STRENGTH OF RESIN / FILLER SYSTEMS USING TWO MODELS i) OF PERFECT and ii) OF LOW ADHESION QUALITY

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Abstract

The tensile strength of particulate composites has been evaluated for the case that adhesion exists between matrix and filler. Two models, each of three components on the basis of cube-within-cube formation, have been used as representative volume elements. By comparing the derived theoretical results with experimental data for treated and untreated particles in epoxy / filler systems, the first model can be characterized as corresponding to perfect adhesion quality between matrix and filler while the second one to low adhesion quality. The strength predicted by the first model is close to that of treated particles corresponding to high strength. This model corresponds to an upper bound of the strength in cube-within-cube models. The strength predicted by the second model is close to that of untreated particles corresponding to low strength, but this model does not correspond to a lower bound of the strength. The systems used for comparison were resin/glass particulate composites. For the case that adhesion exists between matrix and filler, the strengths predicted by the present models are in agreement to those provided by an existing evaluation method in the literature.

Introduction

As pointed out by Nielsen [1], when there is no adhesion between matrix and filler, the tensile strength of particulate composites depends on the tensile strength of the matrix, the filler volume fraction and the stress concentration factor. When adhesion between matrix and filler exists, the tensile strength depends on the fracture deformation and the elastic modulus of the composite. Consequently in this latter case the tensile strength results from a complex interplay between the properties of the individual constituent phases; the resin, the filler and the interface [2].

In general when adhesion exists between matrix and filler the mechanical properties of the composite are affected by a number of parameters; the size, the shape, the aspect ratio (ratio of the length to the side of the base), the distribution of the reinforcing particles, the interaction between the inclusions and the agglomerations of fillers. In the case of nonspherical inclusions the orientations of the fillers with respect to the applied stress is also essential [2]. Some significant parameters play also an important role upon the tensile strength; the quality of adhesion between matrix and filler, air bubbles in the matrix, the stress concentration factor, the plastic behavior of the matrix near the filler and the crack pinning effect (that is when the crack propagation is embedded by a group of particles) [2, 3].

In refs [4, 5], tensile experiments in particulate composites prepared by treated as well as by untreated particles, have shown that the adhesion quality between matrix and filler affect considerably the strength behavior of the composites. The kind of adhesion affects also the values of the stress intensity factor k_{ic} [3, 6].

In this study the tensile strength of particulate composites is evaluated using two cube-within-cube models, each one consisting of three components. The one of these models [7] gives a constant strength of the composite, independent of the filler content, which is equal to the strength of the matrix. Thus this model is characterized as corresponding to perfect adhesion quality between matrix and filler. In the other model [8] the strength decreases as the filler content increases up to 20%, attains a minimum and then increases steadily with a slow rate. Comparison to experimental results [4, 5] shows that this model corresponds to low adhesion quality between matrix and filler. The strength predicted by the above models is in agreement with the theory of Nielsen [1] for the case when adhesion exists between matrix and filler. The theoretical results derived by the presented models are compared to the values of the strength predicted by existing equations in the literature and to experimental results in epoxy/glass particulate composites.

Theoretical Considerations

The theoretical analysis is based on the following assumptions:

1) The particles are perfectly cubic.

2) The matrix volume distribution of each filler is also cubic.

3) The volume fraction of the particles is sufficiently low, so that there is no interaction between the stress fields around neighboring particles.

4) The particles are uniformly distributed in the matrix, so that homogeneity can be assumed.

5) Both the matrix and the inclusion are prepared from perfectly homogeneous, elastic and isotropic materials of known mechanical properties.

6) The matrix is brittle and the stress-strain linearity is maintained up to the failure of the composite.

7) There is no transverse variation of the strains in the components which are connected in parallel and have the same length in the load direction.

8) The stresses do not vary in the direction of the applied load in the components which are connected in series and have the same cross sections.

As shown in Fig. 1 the filler volume fraction is given by

$$u_f = a^3 / c^3 \tag{1}$$

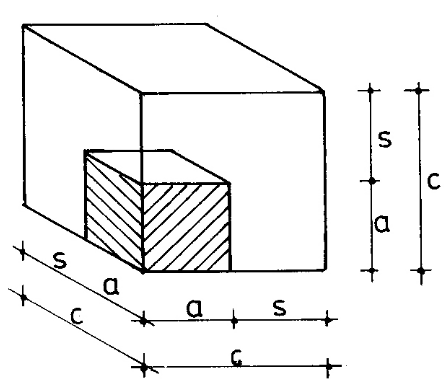

Fig. 1 A schematic representation of the cube-within-cube model.

The components (1) and (2) of the model presented in Fig. (2a), called model 1, are connected in parallel and the resulting element is connected in series with component (3).

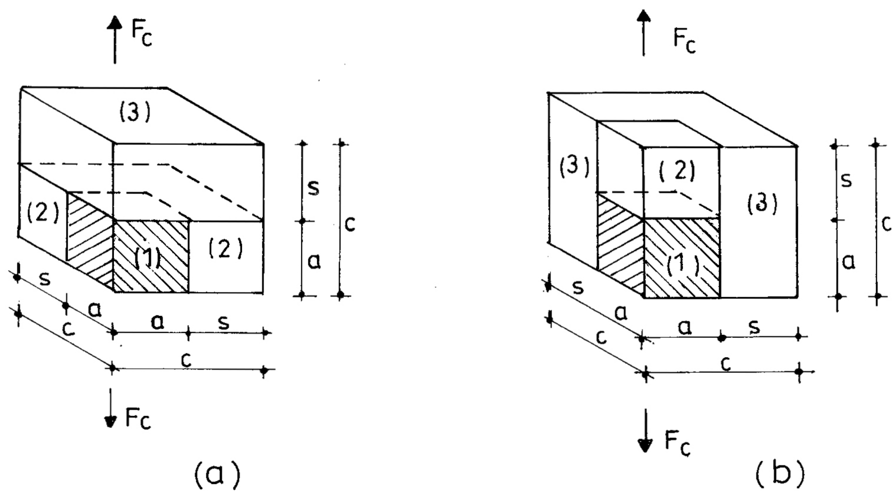

Fig. 2 The two cube-within-cube models each consisting of three components
(a) Paul's model [7]
(b) Ishai-Cohen's model [8]

When an external force acts in the direction shown in Fig. (2a) the stress equilibrium and strain compatibility equations are

$$\sigma_3 = \sigma_c \tag{2}$$

$$\sigma_c = \sigma_1 u_f^{2/3} + \sigma_2 (1 - u_f^{2/3}) \tag{3}$$

$$\varepsilon_1 = \varepsilon_2 \tag{4}$$

$$\varepsilon_c = \varepsilon_1 u_f^{1/3} + \varepsilon_3 (1 - u_f^{1/3}) \tag{5}$$

where the indexes 1, 2, 3 and c correspond to the components (1), (2), (3) and the composite respectively. The constitutive equations are

$$\sigma_1 = \varepsilon_1 \mathrm{E_f} \tag{6}$$

$$\sigma_2 = \varepsilon_2 \mathrm{E_m} \tag{7}$$

$$\sigma_3 = \varepsilon_3 \mathrm{E_m} \tag{8}$$

$$\sigma_c = \varepsilon_c \mathrm{E_c} \tag{9}$$

where the indexes m and f correspond to the matrix and the filler respectively.

Combining eqns (2)-(9) one obtains

$$\sigma_1 > \sigma_3 = \sigma_c > \sigma_2 \tag{10}$$

Assuming that failure in the component (3) corresponds to failure of the whole composite, the strength of the whole composite is given by

$$\sigma_{cu} = \sigma_{mu} \tag{11}$$

Where the index u denotes strength.

Eqns (2)-(9) give

$$\varepsilon_{cu} = \varepsilon_{mu}\left[1 - u_f^{1/3}\left(1 - \frac{1}{(m-1)u_f^{2/3} + 1}\right)\right] \tag{12}$$

Where the index u denotes fracture deformation and $m = E_f / E_m$.

On the other hand in the model presented in Fig. (2b), called model 2, the components (1) and (2) are connected in series and the resulting element is connected in parallel with component (3). When a load acts in the direction shown in Fig. (2b) the governing stress-strain equations are

$$\sigma_c = \sigma_1 u_f^{2/3} + \sigma_3(1 - u_f^{2/3}) \tag{13}$$

$$\sigma_1 = \sigma_2 \tag{14}$$

$$\varepsilon_3 = \varepsilon_c \tag{15}$$

$$\varepsilon_c = \varepsilon_1 u_f^{1/3} + \varepsilon_2(1 - u_f^{1/3}) \tag{16}$$

The constitutive relations are given by eqns (6)-(9). Combining eqns (6)-(9) with eqns (13)-(16) it comes out that

$$\sigma_1 = \sigma_2 > \sigma_c > \sigma_3 \tag{17}$$

Assuming that failure of the composite coincides with failure of the component (2), eqns (6)-(9) and (13)-(16) give

$$\sigma_{cu} = \sigma_{mu}\left[1 - (u_f^{1/3} - u_f)\frac{m-1}{m}\right] \tag{18}$$

and

$$\varepsilon_{cu} = \varepsilon_{mu}\left[1 - \frac{m-1}{m} u_f^{1/3}\right] \quad (19)$$

where $m = E_f / E_m$.

Alternatively in both models (1) and (2) the strength can also be given by the relation

$$\sigma_{cu} = \varepsilon_{cu} E_c \quad (20)$$

where the elastic modulus derived by model (1) is given [7] by

$$E_c = E_m \left(\frac{1 + (m-1)\, u_f^{2/3}}{1 + (m-1)\,(u_f^{2/3} - u_f)} \right) \quad (21)$$

while the elastic modulus derived by model (2) is given [8] by

$$E_c = E_m \left(1 + \frac{u_f}{\frac{m}{m-1} - u_f^{1/3}} \right) \quad (22)$$

Results and Discussion

In Fig's (3) and (4) the tensile strength in epoxy/glass particulate composites versus the filler content is plotted. The strength predicted by eqns (18) and (11) corresponds to the curves (c) and (d) respectively. In the same figures the theoretical values of the strength derived by eqns (A1), (20), (24) and (A2) are also shown. The experimental results come from ref [4, 5] of Spanoudakis and Young for the following cases: i) treated particles with an improvement of the adhesion quality between matrix and filler, ii) untreated particles, and iii) treated particles with a result in a way that there is no adhesion between matrix and filler. From Fig's (3) and (4) it is observed that the straight line which corresponds to model (1), curve (d), is close to the experimental results of treated glass particles with an improved adhesion between matrix and filler. The observed discrepancies are owed to the fact that the adhesion in the specimens is not perfect and also to many parameters which affect the strength

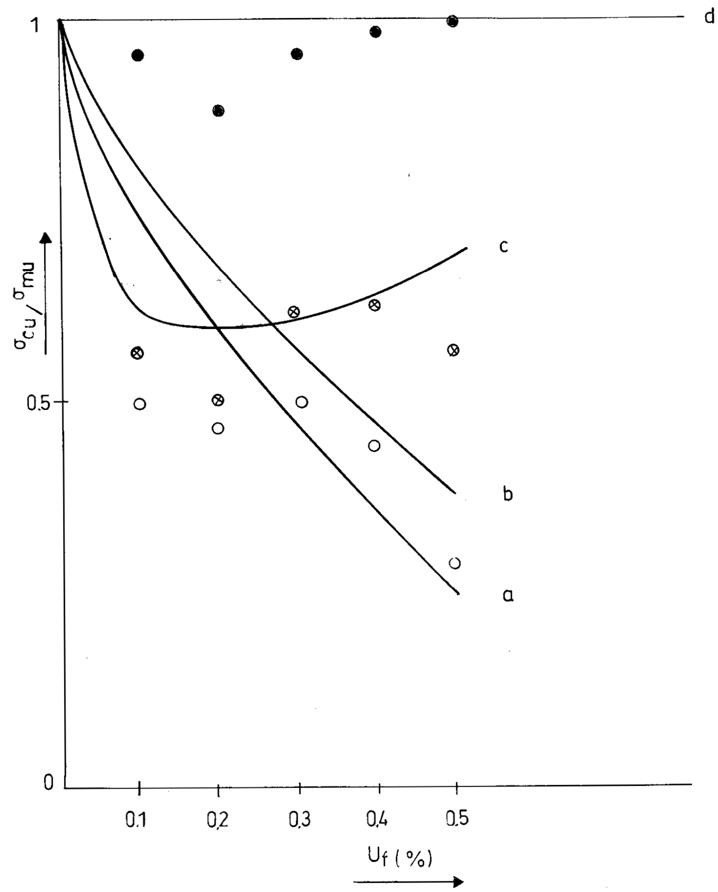

Fig. 3 Ratio σ_{cu}/σ_{mu} (strength of the composite over strength of the matrix) as a function of the volume fraction filler u_f. 4,5 m particles

- • treated with A187 ⊗ untreated
- ∘ DC 1107 treated (after Spanoudakis and Young)
- (a) Nikolais and Narkis, eqn (A2) (b) B. Nielsen, eqn (A1)
- (c) model 2 (d) model 1

and which are referred in the introduction. For the above reasons model (1) is characterized as corresponding to perfect adhesion quality between matrix and filler. From the above Fig's (3) and (4) it is also observed that the strength derived by model (2) is close to the experimental curve corresponding to the lower values of untreated particles. Thus model (2) corresponds to low adhesion quality between matrix and filler. This model does not consist a lower bound of the strength because there can be models with the same geometry of the components but with different dimensions, that give lower values of the strength. In the case of model (2) failure of the component

(2) causes failure of the whole composite in which the local stress concentration factor k is given by

$$\sigma_3' = k\left[\sigma_{mu}\frac{m(1-u_f^{1/3})+u_f^{1/3}}{m}+\sigma_{mu}\frac{u_f^{2/3}}{1-u_f^{2/3}}\right] > \sigma_{mu} \qquad (23)$$

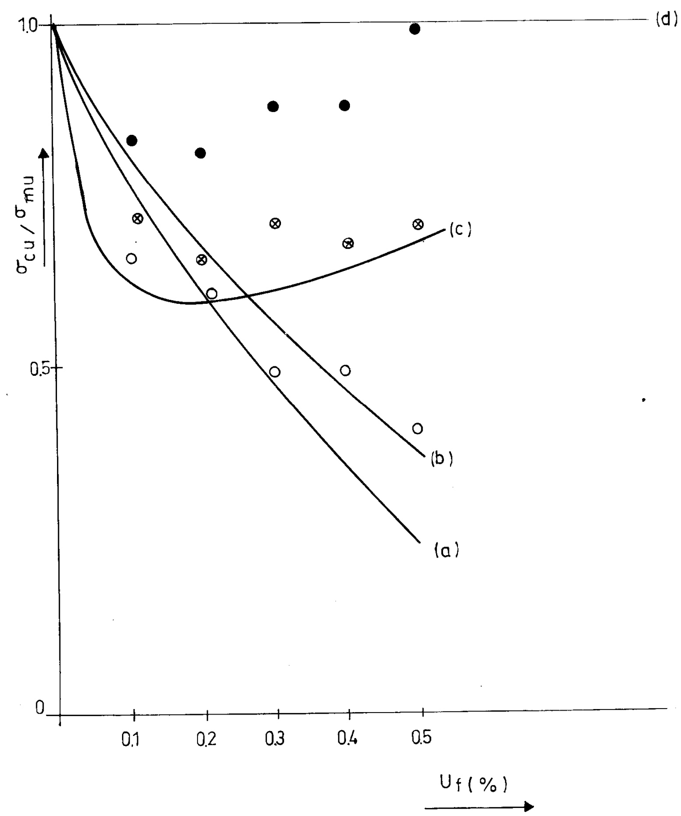

Fig. 4 Ratio σ_{cu}/σ_{mu} as a function of the volume fraction filler u_f. 6,2 □m particles

- • treated with A187 ⊗ untreated
- ◦ DC 1107 treated (after Spanoudakis and Young)
- (a) Nikolais and Narkis, eqn (A2) (b) B. Nielsen, eqn (A1)
- (c) model 2 (d) model 1

The first term in the brackets is the stress in component (3), when failure takes place in the component (2). The second term in the brackets is the stress in component (2) which is now transferred to the component (3). The above procedure is based to the assumption that the stresses can be transferred to the inclusion through the component (2).

From the above expression the values of the stress concentration factor k, for different values of the filler volume fraction u_f, are given in Table 1.

Table 1

u_f	0,05	0,10	0,15	0,20	0,25
k	1,26	1,21	1,13	1,05	0,94

The above values of k determine if failure of component (2) causes failure of the whole composite.

The presented procedure for the stress evaluation in both models is in agreement with Nielsen's theory in which the strength is given by

$$\sigma_{cu} = \varepsilon_{cu} E_c \tag{20}$$

where

$$\varepsilon_{cu} = \varepsilon_m (1 - \mathrm{u_f})^{1/3} \tag{24}$$

Thus, since the predicted strains by the presented models are close to the strains provided by eqn (25), eqns (11) and (18), as well as eqns (20) and (24) give the same values of the strength respectively.

Strength and Ultrasounds

It is known that the strength is related with the hardness predicted by Brinel's method. The existing relation is

$$\sigma = k \cdot BHN_{30} \tag{25}$$

where BHN is Brinel's hardness and k is a constant of the material. The index 30 corresponds to the relation

$$\lambda = 30 = \frac{P}{D^2} \tag{26}$$

where P is the applied load on the specimen and D is the diameter of the penetrator.

It is also known that the hardness can be obtained by means of ultrasonic measurements. The constant k of eqn (25) can be determined by this method.

The present study presents two models for the evaluation of the strength which can be used for comparison to the strength evaluated by ultrasonic measurements and eqns (20) and (24).

Conclusions

2) The strength evaluated by model (1) is independent of the filler volume fraction and equal to the strength of the matrix. This value of the strength compared to experimental results is found to be close to the strength of treated particles and gives an upper bound of the strength of particulate composites predicted by cube-within-cube models. The model is characterized as corresponding to perfect adhesion quality.
3) Comparing the strength obtained by model 2 to experimental results, it is found to be close to the lower values of the strength of untreated particles. Thus the model is characterized as corresponding to low adhesion quality. The strength derived by model 2 does not give a lower bound of the strength of particulate composites.
4) The strength predicted by the presented procedure is in agreement to the strength predicted by Nielsen's theory when there is adhesion between matrix and filler.
5) In model (2) when an initial failure takes place in the matrix, a local stress concentration factor is assumed by means of which the failure in the whole composite is considered.

References

1. L.E. NIELSEN, J. Appl. Plo. Sci., Vol. 10, p. 97 (1966)
2. S. AHMED, F. R. JONES, J. Mat. Sci., Vol. 25, p. 4933 (1990)
3. A. C. MOLONEY, H. H. KAUSCH. T. KAISER, H. R. BEER, J. Mat. Sci., Vol. 22, p. 381 (1987)
4. J. SPANOUDAKIS and R. J. YOUNG, Ibid 19, p. 487 (1984)

5. J. SPANOUDAKIS and R. J. YOUNG, J. Mat. Sci., Vol. 19, p. 487 (1984)
6. A. J. KINLOCH, D. L. MAXWELL, R. J. YOUNG, J. Mat. Sci., Vol. 20, p. 4169 (1985)
7. B. PAUL, Trans. Amer. Inst. Mech. Eng. Vol. 36, p. 218 (1960)
8. O. ISHAI and L. J. COHEN, Int. J. Mech. Sci., Vol. 9, p. 539 (1967)
9. I. N. PRASSIANAKIS, S. K. KOURKOULIS "EXPERIMENTAL STRENGTH OF MATERIALS" SYMMETRIA EDITIONS, ATHENS, GREECE 1999

Appendix

Equations used for comparison:

1) Nielsen's equation (no adhesion) [1]

$$\sigma_{cu} = \sigma_{mu}(1 - u_f^{2/3}) \cdot k \qquad \text{(A1)}$$

where k is a stress concentration factor

2) Nikolais and Narkis equation (no adhesion) [8]

$$\sigma_{cu} = \sigma_{mu}(1 - 1{,}21 u_f^{2/3}) \qquad \text{(A2)}$$