic algorithm is used. As a result, Through the proposed method, the divergence problem can be solved, and also we can exactly identify the structural system. We are verified the effect through numerical simulation.

1. Introduction
Structural damage caused by earthquakes or typhoons can lead to serious loss of life and economic loss. Structural health monitoring studies have been carried out in which monitoring is performed by installing sensors on the main members of the structure to minimize such damage. Structural Health monitoring research has been actively studied to identify systems of structures by measuring the global responses of structures, among which analysis methods vary depending on the major members of the building structure or sensors used. Among them, the identification of the structural system using Kalman filter, which is one of the time domain system identification techniques, is a technique that has been continuously studied because it can estimate real-time response and system parameters in real time. Hoshiya and Saito (1989) investigated the stiffness and damping parameters for shear building\(^2\). The Weighted Global Iteration(WGI) method proposed by Hoshiya and Saito (1989) overcomes the instability of the state vector due to arbitrary setting of the initial error covariance matrix and appropriately sets the weight $W$ to the initial error Covariance matrix $P_0$ for safe convergence of the state vector. This is an algorithm that performs filtering repeatedly\(^3\). However, the disadvantage of the WGI algorithm is that it is derived from the difference between the physical parameters such as the exact parameters stiffness and damping and the initial value set in the initial state vector. The convergence performance is degraded or the state vector is diverted. The Kalman filter has a noise covariance matrix, $Q$, which is the noise of the model, and $R$, the noise of the measurement data. $Q$ and $R$ function to determine the value of the Kalman gain in the Kalman filter according to the data reliability. In most of the system identification studies using Kalman filter, $Q$ and $R$ are determined through the trial and error method in the same way as the error covariance matrix setting. Ming Ge and Eric C. Kerrigan argue that the convergence performance...
of the Kalman filter is important in setting the Error and Noise Covariance Matrix\(^{(3)}\). Therefore, in this study, we propose a method to set \(P_0\), \(Q\), and \(R\) appropriately. Through the proposed method, the test was carried out through a simple 4 - story shear building.

2. Extended Kalman filter, Genetic Algorithm

2.1 Extended Kalman filter

In order to apply the Kalman filter technique to the structure, the equation of motion of the structure should be transformed into the state space equation, and the continuous state space equation is as follows.

\[
\dot{X} = f(X(t), u(t)) + w(t) \quad \text{...........................................(1)}
\]

The discrete time measurements equation with additive noise at \(t = kVt\) can be expressed as

\[
z_k = h(X_k) + v_k \quad \text{...........................................(2)}
\]

Where, \(X_k\) is state vector at \(t = kVt\), \(u(t)\) is the input excitation vector and \(w(t)\) is the process noise vector for zero mean white noise Gaussian processes with covariance matrix \(Q(t)\). Where, \(z_k\) is the measurement vector at \(t = kVt\), \(v_k\) is a measurement noise vector of zero mean white noise Gaussian processes with covariance matrix \(R_k\). The Extended Kalman filter technique can be divided into 'prediction step' and 'update step'. In the prediction step, we can set to prediction state system model as follow equation.

\[
\hat{X}_{k|k-1} = \hat{X}_{k-1|k-1} + \int_{t_{k-1}}^{t_k} f(\hat{X}_{t|k-1}, u(t))dt \quad \text{...........................................(3)}
\]

\[
P_{k|k-1} = \Phi_{k-1} P_{k-1|k-1} \Phi_{k-1}^T + Q_{k-1} \quad \text{...........................................(4)}
\]

In Eq(4), \(\Phi_{k-1}\) is the state transient matrix which can be approximately obtained as In Update step, The predicted state system model estimates the state vector of the next step through the Kalman gain correction, which is the error between the actual measured value and the state vector obtained from the system model.

\[
\Phi_{k-1} = \begin{bmatrix} \frac{\partial f}{\partial X} \end{bmatrix}_{\dot{\hat{X}}_{k|k-1 \text{atis}}^T} \quad \text{...........................................(5)}
\]

The Kalman gain matrix at time step \(k\) derived from the covariance matrix is as follows.

\[
K_k = P_{k|k-1} H_k^T / H_k P_{k|k-1} H_k^T + R_k \quad \text{...........................................(6)}
\]
$H_k$ is the linearized coefficient matrix of the observation equation as The updated system model and the estimated error covariance matrix equation are as follows.

$$X_{k/k} = \hat{X}_{k/k-1} + K_k(z_k + h(\hat{X}_{k/k-1})) \tag{7}$$

$$P_{k/k} = (1 - K_k H_k)P_{k/k-1} \tag{8}$$

2.2 Weighted Global Iteration and Genetic Algorithm

The system identification using Kalman filter has the following problems. (1) The problem of divergence due to the difference between exact structural system parameters and initial stiffness. (2) Divergence problem caused by large initial error covariance matrix due to uncertain initial state vector assumption. (3) The state vector does not converge to a single iteration due to the large difference between the exact structural system parameter and the initial set stiffness. The WGI method is a technique proposed by Hoshiya and Saito to solve the problem that the state vector diverges due to an uncertain assumption of the initial state vector of the extended Kalman filter\(^2\). In order to solve the problem of divergence as the initial error covariance matrix is set to large.

The WGI method first sets the initial error covariance matrix to a range that the Kalman filter's state vector does not diverge, and perform the Kalman filter algorithm. Because the initial error covariance matrix is set to a safe range, the state vector is safe without divergence, even if there is a significant difference between the exact structural system parameters and the initial set stiffness. In the second iteration, the converged state vector is used as the next step state vector, and the initial error covariance matrix is assumed to be about $10^5$ and the iteration is again performed. The $n$ is set differently depending on the difference between the exact system parameter and the initial set stiffness. The $n$ is fixed to $10^3$ by Hoshiya and Saito in WGI, although it is set differently depending on the difference between the exact system parameters and the initial set. According to the article in AL-hussein and Hardar, the WGI method does not guarantee the convergence of the Kalman filter unconditionally\(^3\). For a successful Kalman filter, a reasonable setting of $P_0$, $Q$, and $R$ is very important\(^3\). Therefore, in this study, genetic algorithms are used to set appropriate $P_0$, $Q$, and $R$. The objective function of the genetic algorithm uses the residual response of the system model response and the actual response obtained by $[Z_k - H_x]$ inside the Kalman filter as the objective function by calculating the root mean square error. RMSE can be used as a measure of estimated performance. Therefore, RMSE is calculated to be smaller for $P_0$, $Q$, and $R$, which are set appropriately.

2.3 System identification of 4-Story-shear-building with optimal $P_0$, $Q$, $R$

Consider a four-story structural system subject to the El-Centro earthquake excitation. The exact structural system parameter are mass $2kg$, Stiffness $15000N/m$, and damping $120N·s/m$ and structural system parameter of every story is same. And also initial assume of the Kalman filter refer to Table 1 for reference. The figure 4 is a graph showing the convergence according to the number of iterations of WGI of a four-story shear building model. It can be seen that the state vector of all story converges to the exact stiffness at about the third iteration. Table 1 summarizes the data in Figure 1. And Table 2 summarizes the data in Figure 2. The proposed method in Figure 1 shows that all the state vectors converge to exact stiffness in the first iteration, while the WGI
method converges very unstably at the fourth iteration. This result can be said to be the result of choosing $P_0$, $Q$, and $R$ that are not appropriate. In addition, the result of convergence of damping in Figure 2 also shows that all the state vectors converge to the Exact damping coefficient in the first iteration using the method proposed in this study. However, when WGI method is used, it converges very unstably. As shown in Figure 3, it can be seen that the filtering is proceeding while trusting the data obtained by the system model more reliably because the convergence performance is maximized by setting $P_0$ to a large value for the first iteration and the $Q$ value is smaller. On the other hand, the WGI method is very unstable because it uses a fixed $P_0$, $Q$, $R$. As a result, $P_0$, $Q$, and $R$ must be selected and used appropriately in order to obtain successful Kalman filter convergence.

**Figure 1.** 4-story-shear-building model: Excitation with Earthquake load (El-centro)
(a) Convergence of stiffness Using Optimal $P_0,Q,R$, (b) Convergence of stiffness using WGI

**Figure 2.** 4-story-shear-building model: Excitation with Earthquake load (El-centro)
(a) Convergence of damping coefficient Using Optimal $P_0,Q,R$, (b) Convergence of damping coefficient using WGI
Figure 3. 4-story-shear-building model: Excitation with Earthquake load (El-centro) (a) Parameters used in iteration optimized P0, Q, R (b) Parameters used in iteration WGI

Table 1. Convergence of Stiffness

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Table 2. Convergence of damping coefficient

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3. Conclusions
In this paper, we propose a method to select P0, Q, and R suitable for successful Kalman filter convergence. Through the method proposed in this study, convergence performance through the 4-story shear building was verified and the convergence tendency of the state vector was stable and converged to the exact parametric system. In
this study, it is necessary to continue the verification of this study through experiments on damage search.

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References and footnotes
