Numerical Analysis of Lamb Wave Propagation in a Periodically Stiffened CFRP Plate for Impact Damage Detection

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Abstract

When acousto-ultrasonic health monitoring methods are applied to detection of impact damages in structures constructed by carbon-fibre reinforced plastic (CFRP) stiffened plates, such as skin-stringer structures, the wave propagation behaviour should be clarified. First, the authors attempted to construct a simplified modelling method of an impact damage in a CFRP quasi-isotropic laminate for finite element analysis (FEA). The impact damage was assumed to be homogeneous and a frustum of a cone in shape with the quasi-isotropic stiffness degradation. The five independent stiffness coefficients in the damaged area were determined from the measured velocities of Lamb waves and bulk wave and the theoretical assumption of stress transmission. The FEA results with the simplified modelling method agreed well with both of the experimental results and the results with the precise modelling considering multiple-delamination and fibre orientation of each ply. This agreement indicates the simplified modelling method is appropriate for simulation of ultrasonic propagation through impact damages. Secondly, the velocity dispersion of Lamb waves propagating in the CFRP skin-stringer structure was derived. Here, the stiffened panel was assumed to be a periodic structure of one bay of the skin plate between a pair of stringers. Introducing a periodic boundary condition to a semi-analytical finite element method, the authors successfully obtained the dispersion curves of Lamb waves in the periodically stiffened CFRP plate. The improved modelling methods and the detailed dispersion relation will be effective to clarify the wave propagation behaviour in the complex-shaped CFRP structures with complicated impact damages and helpful to design the optimum configuration of the acousto-ultrasonic system for impact damage detection in the large structure.

1. Introduction

In recent years, carbon fibre reinforced plastic (CFRP) has been put into practical use for various lightweight structural bodies including aircraft structure. The general structure of an aircraft is a semi-monocoque structure, such as skin-stringer structure in which skin panel is reinforced with longitudinal stringers. This is a periodic structure in which stringer reinforcing members are arranged at regular intervals in the transverse direction as shown in Figure 1. This structure often encounters a variety of damages during flight. Among those potential damages, impact damage is one of the fatal damage form that the structure often suffers from. Therefore, it is quite important to construct reliable structural health monitoring systems. Research and development of structural health monitoring (SHM) technology for diagnosing the health status of such a semi-monocoque structure by incorporating sensors has been progressed, and major methods are using ultrasonic guided waves, in particular, Lamb waves to detect damages \textsuperscript{(1)}. In particular, SHM system which combines piezoelectric ceramic
transducer with fibre Bragg grating (FBG) sensors has been developed to detect damages in a wide area of the skin-stringer structure made of CFRP (2).

In order to construct a reliable SHM system based on Lamb waves, we should clarify the wave propagation behavior, such as the change of ultrasonic waves by impact damages of different sizes in various locations of the complex-shaped composite structures (3). To achieve this, two necessary aspects are investigated in this research. One is a simplified modelling method of impact damage for Lamb wave propagation in a CFRP laminate. The other is the calculation of dispersion curves of Lamb waves in a CFRP skin-stringer structure.

In the first study, authors attempted a simplified modelling of an impact damage for FEM simulation for a CFRP laminate. FEM simulation can be efficiently executed with a lower cost, compared to experimental study (4). Therefore, when we establish FEM analysis, we can efficiently investigate optimal configuration of actuators and sensors for monitoring a large structure’s health status. In our model, the damage region is considered to be a quasi-isotropic homogeneous frustum of a cone which has degraded stiffness constants. The stiffness parameters were experimentally determined by measurement of ultrasonic velocities and cross-sectional observation of the damaged region. The validness of this approach is assured through the qualitative and quantitative comparison with the experimental results and the other modelling method.

In the second study, authors attempted to calculate the velocity dispersion curve of the Lamb wave propagating through the periodic section composite structure by assuming the cross section of the CFRP skin-stringer structure to be a periodic repetitive shape. The guided waves in a skin-stringer structure may be different from those in a plane plate, because a wave propagating from a skin to a stringer will be partly reflected. Extending a semi-analytical finite element method to include constraints such as periodicity, we clarify the propagating properties of existing modes under different frequencies.

This paper is organized as follows. In the next section, simplified modelling method of impact damage for FEM is investigated. In the subsequent section, we calculate dispersion curves of Lamb waves in a periodic skin-stringer structure. The last section is devoted to conclusions.

2. Simplified modelling method of impact damage for Lamb wave propagation in composite structures

In order to determine optimal configuration of actuators and sensors for monitoring a large structure’s health status such as skin-stringer structure, we must investigate the change of ultrasonic waves by impact damages with different sizes in various locations
As a first stage of our research, we begin by considering an impact damage on a plane-like skin. In the below, focusing on the modelling of the impact damage, we investigate it on plane plate without stringer. It is difficult to precisely model the impact damages including every delamination, shear crack and transvers crack. However, since the wavelength of Lamb wave is usually longer than the length of microscopic damages, the impact damage could be assumed as the degradation of the anisotropic stiffness in the homogenous damaged area. A much simpler method is proposed in this section.

2.1 Simplified method for modelling an impact damage

2.1.1 Observation of impact-damage area

CFRP quasi-isotropic laminates (T700S/2500, [45/0/-45/90]s) with 3.4mm in thickness was prepared for the observation. An impact damage was generated in the centre of the plate and the diameter of the dent is 4 mm. Besides, the diameter of the damaged area is approximately 28 mm according to the C-scan results. Furthermore, after cutting a plate through the damaged area, we observed its cross section by a microscope for the investigation of the damage form as shown in Figure 2. Delamination can be seen around the impact point, and delaminated area became larger in the deeper interfaces of the plies.

Taking into account the above observation, we consider the impact damage region as a frustum of a cone, in which stiffness constants are degraded. In this research, this frustum of a cone shape modelling of the impact damage region with effective stiffness degradation is called “simplified modelling method”.

2.1.2 Stiffness matrix and velocities of Lamb waves in quasi-isotropic composite plate

Quasi-isotropic composite plate can be expressed with five independent stiffness coefficients $C_{11}, C_{12}, C_{13}, C_{33}$, and $C_{44}$ (3-axis corresponds to out-of-plane direction). The $6 \times 6$ stiffness matrix is as follows:

![Figure 2](image)
Here,
\[
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\]

Here,
\[
C_{66} = \frac{C_{11} - C_{12}}{2}.
\] (1)

The relationship between elastic constants and stiffness coefficients in quasi-isotropic case can be expressed as follows:
\[
\begin{align*}
C_{11} &= \frac{1 - \nu_{23}\nu_{32}}{E_3\Delta}, \\
C_{12} &= \frac{\nu_{21}\nu_{31}\nu_{23}}{E_3\Delta}, \\
C_{13} &= \frac{\nu_{13}\nu_{21}\nu_{32}}{E_2\Delta}, \\
C_{33} &= \frac{1 - \nu_{12}\nu_{21}}{E_1\Delta}, \\
C_{44} &= G_{23}, \quad C_{66} = G_{12}.
\end{align*}
\] (2)

Here,
\[
\Delta = \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13}}{E_1E_2E_3}, \quad \Delta = \frac{\nu_{12}\nu_{21}}{E_1E_2E_3}, \quad \Delta = \frac{\nu_{23}\nu_{32}}{E_3\Delta}.
\] (3)

\(\nu\)s are Poisson’s ratios, \(E\)s are longitudinal elastic moduli, and \(G\)s are traverse elastic moduli.

The stiffness coefficients and density are related to the velocity of elastic waves. For example, the velocity of longitudinal bulk wave propagating in the thickness direction can be expressed by
\[
v_{\text{bulk}} = \sqrt{\frac{C_{33}}{\rho}},
\] (4)

A guided wave in a plate (Lamb wave) has many modes, whose existence and velocity are significantly dependent on frequency. Figure 3 shows a dispersion relation of group velocities of Lamb wave. At low frequency such as \(\leq 220\text{kHz}\), the velocity of the lowest-order symmetric mode (S₀) reaches a constant value known as plate velocity \(v_{\text{plate}}\). In isotropic thin sheet \((^6)\), the plate velocity is expressed as
\[
v_{\text{plate}} = \sqrt{\frac{k(\lambda + \mu)}{\rho(\lambda + 2\mu)}},
\] (5)

where \(\lambda\) and \(\mu\) are Lamé constants.

Similarly, in quasi-isotropic CFRP plate, the plate velocity can be expressed by the following expression
\[
v_{\text{plate}} = \sqrt{\frac{C_{11}C_{33} - (C_{13})^2}{\rho C_{33}}},
\] (6)

Because \(C_{13}\) is much smaller than \(C_{11}\) and \(C_{33}\) for quasi-isotropic CFRP plate, \(C_{13}\) can be neglected. Thus, we can obtain the following equation
\[
v_{\text{plate}} \approx \sqrt{\frac{C_{11}}{\rho}}.
\] (7)
On the other hand, at high frequency such as $f \geq 40$ kHz, the velocities of the lowest-order modes ($A_0$ and $S_0$) approximate to the velocity of Rayleigh surface wave $v_{Rayleigh}$. This is close to the velocity of transverse wave $v_{transverse} = \sqrt{\frac{C_{44}}{\rho}}$ \(^{(6)}\). Therefore,

$$v_{Rayleigh} \approx \sqrt{\frac{C_{44}}{\rho}}. \quad (8)$$

In the frequency range from 40 kHz to 220 kHz, we can assume the ultrasonic group velocity of $S_0$ mode $v_{S_0}$ is close to $v_{plate}$, and that of $A_0$ mode $v_{A_0}$ is close to $v_{Rayleigh}$. Therefore,

$$v_{S_0} \approx \sqrt{\frac{C_{11}}{\rho}}, \quad v_{A_0} \approx \sqrt{\frac{C_{44}}{\rho}}. \quad (9)$$

### 2.1.3 Determination of stiffness constants by measurement of ultrasonic propagation

In order to establish the simplified model, the degradation rates of stiffness coefficients in the impact-damaged area must be determined. Among the five independent stiffness coefficients, three coefficients $C_{11}, C_{33},$ and $C_{44}$ can be determined from the ultrasonic velocities and the other two $C_{12},$ and $C_{13}$ can be deduced according to the previous cross-sectional observation.

Firstly, the degraded value of $C_{11}$ and $C_{44}$ can be determined by Eq. (9). In order to measure $v_{S0}$ and $v_{A0}$, experiments of Lamb wave propagation were conducted. An ultrasonic actuator (AE sensor, made by NF Corporation, NF-AE-900M) was fixed at top of the plate and the other two ultrasonic sensors were set symmetrically on both sides of the plate as receivers as shown in Figure 4. Adding or subtracting the two signals detected by the two sensors, we can obtain symmetric or antisymmetric components of the signals. The two sensors were moved at an interval of 10 mm along the propagation path including the centre of the impact-damaged area. The input signal is generated as three cycles of sine wave with hamming window. The input frequency is chosen to be 180 kHz because between the frequencies ranging from 40 kHz to 220
kHz, it could generate only $A_0$ and $S_0$ modes, and meanwhile, avoiding the frequency range which includes a sharply changing velocity of $A_0$ mode.

![Ultrasonic actuator and sensor setup](image)

**Figure 4.** Experimental setup for Lamb waves propagation to determine $C_{11}$ and $C_{44}$.

The calculated group velocities in the damaged area are as follows.

$$v_{ims_0} = 1.756 \text{ km/s}, \quad v_{imA_0} = 1.342 \text{ km/s}.$$  \hfill (10)

Secondly, the degraded value of $C_{33}$ can be determined by Eq. (4). The propagation time of longitudinal bulk wave in the thickness direction around the impact damaged area was also measured. The experimental setup is shown in Figure 5.

![Longitudinal bulk wave setup](image)

**Figure 5.** Experimental setup for longitudinal bulk wave propagation to determine $C_{33}$.

Since the propagation time varies with the position in the impact-damaged area, the effective value of $C_{33}$ for the entire model is difficult to be determined accurately. However, it should be between the value which is calculated according to the longest and shortest (= in intact area) travel time in the thickness direction. In fact, when applying both the conditions ($C_{33}$ is not degraded/ $C_{33}$ is maximumly degraded) into the simplified models to investigate the Lamb waves behaviour, almost the same waveforms are obtained at each propagation distance. It means the value of stiffness in the thickness direction don’t affect largely the waveform of Lamb waves. Thus, in this paper, we assume $C_{33}$ is constant for convenient.

Finally, the degraded value of $C_{13}$ and $C_{12}$ can be determined based on the observation results in Figure 2. The delamination is found to be the most dominated damage form and it is reasonable to assume that the out-of-plane Poisson’s ratio is close to 0, namely, $\nu_{31}, \nu_{32}=0$, because the delamination cannot transfer the vertical strain. Hence, according to the Eq. (2), $C_{13}$ becomes 0. Besides, the in-plane Poisson’s ratio $\nu_{12}$ can be assumed to be approximately invariant through the impact process. According to the Eq. (2), because of $C_{11} \propto E_{11}$ and $C_{12} \propto E_{11}$, $C_{12}$ can be assumed to be degraded at the same rate as $C_{11}$ (81% degradation).

Through the above analysis, the stiffness matrix of the impact-damaged area has been determined. With determined stiffness matrix, we can establish this simplified model.
2.2 Multiple-delamination modelling method

For comparison, we modelled the impact damage in detail, which is called “multiple-delamination modelling method”. Taking into account a peanuts-shape of delamination, we have established a multi-layered model as shown in Figure 6. In each interface between adjacent plies, a peanuts-shape delamination is considered. The extension is along the ply-direction, and the deeper the interface of plies is, the larger the size is. In the model, each laminate is simulated with uni-direction stiffness constants and a certain ply angle. The delamination is simulated by double-nodes and contact condition is attached into the inner surfaces of delamination.

![Figure 6. Schematic of delamination used in multiple-delamination method for an impact damage.](image)

2.3 Comparison between FEM models and an experimental observation

In order to quantitatively confirm the appropriateness of simplified modelling method, the calculation result with the simplified model, that with the laminated multiple delamination model, and the experiment results are compared in Figure 7. In the experiment, the ultrasonic visualized observation was conducted by Laser Ultrasonic Visualization Inspector system (Tsukuba Technology, LUVI CP-2). The propagation behaviour of Lamb wave and the change of arriving time by impact damage in the simplified model agree well with those in the laminated model and the experimental result.

![Figure 7. Lamb wave propagation behaviour between three kinds of modelling and ultrasonic visualized observation results. LS-DYNA is used for FEM simulation.](image)
For each of three methods above, the time differences (delay) of waveforms are obtained between damaged and intact cases by calculating cross-correlation. The comparison results are shown in Figure 8.

![Figure 8. Difference on propagation time between damaged and intact cases at different propagation distances.](image)

Before passing through the damaged area, the delay is close to 0 for all the three results, which means the waveform of damaged case is approximately the same as that of intact case. However, in the damaged area, especially at the distance of 45mm, the delay obtained for three results do not agree well, mainly because the modelling method did not simulate all the microscopic damage in the impact-damaged area. After passing through the damaged area, the delay tends to stabilize at 3 μs for all the three methods, which means the simplified method is quantitatively appropriate for simulating impact damage when investigating Lamb wave propagation behaviour in quasi-isotropic composite structures.

3. Calculation of dispersion curves of Lamb waves in a CFRP skin-stringer structure

In general, the theoretical calculation of velocity dispersion relation in a flat plate with infinite width is relatively easy, because the wave behaviour is expressed in two-dimensional cross section \(^{(7)}\). However, in the case of skin-stringer structure, reflection between skin and stringer cannot be neglected. Hence, the dispersion relation may be significantly altered. In order to accurately calculate the dispersion relation in an arbitrary cross-sectional shape, semi-analytic finite element (SAFE) method is mostly used \(^{(9)}\). However, when applying the SAFE method to a whole of CFRP skin-stringer structure, the calculation cost becomes very large. Therefore, introducing periodic boundary condition into SAFE method, we performed calculation.

3.1 Derivation of dispersion relation by using SAFE method

The fundamental equation followed by elastic waves is derived from the principle of virtual work. When external force does not work, the principle of virtual work is as follows:

\[
\int_{\Omega} \delta u^t (\rho \ddot{u}) dV + \int_{\Omega} \delta \varepsilon^t \sigma dV = 0
\]  

\( (11) \)
Here, \( u \) is displacement, \( \rho \) is density, \( \varepsilon \) is strain, and \( \sigma \) is stress. \( \dagger \) means complex conjugate and taking transpose, and dot means time differentiation. Integration is about volume.

As shown in Figure 9(a), the length in the Z-axis direction is infinite and the shape of XY cross section is assumed to be uniform along the Z-axis. Then, an elastic wave propagated in the longitudinal direction (Z-axis direction) is considered.

![Elastic waves](image)

**Figure 9.** (a) Columnar objects through which elastic waves propagate and (b) introduction of periodic boundary condition

When the elastic wave propagates in the Z-axis direction with angular frequency \( \omega \) and wave number \( \xi \), the displacement in equation (11) can be expressed as:

\[
\mathbf{u}(x, y, z) = f(x, y)e^{i(\xi z-\omega t)}.
\]

(12)

In the Z direction, \( e^{i(\xi z-\omega t)} \) represents a wave with a certain frequency and it is handled analytically. On the other hand, in the XY cross section, a finite element method of dividing the cross section into meshes is used to obtain the displacement \( \mathbf{u} \).

Let \( \mathbf{U} \) be the vector representing the displacement on all nodes. The displacement of arbitrary point can be interpolated by the displacement of nodes as follows

\[
\mathbf{u}(x, y, z) = \mathbf{N}(x, y)\mathbf{U} e^{i(\xi z-\omega t)},
\]

(13)

where \( \mathbf{N}(x, y) \) is a shape function.

Then the displacement-strain relation formula and the stress-strain relation formula is substituted into the equation (11), it becomes

\[
\delta \mathbf{U}^T \left[-\omega^2 \mathbf{M} + \mathbf{K}_1 + i\xi \mathbf{K}_2 + \xi^2 \mathbf{K}_3\right] \mathbf{U} = 0.
\]

(14)

Here, \( \mathbf{M}, \mathbf{K}_1, \mathbf{K}_2 \) and \( \mathbf{K}_3 \) are all \( 3N_{\text{node}} \times 3N_{\text{node}} \) matrices, where \( N_{\text{node}} \) is a number of node. The nontrivial displacement vector \( \mathbf{U} \) exists on condition that the determinant of the equation (14) is zero. Therefore,

\[
\det\left[-\omega^2 \mathbf{M} + \mathbf{K}_1 + i\xi \mathbf{K}_2 + \xi^2 \mathbf{K}_3\right] = 0.
\]

(15)

This is associating the angular frequency \( \omega \) with the wave number \( \xi \), which is the velocity dispersion relation.

Here, the group velocity representing the velocity of a wave packet including waves with various wave numbers, can be expressed as

\[
\mathbf{v}_g = \frac{d\omega(\xi)}{d\xi}.
\]

(18)

### 3.2 Introduction of periodic boundary condition
The periodic structure is a continuous structure in which the same unit structural elements are connected to each other. Dimensions and shapes of the unit structural elements are completely the same, but it is necessary to match the movement of the left and right shared nodes when connecting them together as shown in Figure 9(b). By introducing this boundary condition into SAFE method, the velocity dispersion relation could be calculated accurately.

### 3.3 Velocity distribution in a skin-stringer periodic structure

The dispersion relation was calculated as shown in Figure 10 (a) (S mode) and Figure 10 (b) (A mode). Furthermore, the propagation time of symmetric and antisymmetric modes at a propagation distance of 100 mm are calculated in Figure 11. As shown in Figure 11, we found the propagation time of modes are obviously concentrated in three areas, which is separately the fastest mode group C of longitudinal waves, the slowest mode group A of transverse waves and the modes group B between them which includes both the characteristics of wave behaviors of longitudinal and transverse waves. The deformed state of these three groups are shown in Figure 12. In the mode group B, X-displacement can be seen. This cannot be included in ordinary Lamb wave behavior. Therefore, derivation by SAFE is necessary.

![Figure 10. Calculated group velocity dispersion curves in a skin-stringer periodic structure. Quasi-isotropic lamination of T700S/2500. Skin thickness: 1.5mm. Thickness of stringer part: 3 mm. The distance between adjacent stringers: 107 mm.](image-url)
In order to make comparison with dispersion curves in an infinite plate, the propagation time of symmetric and antisymmetric modes at a propagation distance of 100 mm of a plate with infinite width are calculated in Figure 13. The comparison between Figure 11 and Figure 13 shows that the number of modes of the skin-stringer periodic structure is extremely larger than those of the flat plate having infinite width. In Figure 13 there are not any modes corresponding to the mode group B in Figure 11.
4. Conclusion

In this research, in order to clarify the propagation behaviour of Lamb waves in a periodic stiffened CFRP plate, two kinds of basic things are investigated. Firstly, an impact damage in a CFRP plate was modelled as a homogeneous frustum-of-a-cone region, in which the quasi-isotropic stiffness constants are degraded. The calculated propagation behaviour of Lamb waves in the simplified model agrees well with that in the multiple-delaminated model and the experiment results. This simplified model could be useful for the wave propagation simulation in complex CFRP structures such as a skin-stringer structure. Secondly, periodic boundary condition is introduced in SAFE method to calculate the dispersion curve of a CFRP skin-stringer periodic structure. Through the comparison of the calculated dispersion curve with the case of a flat plate with infinite width, it was found that this calculation is necessary and will be helpful for recognizing modes in a skin-stringer structure. These knowledges will be useful for investigation of suitable configuration of actuators and sensors for health monitoring system to detect impact damages with various sizes and positions.

References