A random vibration based platform for statistical damage detection in a population of nominally identical composite aerostructures


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Abstract

A random vibration platform for statistical damage detection in a population of composite aerostructures, as developed under the VIBRATION European Commission funded project, is presented and its effectiveness is assessed. The platform is designed to ad-
dress the challenge of achieving high damage detection performance despite the significant member-to-member variability of the population due to material/manufacturing, and the additional environmental and operational uncertainty. It is also designed to cope with in-flight conditions, implying that it exclusively resorts on a low number of limited and low bandwidth vibration response signals and may be implemented in almost real time. To achieve these objectives the platform features two parametric statistical time series type methods based on recently introduced Multiple Model representations. Its effectiveness is assessed via laboratory experiments with four nominally identical composite booms and impact-induced damages. Based on these, the platform is shown to achieve excellent detection performance with no false alarms.

1. Introduction

The economic and life-safety implications of early stage damage detection in the aerospace industry, combined with the increased use of composite parts (1) that often suffer from impact-induced, barely visible or invisible, damage, have stirred significant attention in Structural Health Monitoring (SHM). The aim of the VIBRATION project has been the development of an innovative, almost real time, random vibration based SHM platform for statistical damage detection in a population of aerostructures pertaining to an aircraft fleet. The emphasis of the work has been on accounting for the member-to-member variability in the dynamics among the population, which is due to material/manufacturing, as well as environmental and operational uncertainty, using a small number of sensors and affordable equipment. This aggregate variability may be so significant as to nearly ‘mask’ changes due to damage on which damage detection is to be based (4-12), thus rendering vibration based damage detection highly challenging, and, in general, not possible via conventional statistical time series type SHM methods (4).

The work done on overcoming this problem has been, thus far, quite limited (5, 6), with a number of studies (4, 7-12) performed by some of the present authors. These studies have been based on various concepts, including that of Multiple Model (MM) representations of the variable healthy dynamics through a set of response-only stochastic models, such as AutoRegressive (AR) (4, 8, 9) or transmittance-based Autoregressive with eXogenous (ARX) excitation models (7, 12). AR model parameters (4, 8), their Principal Component Analysis (PCA) (4, 9) based transformed versions, and transmittance-based ARX model residuals (7, 12) have been mainly used as damage sensitive features. Additional methods have been based on probabilistic Random Coefficient (RC) models of the Gaussian (10) or non–Gaussian type (11). Although in principle more appropriate than their MM based counterparts in modelling the dynamics of the healthy population, RC models are more complex and necessitate a sufficiently high number of vibration signal records for proper training. The various methods have been successfully assessed via laboratory experiments with a population of scale composite beams, each one representing the tail boom used in an Unmanned Aerial Vehicle (UAV), and impact induced damage scenarios.

This study is founded on our past works, with the goal being the formulation and assessment of a random vibration based SHM platform, developed under the VIBRATION project, for almost real time detection of damage in a population of nominally identical structures using a low number of limited and low bandwidth vibration response signals. The platform is an autonomous unit capable of acquiring measurements of vibration signals,
signal pre-processing, and damage detection based on two MM based response-only methods: an MM-AR\(^6\) and an MM-ARX\(^7,12\) method. In contrast to our previous studies, which were based on scale beams, the platform's performance is presently assessed via four full-scale composite beams (tail booms used for connecting the tail structure with the main fuselage in a twin-boom UAV; Figure 1) and impact (of 13J, 18J, 24J energy levels) induced damages.

2. The prototype SHM platform

The prototype SHM platform (Figure 2) is an autonomous unit capable of acquiring real time measurements of vibration signals (currently acceleration), performing signal pre-processing operations (such as signal conditioning, normalization, and so on) and analysis in the frequency domain, as well as achieving damage detection based on the MM-AR and MM-ARX methods (see subsections 2.1 and 2.2, respectively). All operations are controlled using a user friendly Graphical User Interface (GUI) which is installed on a typical portable computer. The platform features 4 analog signal generation channels (16 bit D/A resolution, 100 Ks/s/ch simultaneous update rate, ±10V range), 4 acceleration input channels (24 bit A/D resolution, 51.2 Ks/s/ch simultaneous update rate, ±5V range, antialiasing filters, AC/DC coupling, IEPE signal conditioning) and is connected to the computer through a typical USB port. Once a batch of signal responses of pre-selected length is collected and appropriately pre-processed, damage detection is achieved with the activation of the MM-AR and the MM-ARX methods. This procedure may be automated and run continuously or repeated with a user defined period and achieving almost real time health monitoring based on newly acquired signals.
The MM-AR and the MM-ARX methods, which form the core of the platform, are described in the sequel; further details are provided in references (4, 7, 12).

2.1 The Multiple Model AR (MM–AR) method

This a response-only method that employs a set of scalar conventional AR models along with the Gaussian Probability Density Functions (PDFs) of their estimated parameter vectors to define an MM representation of the healthy dynamics (4, 8). The method’s operational phases are presented below.

Baseline phase. A number \( v \) of vibration signals (constant number of signals per structure) are obtained from \( v \) nominally identical healthy structures. Based on these, a Multiple Model representation of the healthy structural dynamics \( m_0 = \{m_{0,1}, \ldots, m_{0,p}\} \) is determined via the estimation of a set of \( p \) AR(\( n_a \)) models (13) of order \( n_a \):

\[
y[t] + \sum_{i=1}^{n_a} a_i y[t - i] = e[t], \quad e[t] \sim \text{NID}(0, \sigma_e^2) \tag{1}
\]

with \( t = 1, \ldots, N \) designating normalized discrete time, \( a_i \) the \( i \)-th AR parameter, and \( e[t] \) the model residuals that should form a white Gaussian zero-mean sequence with variance \( \sigma_e^2 \). Note that NID stands for normally independently distributed with the indicated mean and variance. The respective parameter vectors (each one including the \( a_i \) for \( i = 1, \ldots, n_a \)) are designated as \( a = \{a_{o,1}, \ldots, a_{o,p}\} \); boldface capital/lower letters designate matrices/vectors, respectively, while the subscript \( o \) designates the healthy structural state. Each such vector is asymptotically (\( N \to \infty \)) associated with a Gaussian PDF, with mean equal to each point estimate \( a_{o,i} \) and estimated covariance \( \Sigma_{o,k} \) (\( k = 1, \ldots, p \)).

Inspection phase. Once a fresh vibration response signal is obtained from a specific structure, the objective is to decide whether or not the underlying dynamics may be adequately represented by the MM representation \( m_0 \). In such a case the structure is declared as healthy, otherwise it is declared as damaged. Towards this end, a new AR model, \( m_u \) (the subscript \( u \) designates the current, unknown, structural state), of the same order as those in \( m_0 \), is estimated using the fresh signal. Then a proper test pseudo-statistic, say \( D(m_0, m_u) \), expressing the distance between the current model \( m_u \) and the MM representation \( m_0 \) is obtained in the space defined by the method’s characteristic quantity which is the AR model parameter vector.

Damage detection is then declared if and only if \( D(m_0, m_u) \) is greater than a user specified threshold \( l_{\text{lim}} \), otherwise the structure is declared as healthy:

\[
D(m_0, m_u) \leq l_{\text{lim}} \rightarrow \text{Healthy structure} \\
\text{otherwise} \quad \rightarrow \text{Damaged structure} \tag{2}
\]

\( D(m_0, m_u) \) is presently defined as the minimum of the distances of \( m_u \) from all component models of \( m_0 \), that is:

\[
D(m_0, m_u) \triangleq \arg \min_{k=1,\ldots,p} d(m_{0,k}, m_u) \tag{3}
\]
with \( d(m_{o,k}, m_u) \) designating the Kullback-Leibler (KL) divergence\(^{(14)} \) between two individual models. As the AR model parameter vectors asymptotically follow Gaussian distributions, the KL divergence may be written as:

\[
d(m_{o,k}, m_u) \equiv \frac{1}{2} \left[ \text{tr} \left( \Sigma_{o,k}^{-1} \Sigma_u \right) + (a_{o,k} - a_u)^T \Sigma_{o,k}^{-1} (a_{o,k} - a_u) - 1 - \ln \left( \frac{\det \Sigma_u}{\det \Sigma_{o,k}} \right) \right]
\]

with \( \text{tr}(\cdot) \) designating trace and \( \det(\cdot) \) determinant of the indicated quantity. It is noted that the KL-divergence is a non-symmetric pseudo-distance and thus it should be used as expressed above, without interchanging the two models.

### 2.2 The Multiple Model ARX (MM-ARX) transmittance-based method

This is also a response-only method that is founded on the ARX parametric representation of the transmittance function defined as the ratio of the cross spectral density over the power (auto) spectral density of two vibration response signals, say \( y_1[t], y_2[t] \), measured at two different points on the structure\(^{(7, 8)} \). Thus, in this method a set of transmittance-based ARX models define the MM representation of the healthy dynamics. Its operational phases are presented below.

**Baseline phase.** A number of \( p \) pairs of vibration response signals are obtained from \( v \) nominally identical healthy structures. Based on these, a Multiple Model representation of the healthy structural dynamics \( m_o = \{m_{o,1}, ..., m_{o,p}\} \) is determined via the estimation of a set of \( p \) ARX\((n_{a}, n_{b})\) models of the form\(^{(7)} \):

\[
y_1[t] + \sum_{i=1}^{n_a} a_i y_1[t - i] = \sum_{i=0}^{n_b} b_i y_2[t - i] + e[t], \quad e[t] \sim NID(0, \sigma_e^2) \quad (5)
\]

with \( n_a, n_b \) designating the model orders, \( a_i, b_i \) the \( i \)-th AR and X parameter, respectively, and \( e[t] \) the model residuals which is the method’s damage sensitive feature.

The model residuals statistical independence is a typical requirement for model validation as well as for residual based damage detection\(^{(2, 3)} \). However, in practice the strict satisfaction of this condition leads oftentimes to model over-parametrization and thus to detailed complex models of high sensitivity to various uncertainty sources that affect negatively (increase of false alarms) the performance of a damage detection method. Thus, avoiding model over-parametrization this condition is presently replaced by the less strict (asymptotic) one:

\[
\exists M < \infty: \quad e[t], e[t + r] \text{ statistically independent for } r \geq M \quad (6)
\]

Based on this, the MM-ARX method is formulated using a sequence of residual vectors:

\[
\mathbf{e}_{o,k} \triangleq \left[ \begin{array}{c} e[t_k + 1] \\ \vdots \\ e[t_k + N_q] \end{array} \right], \quad t_k = k \cdot (N_q + M), \quad k = 1, ..., K \quad (7)
\]
where \( \mathbf{e}_{o,k} \) is a zero mean Gaussian distributed random vector with covariance matrix \( \mathbf{R}_{\mathbf{e}_o} \), that is \( \mathbf{e}_{o,k} \sim N(\mathbf{0}, \mathbf{R}_{\mathbf{e}_o}) \). Thus, the sequence of these independent vectors for \( k = 1, \ldots, K \) corresponds to \( K \) non-overlapping windows (residual signal portions) of the residual sequence \( e[t] \), each one with \( N_q \) samples length and distance between two successive windows equal to \( M \) samples. Thus, with the completion of the baseline phase the set \( m_o \) of transmittance-based ARX models along with the corresponding \( \mathbf{R}_{\mathbf{e}_o} \) are available for the inspection phase.

**Inspection phase.** Once a fresh pair of vibration response signals is obtained from a specific structure, the objective is to decide whether or not the underlying dynamics may be adequately represented by the MM representation \( m_o \) so as to be declared as healthy. Towards this, the fresh pair of signals is driven via each ARX model of \( m_o \), and a sequence of vectors \( \mathbf{e}_{u,k} \) (\( k = 1, \ldots, K \)) with covariance matrix \( \mathbf{R}_{\mathbf{e}_u} \) is obtained for each model. Based on these, a series of \( p \) hypothesis tests is considered, with each one corresponding to a specific model \( m_o,j \) (\( j = 1, \ldots, p \)) of the MM representation \( m_o \):

\[
H_0: \{y_1[t], y_2[t]\} \text{ obey } m_{o,j} \\
H_1: \text{ otherwise}
\]

(8)

The Likelihood Ratio (LR) \( L(H_1, H_0 | \mathbf{e}_{u,k}) \) corresponding to \( \mathbf{e}_{u,k} \) and to a single \( \mathbf{e}_{o,k} \) (\( k = 1, \ldots, K \)) residual sequence (from the \( p \) available from the baseline phase) is given by:

\[
\Lambda = L(H_1, H_0 | \mathbf{e}_{u,k}) = \frac{K}{2} \ln \frac{\det \mathbf{R}_{\mathbf{e}_o}}{\det \mathbf{R}_{\mathbf{e}_u}} + \frac{1}{2} \sum_{k=1}^{K} \mathbf{e}_{u,k}^T \mathbf{R}_{\mathbf{e}_o}^{-1} \mathbf{e}_{u,k} \quad (9)
\]

\( H_0 \) is accepted if \( \Lambda \leq \ell \) where \( \ell \) is a user defined threshold and this hypothesis testing is repeated for all \( p \) models of \( m_o \) leading to a set \( \{A_1, \ldots, A_p\} \) of LR values. A structure is then declared as healthy if \( H_0 \) is satisfied for any of the baseline models \( m_{o,j} (j = 1, \ldots, p) \), otherwise the structure is declared as damaged formulating the following decision making:

\[
\text{Healthy structure: } \min_{1 \leq j \leq p} A_j \leq \ell \\
\text{Damaged structure: } \text{otherwise}
\]

(10)

### 3. Procedure for assessing the platform

The platform is assessed via laboratory experiments with four nominally identical composite, full scale, booms that connect the tail structure to the fuselage in the twin-boom Unmanned Aerial Vehicle of Figure 1.

**The nominally identical composite structures.** Each boom (Figures 3, 4) is 2 570mm long and is constructed of two thin walled 'C' section channels, trimmed and assembled together via a set of structural foam ribs and a pair of aluminum backing plates to form a closed rectangular beam. The thickness of the composite boom walls varies along their length, which are optimally designed for both strength and stiffness. The material used is a typical carbon fibre bi-axial (0°/90° and 45°/−45°) dry fabric, hand impregnated with epoxy resin, ply by ply.
The damage scenarios. Damage is induced on three of the booms, at about the same location (see Figure 4) between ribs 6 and 7, using a distinct impact energy level on each, that is 13J, 18J and 24J, corresponding to an invisible, barely visible and visible imprint, respectively (see upper part of Figure 5 and Table 1).

Damage is confirmed via InfraRed Thermography (IRT) NDT employing the Optical Pulse Thermography (OPT) methodology\(^{(16)}\), according to which the material under inspection is heated by an energy pulse of short duration usually carried out using high energy flash lamps. Currently, a heating pulse of approximately 6 kJ is applied for about 15 ms and subsequently the raw data captured by the IR sensor is processed to improve the detection and characterization capability. The IRT results indicate the existence of the stiffeners, differences in thicknesses and material overlapping, thus material/manufacturing variability as well as delamination and small cracks in the impact zones (see lower part of Figure 5).

The experiments. A high number of test cases is considered, initially with the four healthy booms (144 signals / test cases) and, later on, following damage, with the three damaged booms (162 signals / test cases). Of the 144 initial signals (with the four
healthy booms), 4 are exclusively used in the baseline (training) phase and 140 (35 per boom) in the inspection phase. During the experiments, each boom is clamped at one end, representing its connection to the fuselage, while its free end is attached to an electromechanical shaker inducing a (unmeasurable) force excitation to the tail structure (Figures 3, 4). Each exerted force is the realization of a random white Gaussian signal applied vertically at Point X via a Podera electromechanical shaker equipped with a stinger which is attached to the boom via a suction pad. Although not used for damage detection, each exerted force is measured via an impedance head, while vibration acceleration response signals are measured at points Y1 and Y2 via lightweight accelerometers as shown in Figures 3, 4. Further experimental details are provided in Table 1.

**Table 1. The damage scenarios and experimental details**

<table>
<thead>
<tr>
<th>Structural State</th>
<th>Description</th>
<th>No. of booms</th>
<th>No. of signals / boom</th>
<th>Total no. of signals / test cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline phase (training of the platform)</td>
<td>Healthy</td>
<td>---</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Inspection phase (real time operation of the platform)</td>
<td>Healthy</td>
<td>---</td>
<td>4</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Damage A</td>
<td>Impact (13J)</td>
<td>1</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Damage B</td>
<td>Impact (18J)</td>
<td>1</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Damage C</td>
<td>Impact (24J)</td>
<td>1</td>
<td>54</td>
</tr>
</tbody>
</table>

Sampling frequency $f_s = 1\,651.6$ Hz; signal length: 55 068 ($\approx 33$ s)

**Preliminary analysis of the measured vibration signals.** Each measured signal is sample mean corrected and normalized by its own standard deviation. Welch–based Frequency Response Function (FRF) magnitude estimates\(^{(13)}\) are provided in Figure 6. It is evident that the FRFs corresponding to the healthy and damaged booms overlap for most of the frequency bandwidth (see Figure 6(a)), indicating a challenging damage detection problem, as the variability in the healthy boom dynamics may ‘mask’ the subtle changes due to damage; also see Figures 6(b), (c), (d) for the effects of each damage.
4. Assessment of the SHM platform

4.1 Training (baseline) phase

A single 10 000 sample long vibration response signal from point Y1 (see Figures 3, 4) of each of the four healthy booms, that is \( p = v = 4 \), is used for the estimation of four AR(65) models in the context of the MM-AR method. Similarly, four pairs of signals from points Y1 and Y2 are used for the estimation of four transmittance-based ARX(20,19) models \( (\mathbf{\Psi}_A \neq \mathbf{\Psi}_C) \) in the context of the MM-ARX method. All models are estimated using Ordinary Least Squares (OLS) with QR implementation\(^{(13)}\), while model order selection is based on the Bayesian Information Criterion (BIC) and the Residual Sum of Squares / Signal Sum of Squares (RSS/SSS)\(^{(13)}\). Thus, four AR(65) and four ARX(20,19) models are employed as the MM representation \( \mathcal{M} \) of the healthy dynamics by the MM-AR and the MM-ARX methods, respectively. The thresholds \( l_{\text{lim}} \) and \( \ell \) for both methods and the \( N_q = 25, M = 300 \) (thus \( K = 30 \)) for the MM-ARX are also selected.

4.2 Assessment (inspection) phase

Once training has been completed, the platform is ready for monitoring the structural integrity of the booms. For proper performance assessment 140 signals / test cases from the healthy booms and 162 from the damaged ones (also see Table 1) are employed. In Figure 7 screenshots from the platform’s Graphical User Interface are presented. Various parameters, related to data acquisition, signal conditioning and filtering, signal pre-
processing, damage detection and several others may be tuned. Some indicative damage detection results are presented in Figures 7(c), (d). The detection thresholds of the MM-AR and the MM-ARX methods are indicated via a red continuous line, while the employed test statistics (also see equations (3) and (10)) via bars. The overall performance of the platform is excellent, as the detection of all considered damage scenarios is achieved with no false alarms or missed damage cases. Additional damage detection results in terms of a scatter plot of the $D$ test pseudo-statistic are, for the MM-AR method, presented in Figure 8. As the separation between the healthy and damaged structural state is very clear in all cases, flawless damage detection is achieved.

It is worth noting that the number $v$ of the healthy nominally identical structures used in the baseline (training) phase may significantly affect damage detection performance $^{(4, 9)}$. This is due to the fact that the MM representation for modelling the population of the healthy structural dynamics should be based on a representative sample of nominally identical structures under the considered uncertainty sources. Such a sample may be selected using NDT techniques on a number of parts obtained from the production line and/or via the quantification of the uncertainty effects on the structural dynamics based on the vibration response signal power spectral density, the transmittance function and so on$^{(7)}$.

5. Conclusions
A prototype SHM platform for random vibration-based damage detection in composite aerostructures was presented. This platform is autonomous, capable of acquiring and processing vibration signals, as well as of achieving damage detection using a Multiple
Model framework. The platform utilizes two response-only statistical time series methods that employ the parameters and the residuals of multiple AR and transmittance-based ARX models, respectively, as damage sensitive features. The platform’s effectiveness was assessed through almost real time detection of impact-induced damages of different energy in four nominally identical composite booms. The detection results were flawless exhibiting zero false alarms and no missed damages.

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