Propagation of ultrasonic Lamb waves in adhesively bonded lap joints

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Abstract

This paper considers the bonded joint as a multilayer structure that is analysed using the Transfer Matrix method. In this particular case three layers (two adherents and the adhesive) are considered. The study of this propagation problem may be developed from matrix formulations which describe elastic waves in layered media. This technique combines the theory of the dynamics of the continuum within each layer with the conditions for the interaction at the interfaces between layers.

The behaviour of the different modes which propagates in the overlap region is obtained, and is found that the relative amplitudes can be estimated based on the properties of the incident wave mode. It was verified that the excitation of these modes is ruled by the degree to which their mode shapes match the mode shapes of the incident wave. This allows us to explain the physical phenomena that are behind the mode conversion, which could be used to a correct selection of modes for non-destructive evaluation of the bond region.

Another result which must be emphasized consists is the possibility to determine by this method the attenuation of both longitudinal and transversal waves in plates what usually is a difficult by using conventional pulse-echo technique, especially in thin plates. Using an immersion pitch and catch setup, the total attenuation, that is composed by the losses due to the leaking in the fluid and material damping, can easily be obtained doing two measurements at different distances. The leaking losses can be obtained by knowing the bulk properties of both fluid and plate. So, attenuation of longitudinal and transversal waves (material damping) can be evaluated.

In the experimental work two sets of lap joints built from 1mm and 4 mm thickness aluminium plates are tested using the fundamental S0 mode as incident wave. Very good agreement is found when compared with the theoretical predictions.

1. Introduction

Adhesive bonding has been used lately with great success in several types of fabrication processes. The main advantages of this technique results from the favourable stress distribution, the smother appearance and the light weight of the final assembly when compared with other traditional bolting, riveting or spot welding techniques. Bonding gives also the possibility to join different materials in complex shape structures.

Due to manufacturing conditions or degradation during service, different bonding parameters need to be inspected. In industries where automated assembly methods may be used, it is important that the dimensions of adhered joints, such as adhesive layer thickness and overlap length, are within tolerance and are measurable. The detection of defects such as voids in the adhesive or local separation of adhesive from one of the adherents is another need in all industries.

The existing techniques for disbond detection could be divided into normal incidence (time or frequency domain) and guided waves. The conventional normal incidence techniques (point by point) are performed over the area containing the overlap region. This technique is time consuming and access to the joint area is needed. The use of guided waves is a very attractive solution when large structural tests are demanded since they can be excited at one point and propagated over considerable distances.

There are several authors that have study the propagation of guided waves in adhesively bonded lap joints and the influence of bond conditions in wave parameters [1-11]. In some of this studies laser ultrasonic techniques are used to obtain aging degree [2] or large disbonds detection [3,4]. Kissing bonds, which are invisible to longitudinal waves when subjected to high compressive strength, can be detected by the proper Lamb wave mode selection [5]. Other contributions were given to

understanding the transmission of guided waves in lap joints with the help of analytical models [6,7], finite elements [8], wavelet transform [9] or artificial neural networks [10].
In this paper are presented the results of two studies. In the first study, the behaviour of the different modes which propagates in the overlap region is analysed, and is found that the relative amplitudes can be estimated based on the properties of the incident wave mode. In the second study, the possibility to determinate the attenuation of both longitudinal and transversal waves in plates by measuring the total attenuation was analysed. Experimental tests using an immersion pitch and catch setup and 1mm and 4 mm thickness aluminium plates were done using the fundamental S0 mode as incident wave. Results obtained are according with predictions.

2. Background

An adhesively bonded lap joint can be modelled as a three layer system considering two adherents and an adhesive. The geometry of the problem is illustrated in Fig.1 considering the propagation in the *x* direction. The study of this propagation problem may be developed from matrix formulations which describe elastic waves in layered media. Transfer Matrix method is perhaps the most important technique that uses these matrix formulations. To apply this method each layer is considered as a flat isotropic elastic solid and the field equations for displacements and stresses are expressed as the superposition of the fields of four bulk waves existing in the layer. In the first step, the field equations for bulk waves, which are solutions of the wave equation in an infinite medium, are obtained. After, using boundary conditions at the interfaces between layers, the rules for coupling between layers and for superposition of bulk waves are established.

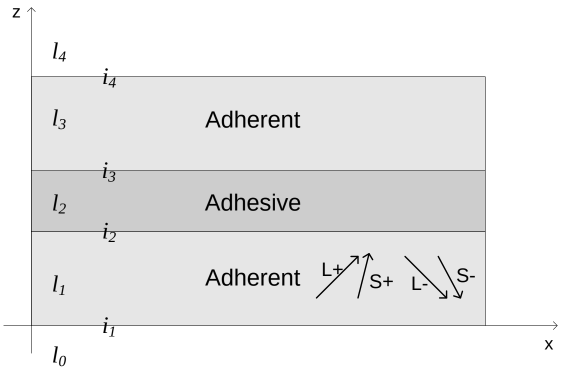


Fig.1 Geometry of the problem

In multilayered plate systems is normally assumed that the analysis is restricted to two dimensions, so plane strain and motion are imposed [12]. In this situation the coordinate system can be reduced to the plane defined by the direction of propagation of the waves and the normal to the plates. In our problem (Fig.1) the plane is defined by *x* and *z* axis. Due to plain strain there is no variation of any quantity in *y* direction ($\partial/\partial y = 0$). Considering that in each layer exist four generic waves: two longitudinal waves (*L*+, *L*-) and two transversal waves (*S*+, *S*-), where signal (+) indicates upwards and (-) indicates downwards, it is possible to define a matrical relation given by

$$\begin{Bmatrix} u_x \\ u_z \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = [D] \begin{Bmatrix} A_{L+} \\ A_{L-} \\ A_{T+} \\ A_{T-} \end{Bmatrix} \qquad (1)$$

where [*D*] is the field matrix that describes the relationship between wave amplitudes (*A*) and the displacements (*u*) and stresses (σ) at any location in any layer.

The Transfer Matrix method is based in the elimination of the intermediate equations of the multilayered system, so the field in all of the layers are described solely in terms of the external boundary conditions. For the system of Fig. 1, if the displacements and stresses are know at the interface 4 (*i4*), the amplitudes in the top of layer 3 (*l3*) can be evaluated. Using a simplified notation of (1) and inverting [*D*]

$$\{A\}_{l3} = [D]^{-1}_{l3,top} \begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,top} . \tag{2}$$

In the interface 3, using (1), the displacements and stresses in the bottom of layer 3 are given by

$$\begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,bottom} = [D]_{l3,bottom} \{A\}_{l3} \tag{3}$$

substituting (2) in (3) we obtain

$$\begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,bottom} = [D]_{l3,bottom}[D]^{-1}_{l3,top} \begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,top} . \tag{4}$$

The matrix product in this equation relates the displacements and stresses between the top and the bottom of each layer and is called the layer matrix [*L*]

$$[L]_{l3} = [D]_{l3,bottom}[D]^{-1}_{l3,top} . \tag{5}$$

In the interface 3 the displacements and stresses must be continuous, therefore

$$\begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l2,top} = \begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,bottom} = [L]_{l3} \begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,top} . \tag{6}$$

As we can see in (6), the displacements and stresses in the top of layer 2 and 3 are related by the layer matrix [*L*], using analogous procedure to the other layers is possible to obtain the relation between the top and the bottom of the multilayer system

$$\begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l3,top} = [S] \begin{Bmatrix} u \\ \sigma \end{Bmatrix}_{l1,bottom} \tag{7}$$

where [*S*] is the matrix system and is obtained by the product of the layers matrices

$$[S] = [L]_{l3} [L]_{l2} [L]_{l1} . \tag{8}$$

In practice two situations will be considered: (1) the system is in vacuum or (2) immersed in a fluid. In the first situation the stresses must be zero at the top of the layer 3 and at the bottom of layer 1. So, equation (7) can be written as

$$\begin{Bmatrix} u_x \\ u_z \\ 0 \\ 0 \end{Bmatrix}_{l3,top} = [S] \begin{Bmatrix} u_x \\ u_z \\ 0 \\ 0 \end{Bmatrix}_{l1,bottom} . \tag{9}$$

Considering the two zero stress terms in left-hand side we obtain

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} S_{31} & S_{32} \\ S_{41} & S_{42} \end{bmatrix} \begin{Bmatrix} u_x \\ u_z \end{Bmatrix}_{l1,bottom} . \qquad (10)$$

To satisfy equation (10) the determinant of the submatrix [*S*] should be zero, which gives rise to

$$S_{31}S_{42} - S_{41}S_{32} = 0 . \qquad (11)$$

This equation is usually called characteristic function and depends of physical parameters of the plate such as: longitudinal velocity, transversal velocity, density and plate thickness. The solutions of characteristic equation depend on frequency. Typical plots that describe the phase velocity or group velocity versus frequency for different propagation modes are obtained using numeric methods. The wavefronts of each of these modes are always parallel to *z* axis.

If the system is immersed in a fluid, the boundary conditions considered in the first situation (stresses equal to zero) are not valid. In this case the equation (7) should take in account the amplitudes in surrounding media, instead of stresses and displacements

$$\{A\}_{l4} = [D]^{-1}_{l4,top}[S][D]_{l0,top}\{A\}_{l0} \qquad (12)$$

Considering that the incoming waves in the boundaries of the system do not exist, the characteristic equation and the guided propagation modes can be obtained in a similar way that was obtained for the system in the vacuum.

The study of the displacement behaviour with plate thickness variation for each propagation mode is also very important, when we work with Lamb waves. If long range propagation is needed for a immersed plate, low out-of-plane displacement must be assured, other way great losses due to leaking to the fluid will be present. In bonded lap joint, as we will see later, the amplitude of different modes which propagates in the overlap region can be estimated based on the properties of the incident wave mode. The excitation of these modes is governed by the degree to which their mode shapes match the mode shapes of the incident wave.

For the determination of the displacements in a bonded lap joint, it is necessary to know the four amplitudes of *L*+, *S*+, *L*- and *S*- existing in each layer. The total number of unknowns for the bonded lap joint (three layers) is twelve. If the system is immersed in a fluid, two additionally unknowns corresponding to the amplitude of *L*+ in the bottom boundary and *L*- in the top boundary must be considered. Using the boundary conditions for all the interfaces and the equation (1), it is possible to obtain a set of equations that can be represented in matricial notation as

$$[G][A] = 0 \qquad (13)$$

where [G] is a 12x12 matrix composed by the amplitudes coefficient and [A] is a column matrix with 12 elements. For homogeneous equations we require that the determinant of the coefficient matrix vanish in order to ensure solutions other than the trivial. Considering one of the amplitudes as unitary and ignoring one equation, the matrix is reduced to 11x11 elements. After knowing all the amplitudes, using equation (1) it is possible to evaluate the displacements in terms of arbitrary units.

In an immersed plate system, another important parameter that needs to be analysed is the influence of the fluid load in the plate. The guided waves travelling in the plate leak energy into the surrounding fluid. These waves in the fluid have been called leaky Lamb waves. In this case the solutions of the characteristic equation obtained from (12) give us complex values of the wavenumber ($k=k_r+ik_i$) and consequently complex values of phase velocity ($c_p=c_{pr}+ic_{pi}$) for the different propagation modes. The relations between these parameters are as follows [13]

$$k_r = \frac{\omega}{c_{pr}}; \qquad k_i = -\frac{\omega\, c_{pi}}{c_{pr}^2} . \qquad (14)$$

The imaginary part of the wavenumber (k_i) represents an exponential decay and is usually called attenuation coefficient α in Neper per meter.

3. Experimental work

The experimental setup used in this work is shown in Fig. 2. Two immersion transducers with 500 kHz central frequency one as transmitter and another as receiver in a pitch and catch configuration were used. A Panametrics pulser/receiver excites the transmitter. After propagation on the plate, the ultrasonic signal is collected by the receiver and amplified and filtered by the pulser/receiver. In the next step the signal is averaged by a digital oscilloscope, in order to increase the signal to noise ratio and finally is transferred to a computer for further processing. Each transducer is connected to a rotation stage to control the correct angular inclination to excite the desirable mode. The use of an immersion method is related to the necessity to guarantee the same coupling between the transducers and the plate: this way is possible to maintain reproducibility conditions for all different measurements.

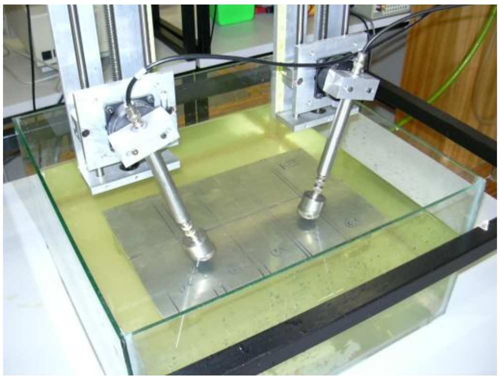


Fig.2 Experimental setup

The lap joint samples were constructed using pairs of aluminium plates with 1 mm (plate A) and 4 mm (plate B) thickness. The bonding zone is preformed in all the width of the plate and has 60 mm of extension. The adhesive used is an epoxy type one with 0.15 mm thickness, that was obtained by action of a uniform stress of 0.1 Kg/m^3. The S0 mode is excited in an aluminium plate (the transmitter plate), travels across the adhesive lap joint and is received in the second aluminium plate (the receiver plate). The wave in the transmitter plate is converted to other modes at the location where it first meets the adhesive layer. These new waves, which travel in the bonded region, are the natural modes of the three-layer system. At the end of the bonded region, further mode conversion takes place giving rise to the original mode existing in the transmitted plate.

To evaluate the behaviour of the different modes, which propagates in the overlap region, the amplitude in the receiver plate was experimentally measured for single plates A and B (A_s) and for bonded plates (A_b) using the same distance between transducers. The attenuation in Np/m due to bonding region (α_b) can be evaluated by knowing its length, which is in our case 60 mm

$$\alpha_b = \frac{1}{0,06} \ln \frac{A_s}{A_b} \ . \tag{15}$$

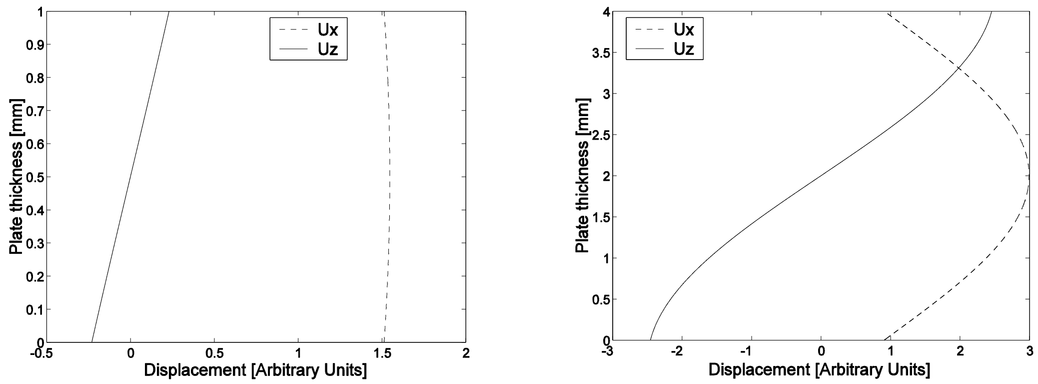
For the plate A it was verified that A_b is lower than A_s, which originates α_b=23.9 Np/m. In the plate B A_b is higher than A_s, which give rise to a negative attenuation α_b=-7.1 Np/m according to our formulation in Eq. (15). To understand this behaviour we need to analyse the theoretical attenuations for single and bonded plates using (14), after the calculation of the phase velocity for a particular propagation mode. For single plates we have found for plate A α=3.1 Np/m and for plate B α=16.3 Np/m. In Table 1 are shown the theoretical attenuation values for the existing modes for the bonded plates.

Based in the attenuation values is possible to predict the dominant propagation mode in the bonding region. In plate A there is an increase of 20.8 Np/m (from 3.1 to 23.9) due bonding zone. So, looking at Table 1 it is evident that the dominant mode should be S0, which has a very similar theoretical attenuation value (21.8 Np/m). In plate B there is a decrease due to bonding zone of 9.5 Np/m (from 16.3 to 6.8). In this case the dominant mode should be A1, because its theoretical attenuation is 10.5 Np/m.

Table 1 Theoretical attenuation in Np/m due to leakage to fluid for the existing modes in the bonded plates

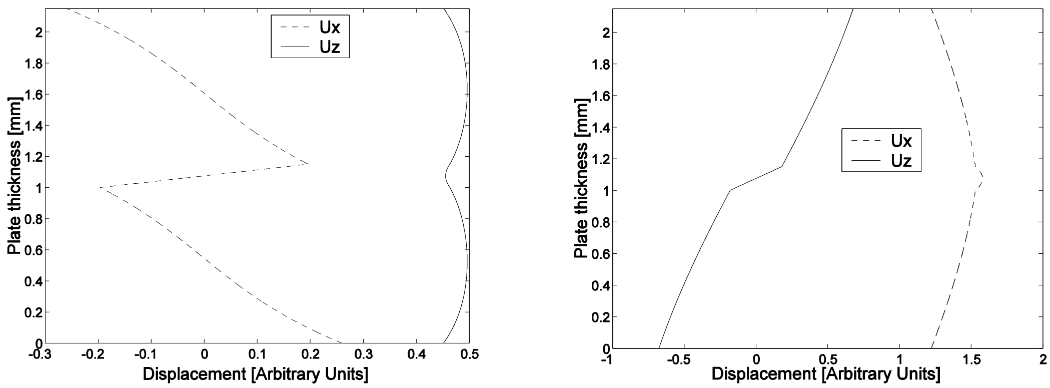
	A0	S0	A1	S1
Plate A	112.9	21.8	-	-
Plate B	22.7	37.5	10.5	0.1

To confirm our assumptions we will analyse the displacement curves for single and bonded plates. In Fig. 3 are presented the displacements of S0 mode for single plates A and B. In Fig. 4 and Fig. 5 are presented the displacements of the existing modes in bonding zone for plates A and B, respectively. From theory [14], it was expected that to excite a certain mode in the bonded zone, the mode shape in the bottom plate of the bonded zone should be similar to the mode shape of a single plate. This concept was also reported by other authors when they stated that a mode will not be excited if its field distribution is orthogonal to the excitation field [15,16].



(a) (b)

Fig.3 Displacements of S0 mode in single plates: (a) plate A; (b) plate B.



(a) (b)

Fig.4 Displacements of the existing modes in bonded plate A: (a) A0; (b) S0.

Analysing the plate A it is possible to see that S0 mode has similar shape in either considered plates (Fig. 3 (a) and Fig.4 (b)), whereas A0 mode is totally different (Fig. 4 (a)). For plate B the A1 mode shape (Fig.5 (c)) is very much like the original S0 mode excited in the transmission plate (Fig. 3 (b)). These results are in completely agreement with our prior experimental analysis and allow us to conclude that is possible to predict the dominant modes in the bonded zone by simple comparison between displacement mode shapes.

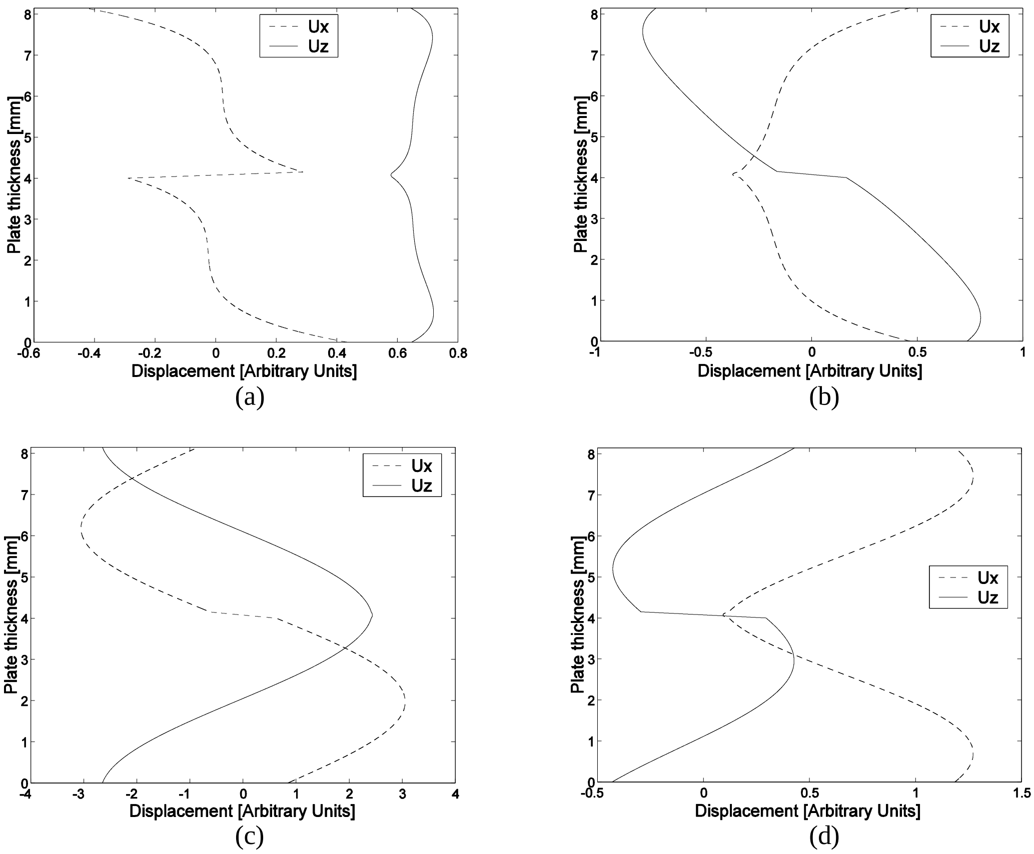


Fig.5 Displacements of the existing modes in bonded plate B: (a) A0; (b) S0; (c) A1; (d) S1.

Another topic analysed in this work, is the possibility of determinate the longitudinal and transversal bulk attenuations of aluminium plates using guided waves parameters. This is interesting, because in the presence of thin plates, accurate attenuation measurements using conventional pulse-echo technique could be a difficult task: under these circumstances high frequencies must be used. In the first step, the total attenuation of two different guided modes is measured using the previous experimental setup. For immersion setup this attenuation is composed by the losses due to energy leaking to the fluid and the losses due to material damping. The leaking losses can be theoretically obtained by knowing the bulk properties of both fluid and plate. After, using transfer matrix method and equation (14) as described previous, the losses due to damping are given by

$$\alpha_{damp} = \alpha_{tot} - \alpha_{leak} \,. \tag{16}$$

Considering in (14) that k_i is coincident with α_{damp}, it is possible to evaluate c_{pi}. In this case, the index d, which is related to damping, is introduced and the new phase velocity, which takes in account only the losses due to material damping, is given by

$$c_p^d = c_{pr}^d + i\, c_{pi}^d \,. \tag{17}$$

As we have two unknowns (longitudinal and transversal attenuations) we need to have two equations to solve our problem. So, the fundamental modes S0 and A0 were used. In the classical problem with the knowledge of the bulk properties it is possible to achieve the phase velocities for each of the propagation modes. In our problem we have the complex phase velocity, and the inverse process should be done to obtain the bulk properties, namely longitudinal and transversal velocities, and later the attenuations. In practice are used S0 and A0 modes complex phase velocities obtained with (16) and (17) in equation (11), that give rise to the bulk wave complex velocities

$$C_L^* = C_L + i\, C_{Li}; \qquad C_T^* = C_T + i\, C_{Ti}, \tag{18}$$

afterwards, the attenuation could be obtained similarly as in (14)

$$\alpha_L = -\frac{\omega C_L}{C_{Li}^2}; \qquad \alpha_T = -\frac{\omega C_T}{C_{Ti}^2}. \tag{19}$$

Experimentally this technique was only used in the plate B (4 mm thickness): because of the low thickness of plate A (1 mm) did not allows to obtain experimentally the values of the attenuation using conventional pulse-echo method with the available transducers. For plate B, transducers with central frequency of 10 MHz were used, and admitting linear dependence with frequency, the predicted attenuations are α_{Lp}=0.5 Np/m and α_{Tp}=2 Np/m at a working frequency of 500 kHz.
To obtain the damping attenuation (α_{damp}) for A0 mode, one additionally difficulty arises due to the fact that this mode has high out-of-plane displacement and consequently high leaking attenuation to the fluid: then, when using previous immersion setup, it was not possible to make any experimental measurements. Instead, a local immersion coupling method [13] was used. The attenuation due to damping for both modes are $\alpha_{damp(A0)}$=1.99 Np/m and $\alpha_{damp(S0)}$=2.1 Np/m, which implicate $c_p^{d}{}_{(S0)}$=4781.8-15.32i m/s and $c_p^{d}{}_{(A0)}$=2684.6-4.58i m/s. Using these complex phase velocities values, the bulk complex velocities are given by C_L^*=6300-7.58i m/s and C_T^*=3130-6.54i m/s and applying (19), the attenuations are α_L=0.594 Np/m and α_T=2.098 Np/m. Comparing these values with those obtained with conventional methods (α_{Lp} and α_{Tp}) we can see good quantitative agreement

4. Conclusions

The propagation of Lamb waves in adhesively lap joints is of great importance in industrial processes. In this work it was verified that is possible to predict the dominant propagation modes that exist in bonded region by comparing the experimental attenuation of single and bonded plates. These results were confirmed by looking at the shape of the displacement curves in single and bonded plates, and verifying that in bonded zone the excitation of the dominant modes is governed by the degree to which their mode shapes match the mode shapes of the incident wave. Good experimental results using 1 mm and 4 mm thickness plates bonded with an epoxy type adhesive were obtained.
Another interesting result was the determination of both longitudinal and transversal attenuations in aluminium plates using guided waves parameters. In practice, damping attenuation of fundamental A0 and S0 modes were experimentally measured and using the inversion process two different complex phase velocities were obtained, using transfer matrix technique. After, complex bulk velocities give rise to bulk (longitudinal and transversal) attenuations. The results were confirmed with reasonable accuracy for 4 mm thickness aluminium plates. This technique could be interesting to use, especially when applied to thin plates, where high frequencies are demanded.

5. References

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