



# A study of the Spectral Behavior of Laser-generated Lamb Waves using Wavelet Transforms

T.Pramila, Rashmi Shukla, N.N.Kishore\*, V.Raghuram\*  
Christ Church College  
\*Indian Institute of Technology Kanpur  
Kanpur, India  
[pramilatabeti@yahoo.co.uk](mailto:pramilatabeti@yahoo.co.uk)

**Abstract** - This paper deals with the study of the spectral components of Lamb waves generated in thin aluminium plate using pulsed Nd-YAG laser and detected using a He-Ne Laser Heterodyne interferometer. Laser generation of Lamb waves leads to simultaneous generation of various frequency component as well as multiple Lamb wave modes. The Lamb wave signals are successfully and comprehensively analyzed using Wavelet transform technique. The velocities of various frequencies present in the Lamb wave signal are calculated and the experimental dispersion curve is plotted. The behavior of various frequency components with respect to reflection from plate ends is explained satisfactorily.

**Key words:** Lamb waves, Laser Generation, Frequency analysis

## 1 Introduction

Lamb waves are the kind of elastic waves that arise from a coupling between shear and longitudinal waves reflected at the top and bottom of plate surface [1]. Lamb waves are waves of plane strain that occur in thin plates. Propagation properties of these waves depend on frequency of vibration and thickness of plate and thus they are highly dispersive in nature [2]. Lamb waves are useful in many different areas of manufacturing, in-service inspection, global seismology, acoustic microscopy, etc. [3]. Structural flaws such as corrosion and fatigue cracks represent changes in effective thickness and local material properties; therefore measurement of variations in Lamb wave propagation can be employed to assess the integrity of plate structures [4]. Lamb wave theory is fully documented in a number of textbooks [5-6]. Worlton [7] provided experimental confirmation of Lamb waves at megacycle frequencies. J.P Monchalin [8] has given an elaborate review on the techniques for the detection of ultrasound by interferometry. Grossman [9] first proposed the concept of wavelets in its present theoretical form. Wavelet analysis and the properties of different wavelets are explained by Daubechies [10]. Cheng et al [11] have presented the study of laser generated Lamb waves in plates. Recently Kim et al [12], Mannan et al [13], and Vajradehi Yadav et al [14] discussed the laser generated Lamb waves.

## 2 Experimental details

The LBU setup consists of an Nd: YAG pulsed laser to generate ultrasonic Lamb waves and an Optical Heterodyne type laser interferometer to detect the Lamb

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waves displacements in the aluminium plates. The set-up utilizes a Yokogawa DL1740 (four channel, one GSa/sec, 500 MHz) Digital Storage Oscilloscope (DSO), having built in Zip drive, Ethernet, USB, GPIB and Serial Ports for communication with external PC's/ systems. The signals are amplified and digitized using the oscilloscope. The schematic layout of the experimental set-up is shown in Figure 1 while Figure 2 show schematic of the sample. The scanning is done manually using two single axis micrometer controlled XYZ translator mounted on the Optical Test Bench.

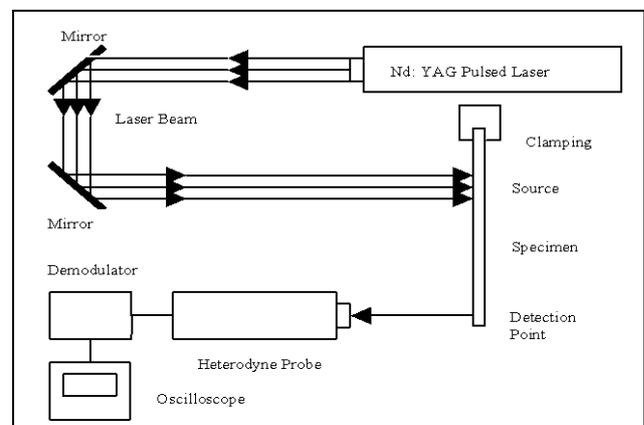


Figure 1: Schematic of experimental set-up

Aluminium plate (421mm x 135mm) of 2 mm thickness is used as a specimen to generate Lamb waves. The laser power is kept as 200 mJ. A precise calibration of



heterodyne interferometer is done prior to the experiment. The signals are recorded in DSO. Wavelet transforms of these signals are carried out. The arrival times of different frequency components are extracted from the Wavelet transforms. Knowing the arrival times, group velocities of these frequency components are calculated.

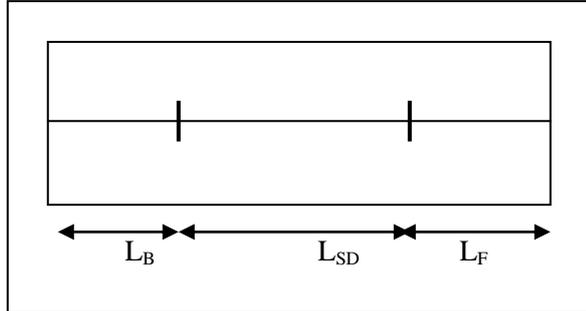


Figure 2 : Schematic of the sample ( $L_B$  – Distance of back edge from the source,  $L_{SD}$  – Source detector separation,  $L_F$  – Distance of the front edge from the detector)

### 3 Results and discussion

#### 3.1 Wavelet Analysis

As the recorded signals contain various frequency components, the wavelet transform of signal is used to obtain arrival times for different frequency components. The wavelets are to be analyzed according to scale. Figure 3 shows the wavelet transform of signal obtained for 2mm plate. There are three windows in this figure. First window shows the signal whose wavelet is to be taken. Second window shows the 3D plot in which X-axis corresponds to time, Y-axis corresponds to frequency according to scale and Z-axis shows the variation of amplitude. In a two-dimensional wavelet transform, this amplitude variation is depicted with the help of colour variation. In the present study, dark blue corresponds to low intensity while dark red corresponds to high intensity. The third window shows the coefficient line for different frequencies, which gives the temporal behaviour of each frequency component during the time of signal detection. By choosing different coefficient lines, one can study the behaviour of different frequency components present in a given signal.

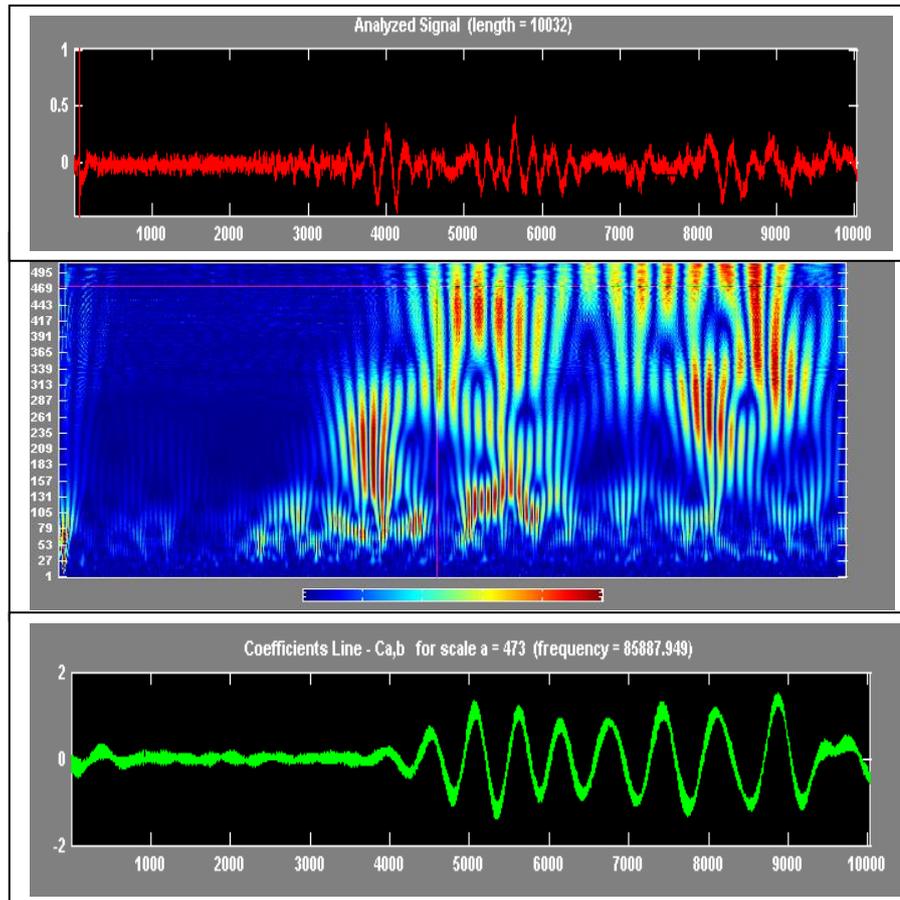


Figure 3 : Wavelet transform of signal and coefficient line corresponding to 85KHz

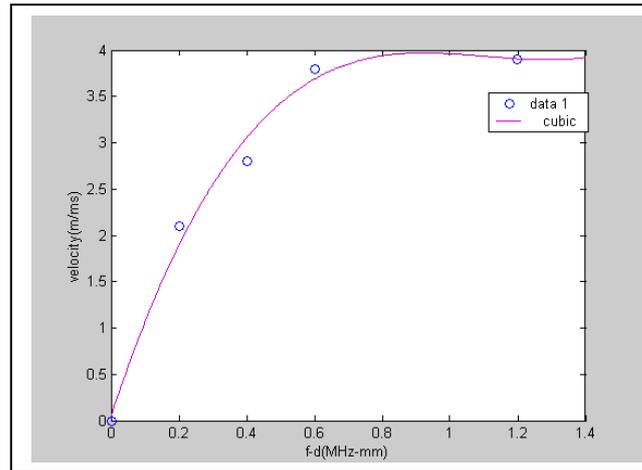


Figure 4 : Dispersion curve for low f-d values of Aluminium

### 3.2 Dispersion curve for the region of low frequency-thickness (f-d) product

Laser generated Lamb wave signal with source detector distance of 200 mm is shown in Figure 3. The wavelet transform of the signal is generated and with the help of the coefficient lines it was observed that a band of frequencies lying in the range 85 - 410 KHz are present in the signal. The arrival times of various prominent frequencies present in the signal are obtained with the help of coefficient lines. Using the times of arrival of these frequencies, their velocities are calculated. These velocities are plotted against the f-d product values to generate the dispersion curve. The dispersion curve thus obtained is shown in Figure 4. The nature of the curve allows one to conclude that these frequency components belong to the  $A_0$  mode.

### 3.3 Spectral analysis

The temporal behaviour of individual spectral components can be studied with the help of coefficient lines. In the present study, the behaviour of two spectral components, one in the low frequency region and one in the high frequency region is taken up. In the wavelet coefficient line corresponding to 179KHz, one can see three well

separated wave packets. From the arrival time of the first wave packet the velocity of this component is calculated to be 3448 m/s. Keeping this velocity constant, the distance traveled by the second and third wave packets are calculated to be 300mm and 542mm respectively. The 300mm distance corresponds to the distance from the source to the fixed back edge of the plate and then from the back edge to the point of detection while the 542mm distance corresponds to the separation between the source and the front edge of the plate and the back to the point of detection. Hence, one can rightly conclude that the second and third wave packets correspond the signals reflected from the back and front edges of the plate respectively. For high frequency signal (406KHz) other than the original, front and back-reflected signals corresponding to  $A_0$  mode, a number of wave packets can be clearly seen. Moreover these wave packets are seen to have more intensities than the original wave packet.

As it is not possible for any mode other than  $S_0$  and  $A_0$  to be present at this particular f-d value (0.8 MHz-mm), it is concluded that these additional wave packets are due to reflections of the Lamb wave from the top and bottom edge of the plate. It is quite reasonable to come to this conclusion as these ultrasonic signals are generated using a point source (unfocused laser beam).

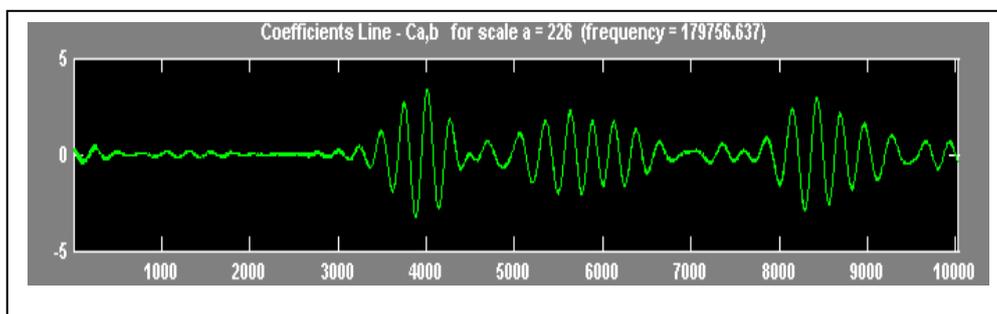


Figure 5 : Coefficient line corresponding to 179KHz frequency

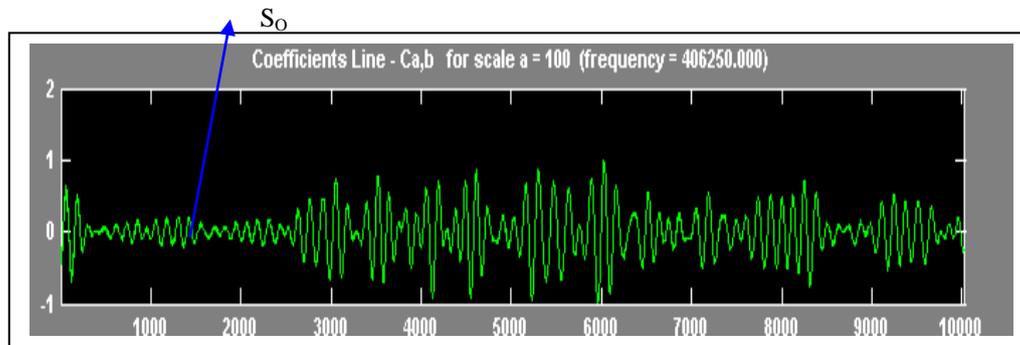


Figure 6 : Coefficient line corresponding to 406KHz frequency

Such ultrasonic source is known to have low directivity. Higher intensity of these additional wave packets can be explained as due to constructive interference of the reflected signals arriving from corresponding points from the top and bottom edges of the plate (Figure 7), which arrive in phase at the point of detection.

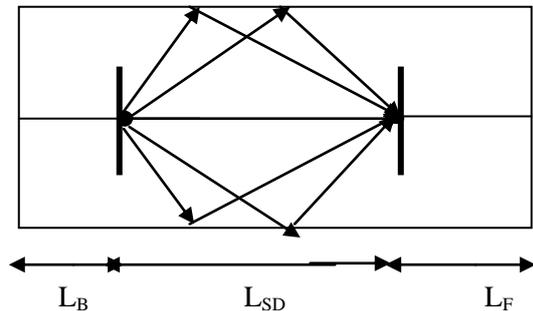


Figure 7 : Top and Bottom Edge Reflections

## 4 Conclusions

In the present work, the spectral analysis of lamb waves generated in aluminium plate (421x135) of 2mm thickness using wavelet transforms is presented. The dispersion curve for low  $f-d$  values is generated. With the help of the coefficient lines corresponding to various frequency components, the temporal behaviour of the spectral components is analyzed. It is seen that while in the case of low frequency components, the temporal behaviour shows the presence of front and back edge reflections, the temporal behaviour of the high frequency components show the presence of additional reflections from the top and bottom edges also. From this result one can conclude that the low frequency spectral component are more suitable for the study of surface integrity of plates.

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