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THE INFLUENCE OF COUPLED THERMAL FIELDS ON THE BEHAVIOR OF CRACKED BODIES

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INTRODUCTION

The properties and the quality of materials are determined most of the times under the coupled action of mechanical and physical fields. A most common characteristic of materials is the appearance of discontinuities such as cracks holes and several types of inclusions that ultimately determine their behavior. Hence the systematic study of the interplay between the applied coupled fields and the discontinuities is of great importance both in material science and engineering.

The purpose of the paper is to investigate the action of thermal fields on materials in the presence of two coupled sources: (a) a heat flow coming from infinity, and (b) a heat dipole. We derive the singular integral equations and we study the behavior of the stress-intensity factors with respect to the thermal factor. The literature used in the present paper is; [1],[2],[3].

§1. BASIC EQUATIONS OF TWO DIMENSIONAL STATIONARY PROBLEMS OF THERMOCONDUCTIVITY AND THERMOELASTICITY

In order to devise the constitutive equations which describe the interaction between thermal and strain-stress fields, we use the model of linear anisotropic and thermoelastic body. The assumptions of the above model are:

a) The components of strains are small.

b) The components of strains and the components of stresses satisfy the generalized Hooke's law.

c) The elastic and thermal properties of the solid vary with the direction and are independent of temperature and stresses.

Firstly, we define the vector of thermoconductivity $\overrightarrow{K_n}$ at the given point, which refers to the elementary surface with normal vector $\vec{n}$:

$$\overrightarrow{K_n} = a_1 k_{11}\vec{i} + a_2 k_{22}\vec{j} + a_3 k_{33}\vec{k} \qquad (1)$$

where: $(\vec{i},\vec{j},\vec{k})$ are the unit vectors of curvilinear coordinate system

a_i : the direction cosines

k_{ii} : the conductivity coefficients

Using the vector $\overrightarrow{K_n}$, the heat flux density q_n , crossing the elementary surface, can be expressed by the relation:

$$q_n = -(\overrightarrow{K_n} \cdot \overrightarrow{gradT}) \qquad (2)$$

where T is the temperature.

If the axes of the system coincide with the principal axes of conductivity, the thermal field is described by the following equation of thermoconductivity:

$$k_{11}\frac{\partial^2 T}{\partial x^2}+k_{22}\frac{\partial^2 T}{\partial y^2}+k_{33}\frac{\partial^2 T}{\partial z^2}=c\rho\frac{\partial T}{\partial t}-Q \qquad (3)$$

where: c : the bulk heat capacity

ρ : the density of the material

Q : the heat transmited per unit volume per unit time.

As far as the boundary conditions of thermoconductivity are concerned, we have the following possible conditions:

a) The temperature of the whole surface of the body is prescribed:

$$T=f_1(x,y,z,t) \qquad (4)$$

b) The heat flux density across the surface is prescribed:

$$\overrightarrow{K_n}\cdot\overrightarrow{gradT}=f_2(x,y,z,t) \qquad (5)$$

c)The flux across the surface is proportional to the temperature difference between the surface and the surrounding medium.

$$\overrightarrow{K_n}\cdot\overrightarrow{gradT}=\lambda(T-T_o) \qquad (6)$$

where: T_o is the temperature of the surrounding medium

λ is the heat transfer coefficient.

If the plane body is homogeneous and does not contain any heat source, the equation (3), in an arbitrary coordinate system becomes:

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2}+2\lambda_{12}\frac{\partial^2 T}{\partial x\partial y}+\lambda_{22}\frac{\partial^2 T}{\partial y^2}=0 \qquad (7)$$

where: $\lambda_{11}=k_{11}\cos^2 a+k_{22}\sin^2 a$

$$\lambda_{22}=k_{11}\sin^2 a+k_{22}\cos^2 a$$

$$\lambda_{12}=(k_{11}-k_{22})\sin a\cos a$$

a is the angle between x-axis and principal direction of the axe of conductivity

$$\lambda_{11}+\lambda_{12}=k_{11}+k_{22}\ ,\quad \lambda_{11}\lambda_{22}-\lambda_{12}{}^2=k_{11}k_{22}$$

The general solution of equation (7) is given by:

$$T=F(z_3)+\overline{F(z_3)} \qquad (8)$$

$F(z_3)$ is an analytical complex function and μ_3 is one of the roots of the characteristic equation:

$$\lambda_{22}\mu^2+2\lambda_{12}\mu+\lambda_{11}=0$$

with $\mu_3=-\lambda_{12}+i(k_{11}k_{22})^{1/2}/\lambda_{22}$

The flux of heat, in terms of $F(z_3)$, is expressed by:

$$\overrightarrow{K_n}\cdot\overrightarrow{gradT}=(\lambda_{11}\frac{\partial T}{\partial x}+\lambda_{12}\frac{\partial T}{\partial y})\beta_1+(\lambda_{12}\frac{\partial T}{\partial x}+\lambda_{22}\frac{\partial T}{\partial y})\beta_2 \qquad (9)$$

Where: β_1,β_2 are the direction cosines between elementary surface ds, and the normal vector $\vec{n}$.

Relation (9), on the basis of (8) becomes:

$$\overrightarrow{K_n}\cdot\overrightarrow{gradT}=A_1^*F'(z_3)+\overline{A_1^*F'(z_3)} \qquad (10)$$

where: $A_1^*=(\lambda_{12}+\mu_3\lambda_{22})(-\beta_2+\mu_3\beta_1)$

At any point of the solid, when the thermal potential $F(z_3)$ is known, relations (8) and (10) give us the possibility to evaluate the temperature and the flux of heat respectively.

In the case of plane strain of an homogeneous anisotropic body, when for each point of the body there's a plane of elastic symmetry normal to the z-axis (which coincide with one of the principal directions of conductivity), generalized Hooke's law takes the form:

$$\begin{aligned}
\varepsilon_{xx} &= \alpha_{11}\sigma_{xx} + \alpha_{12}\sigma_{yy} + \alpha_{13}\sigma_{zz} + \alpha_{16}\sigma_{xy} + \beta_{11}T \\
\varepsilon_{yy} &= \alpha_{12}\sigma_{xx} + \alpha_{22}\sigma_{yy} + \alpha_{23}\sigma_{zz} + \alpha_{26}\sigma_{xy} + \beta_{22}T \\
\gamma_{xy} &= \alpha_{16}\sigma_{xx} + \alpha_{26}\sigma_{yy} + \alpha_{36}\sigma_{zz} + \alpha_{66}\sigma_{xy} - 2\beta_{66}T \\
\varepsilon_{zz} &= \alpha_{13}\sigma_{xx} + \alpha_{23}\sigma_{yy} + \alpha_{33}\sigma_{zz} + \alpha_{36}\sigma_{xy} + \beta_{33}T = 0 \\
\gamma_{yz} &= \alpha_{44}\sigma_{yz} + \alpha_{45}\sigma_{xz} = 0 \\
\gamma_{xz} &= \alpha_{45}\sigma_{yz} + \alpha_{55}\sigma_{xz} = 0
\end{aligned} \qquad (11)$$

or:

$$\begin{aligned}
\varepsilon_{xx} &= c_{11}\sigma_{xx} + c_{12}\sigma_{yy} + c_{16}\sigma_{xy} + \alpha_1 T \\
\varepsilon_{yy} &= c_{12}\sigma_{xx} + c_{22}\sigma_{yy} + c_{26}\sigma_{xy} + \alpha_2 T \\
\gamma_{xy} &= c_{16}\sigma_{xx} + c_{26}\sigma_{yy} + c_{66}\sigma_{xy} - 2\alpha_6 T
\end{aligned}$$

where: a_{ij}, c_{ij} are the elastic constants which satisfies the following relation:

$$c_{ij} = a_{ij} - \frac{a_{i3}a_{j3}}{a_{33}} \qquad i, j = 1,2,6 \qquad (13)$$

and β_{ij} are the coefficients which determine the components of strain tensor of a body, free of external loads, when the temperature changes one degree.
For the above coefficients the relations:

$$a_i = \beta_{ii} - \frac{\beta_{33}a_{i3}}{a_{33}} \quad (i = 1,2)$$

$$a_6 = \beta_{66} + \frac{\beta_{33}a_{36}}{2a_{33}} \qquad (14)$$

For the orthotropic solid, as long as the coefficients $c_{ij}, a_{ij}, \lambda_{ij}$ are stable, the components of stress and strain tensor through the 3 complex potentials: $\Phi(z_1), \Psi(z_2), F(z_3)$ are:

$$\begin{aligned}
\sigma_{xx} &= 2\,\mathrm{Re}[\mu_1^2\Phi(z_1) + \mu_2^2\Psi(z_2) + \eta_o\mu_3 F(z_3)] \\
\sigma_{yy} &= 2\,\mathrm{Re}[\Phi(z_1) + \Psi(z_2) + \eta_o F(z_3)] \\
\sigma_{xy} &= -2\,\mathrm{Re}[\mu_1\Phi(z_1) + \mu_2\Psi(z_2) + \eta_o\mu_3 F(z_3)] \\
u &= 2\,\mathrm{Re}[p_1\phi(z_1) + p_2\psi(z_2) + p_*\psi(z_3)] \\
v &= 2\,\mathrm{Re}[q_1\phi(z_1) + q_2\psi(z_2) + q_*\psi(z_3)]
\end{aligned}$$

where:

$$p_j = c_{11}\mu_j^2 + c_{12} - c_{16}\mu_j$$

$$\mu_j q_j = c_{12}\mu_j^2 + c_{22} - c_{26}\mu_j \qquad j = 1,2$$

$$p_* = a_1 + \eta_o(c_{11}\mu_3^2 - c_{16}\mu_3 + c_{12})$$

$$\mu_3 p_* = a_2 + \eta_o(c_{11}\mu_3^2 - c_{26}\mu_3 + c_{22})$$

$$\eta_o = -(a_1\mu_3^2 + 2a_6\mu_3 + a_2)/\Delta(\mu_3)$$

$$\Delta(\mu_3) = c_{11}(\mu_3 - \mu_1)(\mu_3 - \mu_2)(\mu_3 - \overline{\mu_1})(\mu_3 - \overline{\mu_2})$$
$$\Phi(z_1) = \phi'(z_1)$$
$$\Psi(z_2) = \psi'(z_2)$$
$$F(z_3) = \psi'(z_3)$$

μ_j $(j = 1,2,3,4)$, $(\mu_1 = \overline{\mu_3}, \mu_2 = \overline{\mu_4}$) are the roots of the characteristic equation:

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0$$

which correspond to the following biharmonic equation:

$$a_{22}\frac{\partial^4 U}{\partial x^4} - 2a_{26}\frac{\partial^4 U}{\partial x^3 \partial y} + (2a_{12} + a_{66})\frac{\partial^4 U}{\partial x^2 \partial y^2} - 2a_{16}\frac{\partial^4 U}{\partial x \partial y^3} + a_{11}\frac{\partial^4 U}{\partial y^4} = 0$$

$$\left(\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}, \sigma_{yy} = \frac{\partial^2 U}{\partial x^2}, \sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y}\right)$$

For the isotropic solid the relations (8) and (15) have the form:

$$\begin{aligned} &\sigma_{xx} + \sigma_{yy} = 2[\Phi(z) + \overline{\Phi(z)}] \\ &(\sigma_{yy} - \sigma_{xx}) + 2i\sigma_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)] \\ &2\mu(u + iv) = \kappa\phi(z) - z\overline{\Phi(z)} - \overline{\psi(z)} + \beta\int F(z)dz \\ &T(x,y) = 2\,\mathrm{Re}\,F(z) = F(z) + \overline{F(z)} \end{aligned} \qquad (16)$$

where $\begin{cases}\kappa = 3 - 4\nu \\ \beta = \alpha E\end{cases}$ in the case of plane strain

and $\begin{cases}\kappa = \dfrac{3-\nu}{1+\nu} \\ \beta = \dfrac{\alpha E}{1+\nu}\end{cases}$ in the case of plane stress

where: ν: Poisson's ratio
E: elastic modulus
a : linear thermal expansion coefficient

In order to solve the problem of plane stationary thermoelasticity two successive steps have been used. In the case of anisotropic solid, in the first step we devise the thermal field which satisfies one of the boundary conditions (4)-(6) and equation (7).While, in the case of isotropic solid we have to solve the Laplace equation:

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

In second step we devise the stress-strain field, using the relations (15) or (16).

§2. CONSTITUTIVE EQUATIONS OF THERMOCONDUCTIVITY AND THERMOELASTICITY FOR A CRACKED ISOTROPIC MEDIUM

We consider an infinite isotropic plate, containing one crack. The plate is submitted to the normal stresses N_1, N_2 at infinity, and it is under the influence of homogeneous flux heat q_∞. Besides the given forces, concentrated forces $P_j + iQ_j$ at z_j^* $(j=1,...,K^*)$, moments at z_j^{**} $(j=1,...,K^{**})$ and K heat sources q_j at z_j $(j=1,...,K)$ can act in the plate as it is indicated in Figure 1.

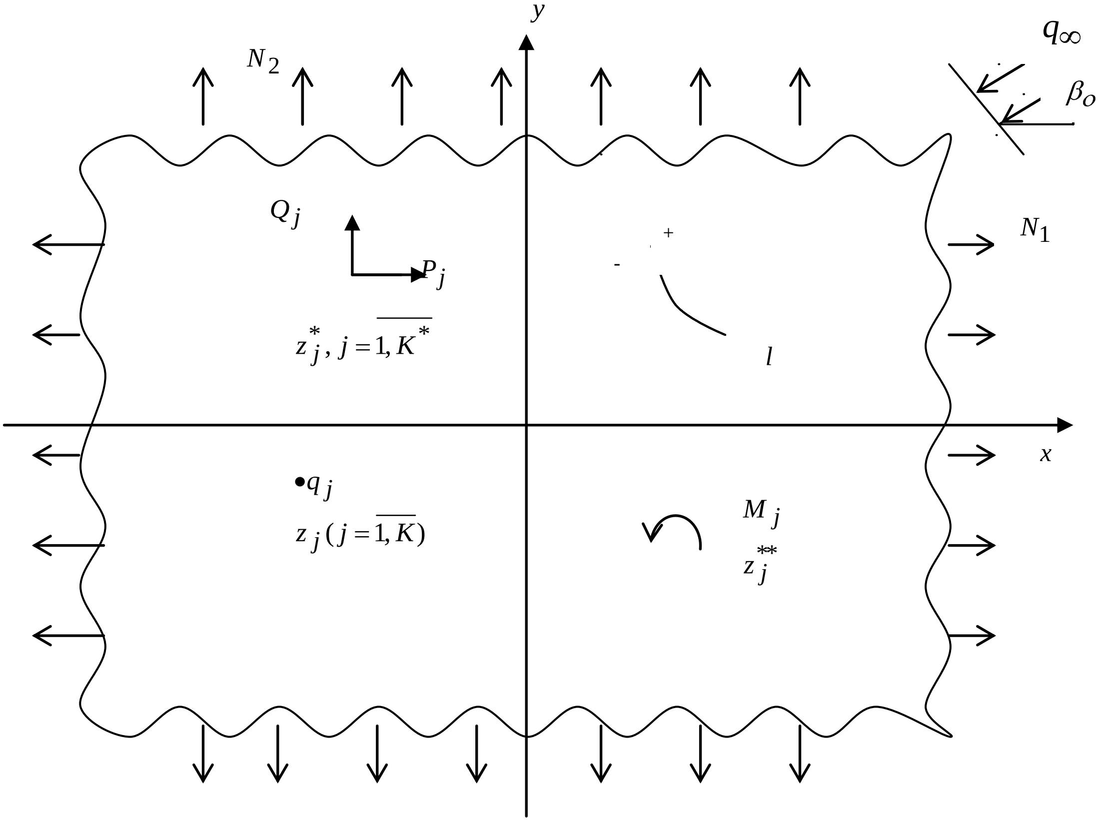


Fig (1)

Before constructing the constitutive equations of thermoelasticity , for the plane cracked body, we should determine the thermal boundary conditions, along the lips of the crack. In the case of isotropic medium, when the thermal contact along the lips of the crack is assumed as thermally not ideal, the following conditions are valid:

$$\lambda_s \frac{\partial^2}{\partial s^2}(T^+ + T^-) + 2\lambda^* \left[\left(\frac{\partial T}{\partial n}\right)^+ - \left(\frac{\partial T}{\partial n}\right)^-\right] = 0$$

$$\lambda_s \frac{\partial^2}{\partial s^2}(T^+ - T^-) + 6\lambda^* \left[\left(\frac{\partial T}{\partial n}\right)^+ + \left(\frac{\partial T}{\partial n}\right)^-\right] - 12\lambda_n (T^+ - T^-) = 0$$

where λ^* is the coefficient of thermal conductivity of the isotropic medium.

Concerning thermoconductivity there are three crack categories:

a) Thermoconductive crack: when $\lambda_s \neq 0$ and $\lambda_n \neq 0$

b) Longitudinal thermoinsulating crack: when $\lambda_{s=0}$ and $\lambda_n \neq 0$

c) Thermoinsulating crack : $\lambda_s = \lambda_{n=0}$

For the solution of the problem of thermoconductivity of a multiconnected cracked body, the temperature may be expressed by:

$$T(x,y) = T_o(x,y) + T_*(x,y)$$

where: $T_o(x,y)$ is the submitted thermal field which is considered as known

and : $T_*(x,y)$ is the demanding thermal field which appears due to the existence of discontinuities at the body.

Consequently, depending on the kind of the thermal contact at the boundary of the crack l, we can have one of the three following thermal boundary conditions.

$$T_*^{\pm} = f^{\pm}(t) - T_o(t) \qquad ,t \in l \qquad (19)$$

$$\lambda^* \left(\frac{\partial T}{\partial n}\right)^{\pm} = Q^{\pm}(t) - \lambda \frac{\partial T_o}{\partial n} \qquad ,t \in l \qquad (20)$$

$$\begin{cases} \lambda_s \dfrac{\partial^2}{\partial s^2}(T_*^+ + T_*^-) + 2\lambda^* \left[\left(\dfrac{\partial T_*}{\partial n}\right)^+ - \left(\dfrac{\partial T_*}{\partial n}\right)^-\right] = 2\lambda_s \dfrac{\partial^2 T_o}{\partial s^2} \\ \lambda_s \dfrac{\partial^2}{\partial s^2}(T_*^+ - T_*^-) + 6\lambda^* \left[\left(\dfrac{\partial T_*}{\partial n}\right)^+ + \left(\dfrac{\partial T_*}{\partial n}\right)^-\right] - 12\lambda_n (T_*^+ - T_*^-) = 12\lambda^* \dfrac{\partial To}{\partial n} \end{cases} \qquad (21)$$

Relationships (19) to (21) constitute the basis for the solution of the problem of thermoconductivity and evaluation of thermal field of the isotropic plate. In order to simplify the procedure, we will examine the case of relation (19).

The thermal potential $F_o(z), (T(x,y) = 2\,\mathrm{Re}\, F_o(z))$ describing the thermal field $T(x,y)$ is:

$$F_o(z) = \frac{q_\infty}{2} z e^{-i\beta_o} - \sum_{j=1}^{K} \frac{q_j}{2\pi\lambda^*} \ln(z - z_j) + F(z) \qquad (22)$$

$$\text{where:} \quad F(z) = \frac{1}{2\pi i} \int_l \frac{\gamma(\tau)}{\tau - z} d\tau \qquad (23)$$

and $\gamma(t)$ is the density of the Cauchy integral in the case of the thermal boundary condition (19).

The other two complex potentials have the form:

$$\Phi_o(z) = \Gamma - \sum_{j=1}^{K^*} \frac{P_j + iQ_j}{2\pi(1+\kappa)} \frac{1}{z - z_j^*} + \frac{\beta}{1+\kappa} \sum_{j=1}^{K} \frac{q_j}{2\pi\lambda^*} \ln(z - z_j) + \Phi(z) \qquad (24)$$

$$\Psi_o(z) = \Gamma' + \sum_{j=1}^{K^*} \left[\frac{\kappa(P_j - iQ_j)}{2\pi(1+\kappa)} \frac{1}{z - z^*} - \frac{\overline{z_j^*}(P_j + iQ_j)}{2\pi(1+\kappa)} \frac{1}{(z - z_j^*)^2} \right] - \sum_{j=1}^{K^{**}} \frac{M_j}{2\pi} \frac{1}{(z - z_j^{**})^2} - \frac{\beta}{1+\kappa} \sum_{j=1}^{K} \frac{q_j}{2\pi\lambda^*} \frac{\overline{z_j}}{z - z_j} + \Psi(z) \qquad (25)$$

where: $\Gamma = \frac{1}{4}(N_1 + N_2)$, $\Gamma' = -\frac{1}{2}(N_1 - N_2)$

$$\Phi(z) = \frac{1}{2\pi i}\int_l \frac{\phi(\tau)}{\tau - z}\,d\tau \qquad (26)$$

$$\Psi(z) = \frac{1}{2\pi i}\int_l \frac{\psi(\tau)}{\tau - z}\,d\tau$$

The thermal boundary condition (19) based on the Plemely formulas and the relations (16d), (22), (23) becomes:

$$2\,\mathrm{Re}\left[\pm\gamma(t) + \frac{1}{2\pi i}\int_l \frac{\gamma(\tau)}{\tau - t}\,d\tau\right] = f^{\pm}(t) - 2\,\mathrm{Re}\left[\frac{q_\infty}{2}\,t e^{-i\beta_o} - \sum_{j=1}^{K}\frac{q_j}{2\pi\lambda^*}\ln(t - z_j)\right] \qquad (27)$$

where $f^{\pm}(t)$ the known temperatures along the lips of the crack l.

Subtracting relations (27) by parts we obtain:

$$2\,\mathrm{Re}\,\gamma(t) = f^{+}(t) - f^{-}(t) \qquad (\gamma(t) = \gamma_1(t) + i\gamma_2(t))$$

$$\text{or} \quad \gamma_1(t) = \frac{f^{+}(t) - f^{-}(t)}{2} \qquad (28)$$

Adding relations (27) by parts we have:

$$2\,\mathrm{Re}\,\frac{1}{\pi i}\int_l \frac{\gamma(\tau)}{\tau - t}\,d\tau = f^{+}(t) + f^{-}(t) - 2\,\mathrm{Re}\left[q_\infty t e^{-i\beta_o} - \sum_{j=1}^{K}\frac{q_j}{\pi\lambda^*}\ln(t - z_j)\right] \qquad (29)$$

Equations (28) and (29) are the constitutive equations of the problem of thermoconductivity.

Considering the stresses along the lips of the crack as known, the following boundary condition is satisfied:

$$\left(\sigma_n^{\pm} - i\sigma_t^{\pm}\right)_{t\in l} = \Phi_o^{\pm}(t) + \overline{\Phi_o^{\pm}(t)} + \frac{d\bar{t}}{dt}\left[\bar{t}\,\Phi_o^{\pm\prime}(t) + \Psi_o^{\pm}(t)\right] \qquad (30)$$

Subtracting relations (30) by parts and taking into consideration Sohotsky-Plemely formulas, we derive the following expression for $\Psi(z)$:

$$\Psi(z) = \frac{1}{2\pi i}\int_l \frac{q_1(\tau)}{\tau - z}\,\overline{d\tau} - \frac{1}{2\pi i}\int_l \frac{\overline{\phi(\tau)}}{\tau - z}\,\overline{d\tau} - \frac{1}{2\pi i}\int_l \frac{\bar{\tau}\phi(\tau)}{(\tau - z)^2}\,d\tau \qquad (31)$$

Adding relations (30) by parts and taking account of (31) we obtain the following singular integral equation:

$$\frac{1}{\pi i}\int_l \frac{\phi(\tau)}{\tau - t}\,d\tau - \frac{1}{\pi i}\int_l \frac{\overline{\phi(\tau)}}{\bar{\tau} - \bar{t}}\,\overline{d\tau} - \frac{\overline{dt}}{dt}\left[\frac{1}{\pi i}\int_l \frac{\overline{\phi(\tau)}}{\tau - t}\,\overline{d\tau} + \frac{1}{\pi i}\int_l \frac{\bar{\tau} - \bar{t}}{(\tau - t)^2}\,\phi(\tau)\,d\tau\right] = A(t,\bar{t}) + \frac{1}{\pi i}\int_l \frac{q_1(\tau)}{\tau - t}\,\overline{d\tau}$$

$$t \in l \qquad (32)$$

where:

$$q_1(t) = (\sigma_n^+ - \sigma_n^-) - i(\sigma_t^+ - \sigma_t^-)$$

$$A(t,\bar{t}) = (\sigma_n^+ + \sigma_n^-) - i(\sigma_t^+ + \sigma_t^-) - 2\left[\Gamma + \bar{\Gamma} + \frac{\overline{dt}}{dt}\Gamma'\right] + 2\,\mathrm{Re}\left[\sum_{j=1}^{K^*}\frac{P_j + iQ_j}{2\pi(1+\kappa)}\frac{1}{t - z_j^*} - \frac{\beta}{1+\kappa}\sum_{j=1}^{K}\frac{q_j}{2\pi\lambda^*}\ln(t - z_j)\right]$$

$$-\frac{\overline{dt}}{dt}\left[\sum_{j=1}^{K^*}\frac{\kappa(P_j - Q_j)}{\pi(1+\kappa)}\frac{1}{t - \bar{z}_j^*} + \sum_{j=1}^{K^*}\frac{P_j + iQ_j}{\pi(1+\kappa)}\frac{\bar{t} - z_j^*}{(t - z_j^*)^2} - \sum_{j=1}^{K^{**}}\frac{M_j}{\pi}\frac{1}{(t - z_j^{**})^2} - \frac{\beta}{1+k}\sum_{j=1}^{K}\frac{q_j}{2\pi\lambda^*}\frac{\bar{z}_j}{t - z_j}\right]$$

§3 NUMERICAL SOLUTION AND APPLICATION

In order to solve the singular integral equation, we use the numerical method: 'Lobbato Chebyshev' . The essence of the chosen numerical method, consists in the substitution of the singular integral equation by a discrete analogue, on the basis of approximate formulae. In order to find the unknown function, in advance, the singularity at the ends of the crack is determined. The required unknown function is expressed as the product of two functions, one of which ,(the weight function),expresses its singularities and determine the selection of the orthogonal polynomial. The discrete points used (both integration and collocation points) are the nodes of the approximate formulae and they are determined on the basis of some functional relation.

Many problems of the theory of elasticity, thermoelasticity, electroelasicity, mechanics of cracks etc, are reduced to singular integral equation of the following form:

$$A(x)w(x)\phi(x) + \beta(x)\int_{-1}^{1}\frac{w(t)\phi(t)}{t - x}dt + \int_{-1}^{1}w(t)K(t,x)\phi(t)dt = f(x), \quad -1 \le x \le 1 \qquad (33)$$

Using the approximate integration formulae, for the normal and singular integral, the above equation is transformed into :

$$\left[A(x)w(x) - 2\mathrm{B}(x)\frac{Q_n^{(a,\beta)}(x)}{P_n^{(a,\beta)}(x)}\right]\phi(x) + \sum_{k=1}^{n}A_k\left[\frac{\mathrm{B}(x)}{t_k - x} + K(t_k,x)\right]\phi(t_k) = f(x), \qquad -1 \le x \le 1 \qquad (34)$$

where: $w(t) = (1-t)^a(1-t)^\beta$ - the weight function

A_k the coefficients of integration

$P_n^{(a,\beta)}(x)$, $Q_n^{(a,\beta)}(x)$ orthogonal polynomials first and second kind, depending on the singularity (Chebyshev,Legendre, Jacobi, etc.)

In our case , we use Chebyshev polynomial.($a = \beta = -1/2$)

Substituting, in relation (32), the variable x by the roots of the equation:

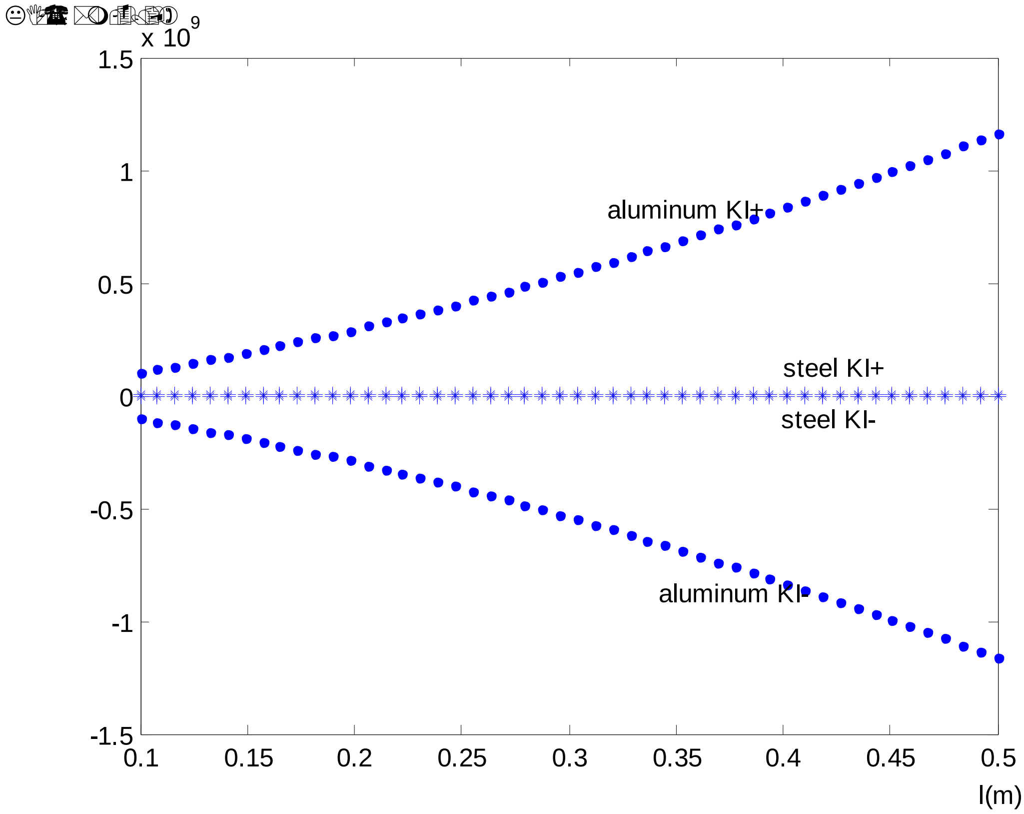
$$A(x)w(x) - 2\mathrm{B}(x)\frac{Q_n^{(a,\beta)}(x)}{P_n^{(a,\beta)}(x)} = 0 \qquad (35)$$

we formulate the following system of linear algebraic equations:

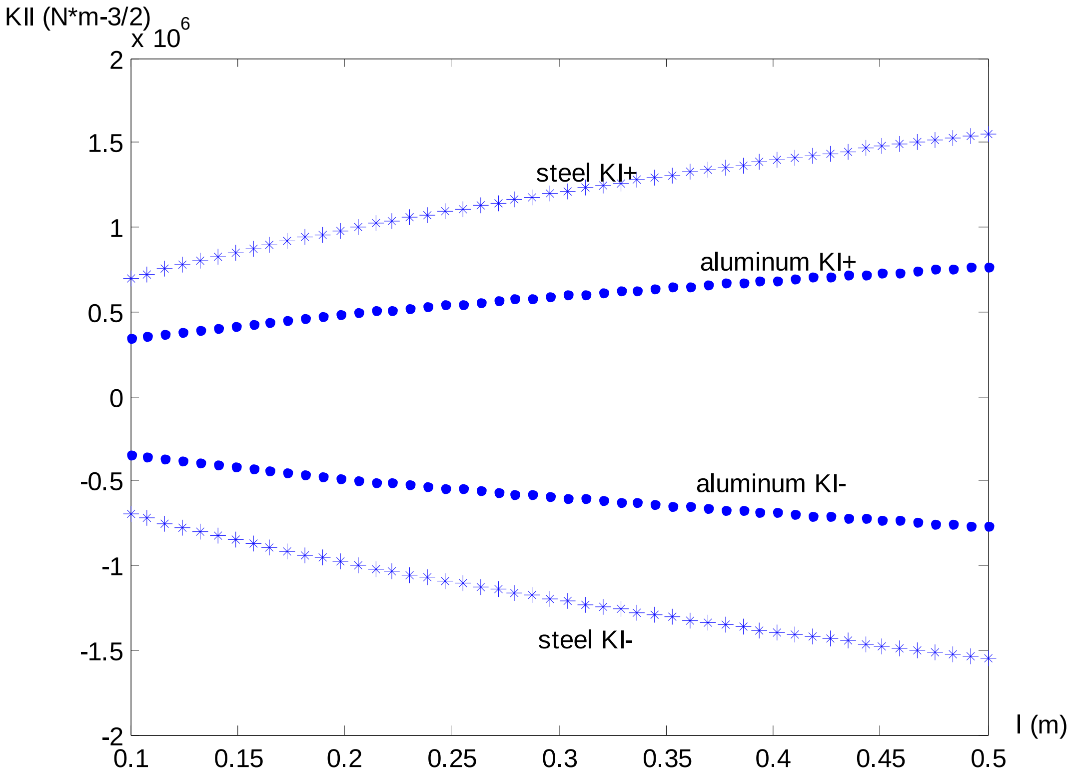
$$\sum_{k=1}^{n} A_k \left[\frac{B(x_r)}{t_k - x_r} + K(t_k, x_r) \right] \phi(t_k) = f(x_r) \qquad r = 1,\ldots, n \qquad (36)$$

Next, several figures present the behavior of the stress-intensity factors K_I and K_{II}.

As examples of application of the method developed we present two different problems. We assume an infinite plate, containing one rectilinear crack of a length l. The Ox –axis coincides with the axis of the crack and its origin with the mid-point of the crack. The temperature To along the lips of the crack is known. Furthermore the plate is under the influence of homogeneous flux heat q_∞ , at infinity. Two different materials have been used: aluminum and steel.



Fig(2a). The variation of K_I stress intensity factor, when the length l of the crack is increasing . The temperature To along the lips of the crack is 300K and the heat flux is $10^4\ J$



Fig(2b). The variation of K_{II} stress intensity factor, when the length l of the crack is increasing . The temperature To along the lips of the crack is 300K and the heat flux is $10^4\ J$

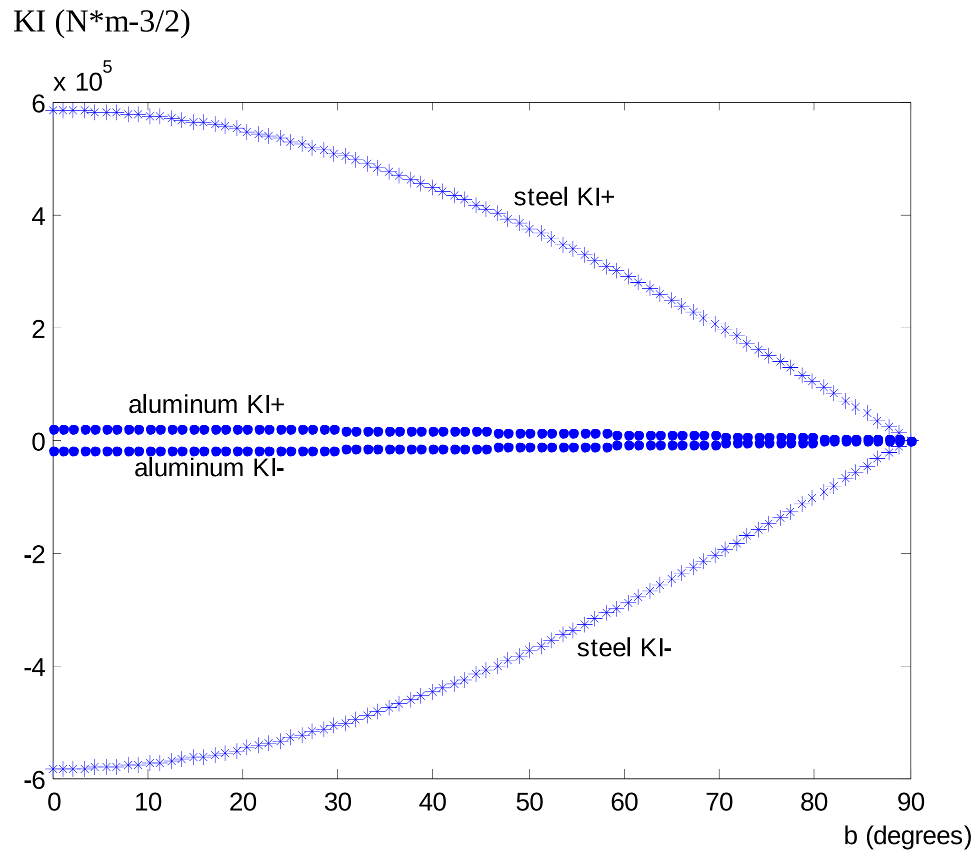
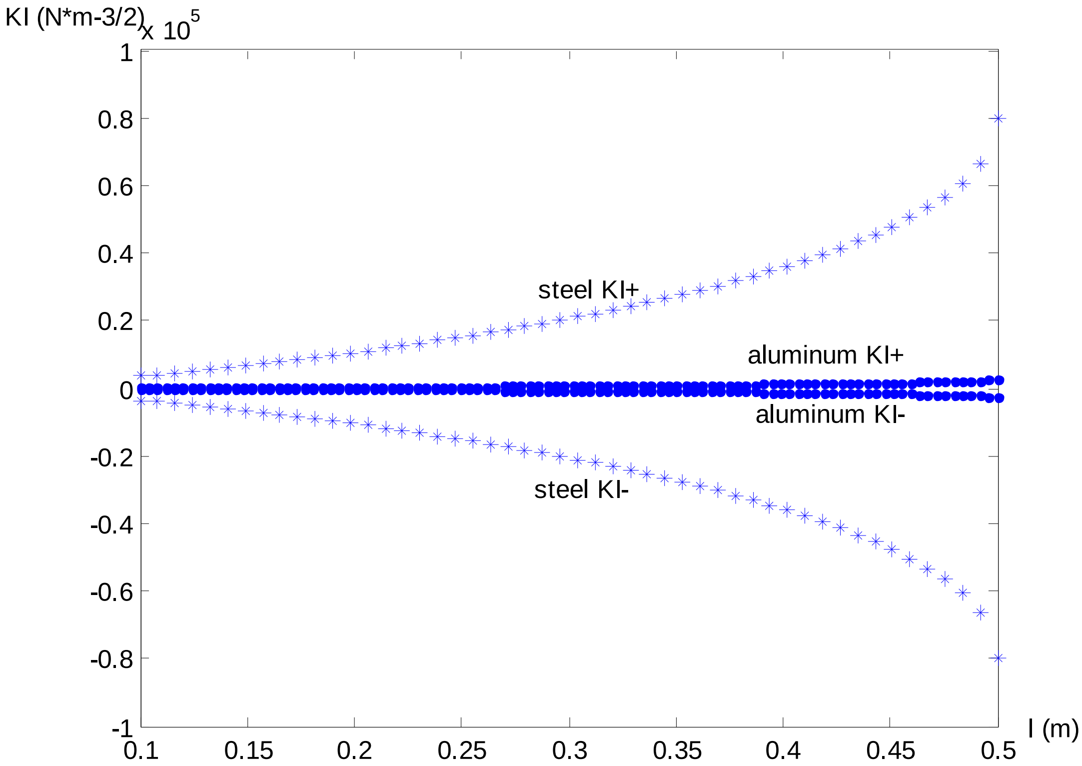
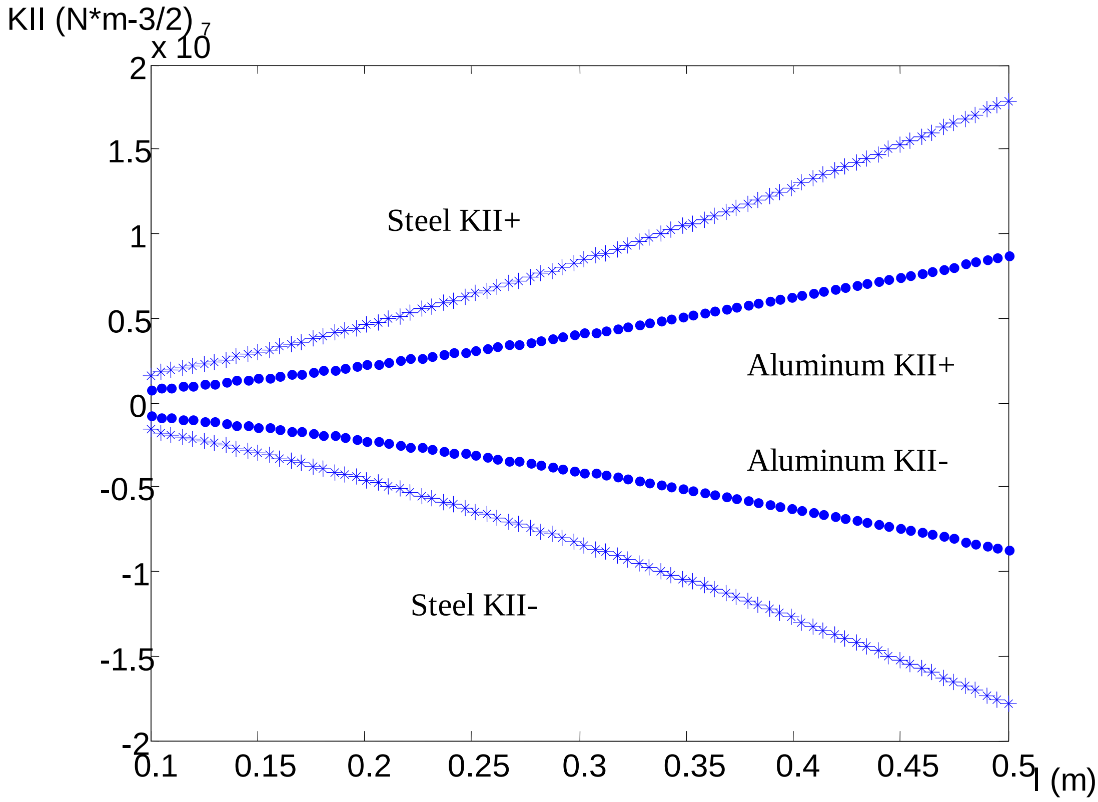


Fig.(3): The variation of K_I stress intensity factor when the angle between flux of heat and Ox axis is increasing. The length of the crack is 0.5 m, the flux of heat at infinity is $10^4\,J$ and the temperature To is 300K.

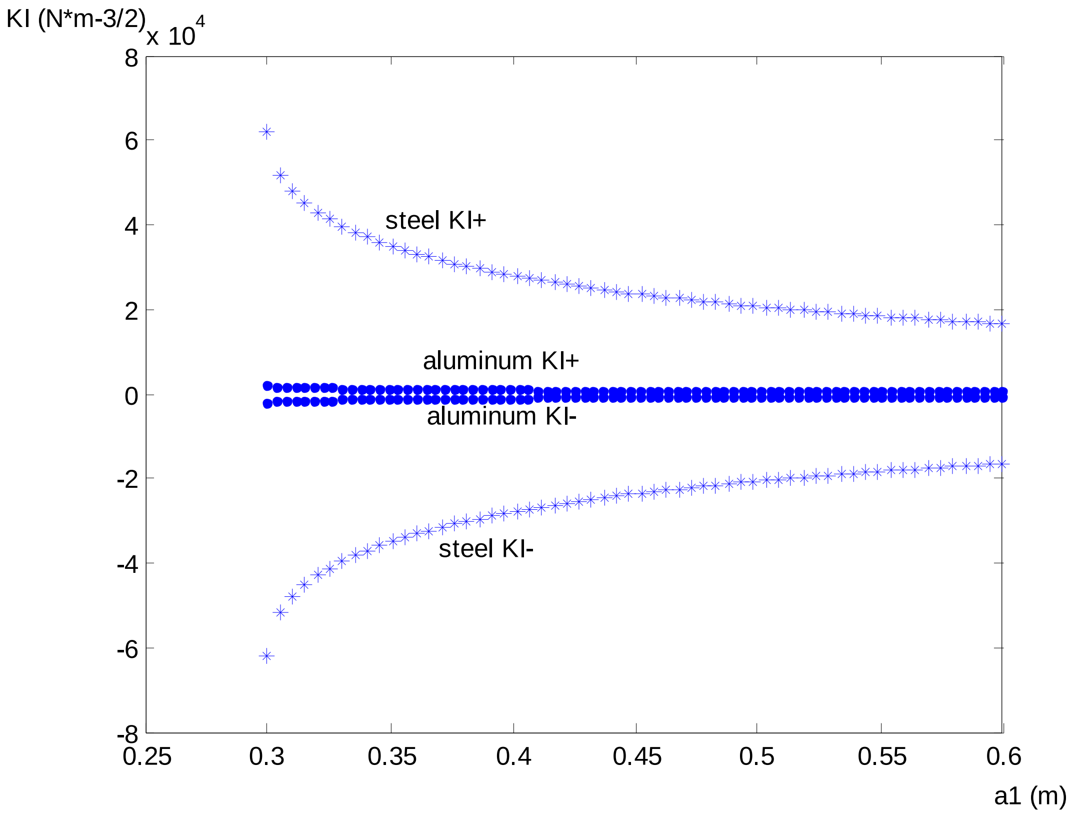
In sequel we assume an infinite plate which contains one rectilinear crack of a length l. The Ox axis coincides with the axis of the crack and its origin with the mid-point of the crack. The plate is under the influence of homogeneous flux heat q_∞ at infinity. Furthermore, heat sources $+q_1,-q_1$, $+q_2,-q_2$ act at points $(a_1,0)$, $(-a_1,0)$, $(0,a_2)$ and $(0,-a_2)$ respectively. In this case, the behavior of the stress intensity factors is given below:



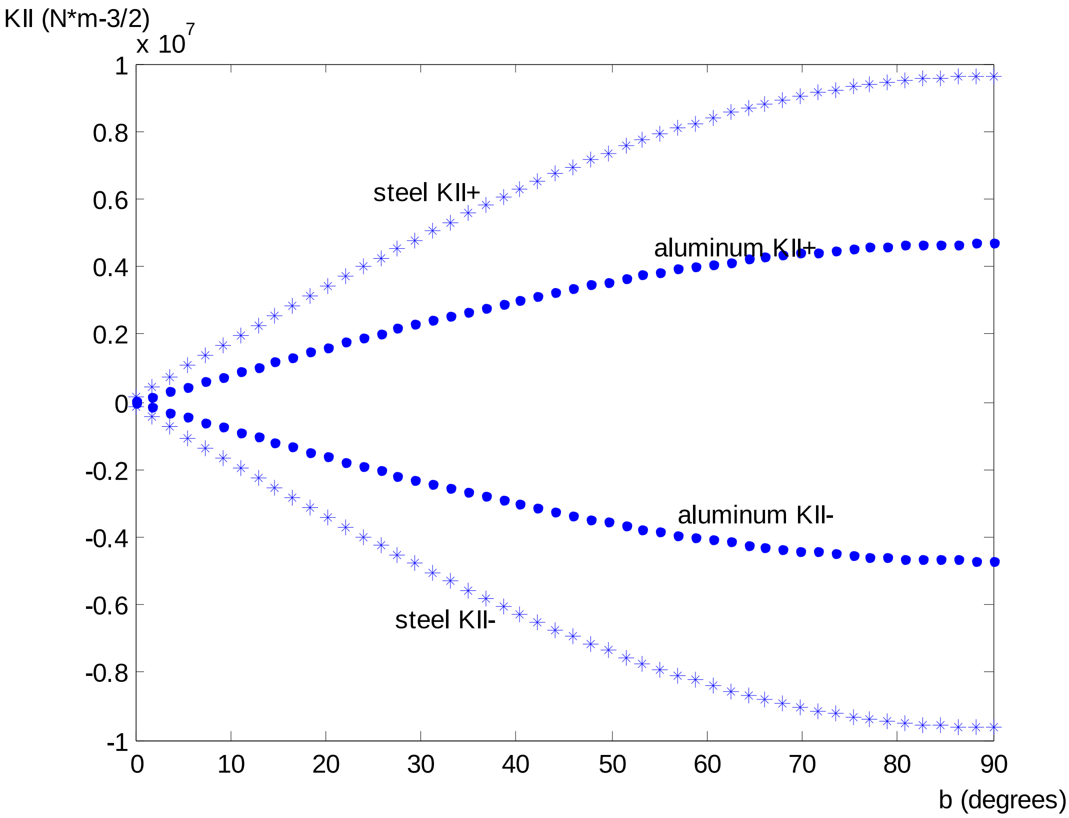
Fig(4a): The variation of K_I stress intensity factor when the length of the crack is increasing



Fig(4b):The variation of K_{II} stress intensity factor when the length of the crack is increasing



Fig(5a): The variation of K_I intensity factor when the distance of the heat source of the point O, is increasing



Fig(5b): The variation of K_{II} intensity factor, when the angle b is increasing

CONCLUSIONS

The solution of the problem of two dimensional stationary problem of thermoelasticity , give us the possibility to evaluate the thermal and stress-strain field, as well as to know the stress intensity factors at the ends of the crack, which can be further used in order to study of resistivity of cracked body.

LITERATURE

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