Damage Indices as a Measure for Optimal Sensor Placement in a Guided Wave Monitored Cracked Notched Plate Based on Numerical Simulation versus Scanning Lased Doppler Vibrometry

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Abstract

Optimum placement of piezoelectric wafer acoustic sensors onto a structure with a given tolerable damage in terms of guided wave-based structural health monitoring (GW-SHM) can become a challenge. The tolerable damage has to be detected by 100% for the GW-SHM system in terms of probability of detection in case the GW-SHM system is acceptable. Damage indices are scalar quantities that can be obtained from differences of guided wave signals between a pristine and a damaged condition. This data can be either generated through waveform simulations such as using COMSOL-Multiphysics or experimentally by use of Scanning Laser Doppler Vibrometry (SLDV). An attempt has been made by numerical simulations to first understand the correlation within the different damage indices considered and the differential signals resulting from the pristine and the tolerated damaged condition of a notched plate. The locations of maximum differential signal are considered to be the locations of optimum sensor placement. Validation of those locations is made through 3D SLDV. However, the SLDV method is also explored as the single evaluation method to generate the differential signals in terms of components of higher structural complexities such as a structural repair, where simulations may be more cumbersome than 3D laser scans.

1. Introduction

In an active structural health monitoring (SHM) approach using guided waves (GW), one of the prime requirements is to deploy actuators/sensors for reliably determining damage of a tolerable size. For a given appropriate loading condition, the likely location of a damage can be obtained from the result of a load (stress/strain) distribution simulation analysis. Based on the inputs of location and size of the tolerable damage, the number of actuators/sensors required to be placed on the structure for reliably detecting the respective damage can be determined. There are numerous ways in which one can achieve the optimum actuator-sensor pattern for the given allowable damages to be present in the structure for a general SHM system where some are reported in references [2, 6, 14, 15, 17]. Theoretically, a dense sensor network is considered to detect the damages more reliably. However, irrespective of this consideration’s truth this may be highly impractical to implement since the volume of data to be processed will turn out to be big for a large structure. A sparse network with a high probability of detecting the tolerable damage has to be opted for with respect to the SHM applications to be found.
Early work on optimum sensor positions for damage detection was performed by Hemez et al. [6]. In his work he used strain energy distribution as a criterion to decide the optimum sensor placement. Lee [2] describes in his paper that the best possible solution is to place the sensors close to the damage while a first wave packet in a signal has to be considered. Stepinski et al. [13] mentioned that at least three sensors are required to triangulate a damage instance with more accuracy. In a recent study, Wandowski et al. [14] used laser vibrometry along with a piezoelectric actuator - often confusingly specified as a piezoelectric wafer active sensor (PWAS) - for non-contact sensing that allows to speed up the measurement process for a circular sensing network in order to verify if a sensing network is a valid candidate for damage detection or not. Ewald et al. developed a blob detection algorithm for locating hot spot areas in GW-SHM [15].

In this paper, optimal sensor locations are identified with the aid of numerical simulation by means of differential imaging and Damage Indices-map (D.I. map) methods for a given plate with holes and cracks around the holes. While understanding the characteristics of wave propagation using numerical simulation on simple plate-like structures, Scanning Laser Doppler Vibrometry (SLDV) is an efficient method to realize the wave propagation in a realistic situation in terms of identifying the optimal sensor positions for a given tolerable damage. Numerical simulations developed are validated with experiments in the end.

2. Models and properties

2.1 Model geometry

The material of the plates considered is aluminium grade 2024 and its mechanical properties are given in Table 1. The plates are shown in Fig. 1(a) considered as model-1 being a plate with three holes and a crack of 25mm at the middle hole and as model-2 being a plate with a crack of 14 mm at the right side which is shown in Fig. 1(b).

![Figure 1(a). Model-1 with crack at the centre of the hole and (b) Model-2 with crack at the right side of the hole](image)

Table 1. Mechanical properties of Aluminium Grade 2024

<table>
<thead>
<tr>
<th>Property</th>
<th>Model-1</th>
<th>Model-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [kg/m³]</td>
<td>2700</td>
<td>2700</td>
</tr>
<tr>
<td>Young’s Modulus [GPa]</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Poisson’s Ratio [-]</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>
2.2 Piezoelectric material

The GW in the aluminium plate is generated by surface mounted PWAS transducers. In this study a Lead Zirconate Titanate PZT-5A material as a PWAS transducer has been considered. The properties of PZT are shown in Table 2.

### Table 2. Electrostatic properties of Lead Zirconate Titanate PZT-5A

<table>
<thead>
<tr>
<th>Elasticity matrix [GPa]</th>
<th>Coupling matrix [C/m2]</th>
<th>Relative permittivity [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$, $E_{22}$</td>
<td>$E_{33}$, $E_{44}$, $E_{55}$</td>
<td>$E_{66}$, $E_{12}$, $E_{13}$</td>
</tr>
<tr>
<td>120.4</td>
<td>110.9</td>
<td>21.1</td>
</tr>
</tbody>
</table>

3. Numerical simulation of guided waves using FEM

A 3D-FEM model has been developed by means of COMSOL-Multiphysics to generate and receive GW in the given plates (model-1 and model-2). This is achieved by coupling the piezoelectric devices and the structural mechanics modules in the COMSOL software. The material behavior due to applied force is studied within the structural mechanics module and the piezoelectric effect is studied within the electrostatics domain. This is shown in Fig. 2 where a piezoelectric material is being applied as a common interface coupling the two physical models. In an early study, a 2D-FEM model was analysed by Nieuwenhuis et al. [7] regarding the generation and detection of guided waves using PZT wafer transducers. Similarly, Zennaro et al. has developed a numerical simulation to study and model the transducers for GW [9].

The governing equations for the piezoelectric effect, i.e. the generated electric field for the applied stress, and the inverse piezoelectric effect, i.e. the total strain generated for the given electric field, can be written as follows:

\[
E = -g \cdot T + \beta^T \cdot D \tag{1}
\]

\[
S = s_E \cdot T + d^T \cdot E \tag{2}
\]

where the following parameters represent,
- **S, T** - strain and stress
- **E, D** - electric field and electric displacement (charge)
- **d, d^T** - coupling between electrical and mechanical variables
- **s_E** - Strain per unit stress
- **$\varepsilon_T$** - Electric permittivity
- **g, $\beta^T$** - Piezoelectric coefficients and permittivity.

![Figure 2. Finite Element multi-physics model using COMSOL-Multiphysics](image)

Eq. (2) is used for modelling the actuator and Eq. (1) for modelling the sensor behaviour.
in the absence of the temperature influence. GW simulation can be performed without the multi-physics option by considering only the structural mechanics module and then considering plain strain conditions. In this case the respective transducer geometry is given a time dependent boundary load. The application of such models can be referred to in [3, 15].

3.1 Excitation signal function

Since GWs are dispersive waves, a tone burst excitation was preferred in order to excite coherent single-frequency waves to minimize the dispersion effects. However, the raw tone burst excitation has side frequencies or side lobes associated with the sharp transition at the start and the end of the signal. A Hanning window function of the form described with Eq. (3) is therefore used to smoothen the raw tone burst excitation and to apparently reduce the side lobes [16]. The excitation signal is shown in Fig. 3.

![Figure 3. Five cycles Hanning window excitation function](image)

\[ V(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{\omega_0 t}{N_C} \right) \right] \sin(\omega_0 t), t \in \left[ 0, \frac{N_C}{f} \right] \]  (3)

where the following parameters represent:
- \( V(t) \): excitation voltage
- \( \omega_0 \): angular frequency
- \( N_C \): number of counts that matches the length of the Hanning window.

3.2 Mesh and critical time step

The mesh element size used for the simulation in the FEM models is critical for the accuracy of the solution. The definition of the maximum length element (\( \Delta x_{\text{max}} \)) is presented in Eq. (4).

\[ \Delta x_{\text{Max}} = \frac{\lambda}{R} \]  (4)

where \( \lambda \) is the wavelength of the guided wave mode propagated in the direction of incidence and \( R \) is a number between 5 to 10. The use of a value of \( R \geq 8 \) was recommended by Ghose et al. [3]. For the evaluation of the critical time step, the CFL (Courant-Friedrichs-Lewy) condition should be met to achieve a stable solution. It is defined by the following relationship using Eq. (5).

\[ \Delta t_{\text{critical}} \leq \frac{\Delta x_{\text{max}}}{C} \]  (5)
where C is the longitudinal ultrasonic wave of the material. In the model used here, \( \Delta x_{\text{Max}} \) of 1 mm has been used in the structural mechanics domain and \( \Delta x_{\text{Max}} \) of 0.2 mm is used for the electrostatic domain. A time step of 0.1 \( \mu \text{s} \) satisfies the CFL criteria.

### 3.3 Results of numerical simulations

![Figure 4. GW propagation in an undamaged plate after a) 30 \( \mu \text{s} \), b) 50 \( \mu \text{s} \), c) 70 \( \mu \text{s} \) & d) 90 \( \mu \text{s} \)](image1)

![Figure 5. GW propagation in a damaged condition after a) 30 \( \mu \text{s} \), b) 50 \( \mu \text{s} \), c) 70 \( \mu \text{s} \) & d) 90 \( \mu \text{s} \)](image2)

### 4. Optimal sensor placement using differential imaging and damage indices map (D.I. map)

#### 4.1 Differential imaging

The basic principle of the differential imaging method is that for a time incident x the wave propagation image for a given damage configuration is subtracted from the wave propagation of a pristine condition. Thus, the resultant image shows the areas of maximum and minimum displacements caused due to the scattering of GW in the presence of damage.

For the known elastic constants and by means of Hooke’s law expressed as \( \sigma_{ij} = C_{ijkl} e_{kl} \), the following constitutive relation is established between strain and the displacements (Eqs. 6 & 7), and is implemented in the COMSOL structural mechanics module:

\[
\varepsilon_{i,j} = \frac{1}{2} (u_{i,j} + u_{j,i})
\]  

(6)
\[
\varepsilon_{zz} = \frac{\partial u_z}{\partial z}
\]

(7)

\[
\frac{\partial \Delta u}{\partial z \cdot \partial t} = \left( \frac{\partial u_{\text{crack}}}{\partial z \cdot \partial t} - \frac{\partial u_{\text{pristine}}}{\partial z \cdot \partial t} \right)
\]

(8)

where \( \sigma \) and \( \varepsilon \) are Cauchy’s stress and strain tensors and \( u \) is the displacement. One can write the differential displacements in the form of Eq. (8). Results of such differential displacements for different time intervals have been determined for model-1 and model-2 as shown in Fig.s 6 and 7. It can be seen that the areas where the differences vary significantly depend on location and time. What therefore looks to be useful is to place sensors along the vertical symmetry axis and possibly also adjacent and to record the signals during the time interval between 30 and 90 \( \mu s \). The application of differential imaging for composite materials can be referred to in [12]. The method has also been used to obtain the optimum sensor location for structural repairs [1].

4.2 Damage Indices map (D.I map)

Damage detection in SHM using GW signals has been widely reported in literature [2, 5, 13, 16, 17]. Damage indices or damage metric values are used to measure the amount of damage present in the structure through the statistical processing of the resultant signal with the signals of the pristine state. If any damage is present in the structure, the spectral content of the signal will be affected. The resultant scalar value is called a damage index. In this paper, Differential Energy (DE), Correlation Coefficient Deviation (CCD), Root Mean Square Deviation (RMSD) and Mean Absolute Error (MAE) are used as damage indices. The procedure to obtain a D.I. map is to virtually assume a sensor on each point of the plate where the displacements are recorded as a result of the GW propagation. For the given time range, all the sensor points in the plate record displacement signals and subsequently D.I.s are evaluated for all points to obtain a D.I. map as shown in Fig. 8. The evaluation of D.I. needs two states (pristine and damaged) as given below:

![Figure 6. Differential images of model-1 (centred crack) at a) 30 \( \mu s \), b) 50 \( \mu s \), c) 70 \( \mu s \) & d) 90 \( \mu s \)](image)

DE is the resulting energy of the signal obtained between the damaged and the reference condition. The energy of the given signal is the sum of the squares of the resulting amplitude in time-domain. It is defined as follows:
DE = \sum_{i=1}^{N} (S_{id} - S_{iu})^2 \quad (9)

Figure 7. Differential images of model-2 (crack right side) at a) 30 µs, b) 50 µs, c) 70 µs & d) 90 µs

Figure 8. Generation of damage indices map

CCD consists of the statistical scalar value to compare the similarity of two sets of data. The value of ‘1’ means there is a total similarity and the value ‘0’ means there is an absolute dissimilarity between the two data sets. It is defined as follows:

\[ \text{CCD} = 1 - \left( \frac{\text{cov}_u}{\sigma_u} \right) \left( \frac{\text{cov}_d}{\sigma_d} \right) \quad (10) \]

cov\_u and cov\_d are the co-variances of the time-domain data for the undamaged and the damaged conditions. \( \sigma_u \) and \( \sigma_d \) are the standard deviations of the undamaged and the damaged conditions.

RMSD is used to obtain variations in phase and amplitude of damaged and undamaged conditions. It is given in Eq. (11).

\[ \text{RMSD} = \sqrt{\frac{\sum_{i=1}^{N} (S_{id} - S_{iu})^2}{\sum_{i=1}^{N} S_{iu}^2}} \quad (11) \]

MAE is the mean absolute difference between data of undamaged and damaged conditions. It is given as follows:
MAE = \frac{1}{N} \sum_{i=1}^{N} |S_{id} - S_{iu}| \quad (12)

$S_{ud}$ and $S_{id}$ are time-domain signal amplitudes for pristine and damaged conditions while for RMSD it means the frequency-domain signal amplitudes for the pristine and the damaged conditions. Fig.s 9 and 10 show the D.I. map as a result of plotting the damage indices on the given models.

Figure 9. Damage Indices (D.I) map for model-1; a) Differential energy, b) Correlation Coefficient Deviation (CCD), c) Root Mean Square Deviation (RMSD), d) Mean Absolute Error (MAE)

Figure 10. Damage Indices (D.I.) map for model-2; a) Differential energy, b) Correlation Coefficient Deviation (CCD), c) Root Mean Square Deviation (RMSD), d) Mean Absolute Error (MAE)

Similar to differential images, the D.I. map also shows points of high intensity (red colours) representing locations where the influence of damage can be significantly recorded. By correlating the differential images and the D.I. maps, an optimum of sensor positions can be identified for model-2, which is at $x = 0.040$ m and $y = 0.135$ m respectively, as shown in Fig. 11. The D.I. based optimum sensor locations for model-1 can be referred to in [11]. In Fig. 11 sensor-1 is kept at a position of high differential displacement and sensor-2 is kept at a position of low differential displacements. Their respective D.I.s are shown in the histogram plot in Fig. 12. It can be seen that the location of sensor-1 is indeed the location for a best sensor placement irrespective of differential imaging or the application of a D.I.
5. Experimental validation

SHM hardware consists of an integrated function generator and data acquisition modules as shown in Fig. 15(a). The description of the SHM system can be referred to in [1,8]. Fig. 15(b) shows the plate (model-2) integrated with sensors and their positions are taken as per Fig. 11. The comparison of the simulation with the experimental signals are shown in Fig. 15(c) for sensor-1 and in Fig. 15(d) for sensor-2.

![Figure 11. Optimum sensor locations marked in red for model-2](image)

![Figure 12. Histogram of the damage indices for sensor 1 and sensor 2: a) Cross-correlation deviation, b) Root mean square deviation](image)

![Figure 15. Experimental validation: a) SHM hardware, b) Aluminium plate with embedded PWAS, c) Comparison of simulated and experimental signal for sensor-1 and d) Comparison of simulated and experimental signal for sensor-2](image)

The first packet of the Hanning windowed signal arriving matches closely with phase and amplitude between the experiment and the simulation. However, the wave packets to follow rather differ being a result of boundary reflections of the symmetric modes,
which differ in amplitudes due to the fact that the boundaries in the FEM models have been considered smooth when compared to edges of the actual plate considered here.

6. Potentials of SLDV in GW-SHM

Experimental validation of wave generation, propagation and interaction with features of the plate has also been carried out by means of a SLDV setup as shown in Fig. 13. It consists of a non-contact technique based on the Doppler effect, which measures vibrations in structures. Surface wave velocity (i.e. of the GW) as a result of out-of-plane displacements has been recorded in a grid of points at the surface of the plate considered, and time-domain surface velocity has been plotted, analysed and compared with the results from the simulation. Fig. 14(a) shows simulation results at 35 µs after launching the GW where the incident GW interacts with the sensor and generates secondary waves that further interfere with the incident symmetrical GW mode.

![Scanning Laser Doppler Vibrometry (SLDV) setup](image)

Figure 13. Scanning Laser Doppler Vibrometry (SLDV) set up

This effect could be verified with the SLDV results for the same interval in time. When GW further travels and interacts with the crack (model-2), the incident wave undergoes scattering of GW energy and as a result, the incident wave energy splits after transmitting through the crack. This is shown in Fig. 14(c) and (d) for simulation and SLDV results at 60µs.

![Wave propagation results](image)

Figure 14. Wave propagation results; a) Simulation at 35 µs, b) SLDV results at 35 µs, c) Simulation at 60 µs and d) SLDV results at 60 µs

7. Discussion and conclusion

Differential imaging and D.I. map methods have been applied on two notched plates with two different artificial cuts (model-1 and model-2) simulating a damage. The
results of differential imaging shown in Figs. 6 and 7 reveal that the locations where a
difference between two damaging conditions are measured very much depend on where
the location and size of the damage to be monitored is. A similar result can be obtained
when considering damage indices (D.I.) published in literature and mapping those as
shown in Figs. 9 and 10 in the form of D.I. maps. Although the maximum D.I. values
are close to the damage to be monitored and are therefore uninteresting as a parameter
for damage localisation in this specific case, there is a decent wide field on the specimen
where the D.I. is at least half of the maximum, representing locations where a damage
considered could be reliably detected at a decent distance away from the damage’s true
location. Fig. 10 shows to which extent this is the case and the results in Fig. 12 show
that the respective quantitative difference in the signal is obtained with regard to the two
conditions (high and low signal) to be monitored. These differences are significant and
representative, and can therefore be taken as a means for damage monitoring. They are
not just of a numerical nature but can also be validated by experiments as done with
respect to the SLDV based experimental results shown. As such it has been shown that
numerical simulation is of a significant help to determine the locations where sensors
should be placed to detect a tolerable damage with regard to location and size. Reliable
detection of damage based on an SHM system therefore needs a definition with respect
to damage location and tolerable size because only under these conditions a true sensor
pattern can be identified in terms of the sensor locations as well as the time intervals
during which the sensors will have to record the respective signals. What can also be
observed from the differential imaging method and the D.I. mapping methods is the
pattern (shape and size) a sensor may have to monitor the maximum of differential
energy distribution. This can become important with respect to coating a structure with
sensor patterns as it has been described in [10]. Principally SLDV could even be
thought to be solely used as a means for determining differential images experimentally.
In that case data would have to be determined for a pristine and a damaged condition of
the component to be monitored and the images would have to be merged accordingly.
However, this idea still needs to be proven in reality. Damage tolerance design can be
very well associated with SHM systems in a way that structures can be continuously
monitored regarding a tolerable damage (diagnostics). The information retrieved can
then be used to predict the structure’s remaining useful life (prognostics). The
probability of detecting such a tolerable damage needs to be at least 90% with a 95% of
confidence level. Finding the appropriate actuator and sensor configuration is therefore
of significant importance. The differential imaging and D.I. methods described here are
expected to provide a means on how to achieve this requirement of reliable detection
and identification of SHM potentials shown for the example of GW-SHM. Besides this,
SLDV is realized as a potential method for understanding GW interferences regarding
optimal sensor placement in real structures. This may become specifically interesting
with ageing and repaired structures not only in aerospace applications but also in
infrastructures such as bridges, building and others. Beyond this the approach may even
be extended to structures of complex geometric shape and anisotropic material
behaviour, underlining the generality of the approach in the end.

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References