



## Ultrasound interferometry for the evaluation of thickness and adhesion of thin layers.

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### Abstract

This work deals with the evaluation of thin layers by a technique called ultrasound interferometry, which is based on the resonance and antiresonance of ultrasound waves verified in the presence of some interfaces having different acoustical impedances. A Graphical User Interface (GUI), developed in matlab was used to simulate the calculation of several coating thicknesses over different substrates, like iron and aluminium. The results are based on the representation of reflection or transmission coefficients against frequency, thickness and velocity in the coatings.

### Introduction

Ultrasound interferometry is a well known method, which is, frequently used to measure phase velocities. It is easily proved that changing the frequency of a probe transmitting into a medium with parallel faces, resonance and antiresonance of ultrasound waves can be visualised, in a continuous mode. Frequency values corresponding to two consecutive resonances and two consecutive antiresonances are directly correlated with the layer thickness and with the propagation wave velocity. The proposed goal required first the development of equations relating to the multiple internal reflections and transmissions of waves against frequency [1, 2].

In order to simulate this technique for an effective calculation of thicknesses of very thin layers, some tests were accomplished considering three propagation media: coupling, layer and substrate. The results illustrate essentially painting layers on different substrates like; iron, aluminium.

A Graphical User Interface (GUI) was created to help simulating the behaviour of the frequency or abstract variable dependent reflection and transmission coefficients [3]. Any number of layers can be tested in the simulation tool.

This technique has also great potential to test laminate adhesion, besides coating thickness evaluation.

### Theory

#### *Wave Propagation in three layers systems*

Figure 1 shows the schematic representation of the three layer system. In the absence of dispersion, the reflection (R) and transmission (T) amplitude coefficients from the medium 1 to medium 2 with acoustic impedance  $Z_1$  and  $Z_2$  respectively, are given by

$$R_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (1)$$

and

$$T_{12} = \frac{2Z_2}{Z_2 + Z_1} \quad (2)$$

considering acoustic waves normally incident. Equations like these can also be obtained in the interface between media 2 and 3.



Since more than two media are involved, the interference effects give rise to an infinite series of reflected and transmitted frequency dependent pulses. Let one consider the acoustic plane wave normally incident with a longitudinal propagation in the x-direction given by [2, 4],

$$u(x,t) = u_0 e^{-i(\omega t - kx)} \quad (3)$$

where  $u_0$  is the amplitude and  $k$  is the wavenumber. Assuming that  $x = 0$  in the first interface (among layer 1 and layer 2) the input wave may be written as

$$u(0,t) = u_0 e^{-i(\omega t - \phi_0)} \quad (4)$$

where  $\phi_0$  is the phase possessed by the wave when it enters the specimen ( $x=0$ ).

A portion of the pulse  $u(x,t)$  incident on the first interface will be reflected back to the medium 1 with amplitude  $R_{12} u(x,t)$  and the other portion will be transmitted into medium 2 with amplitude  $T_{12} u(x,t)$ . At the interface between media 2 and 3 the acoustic wave, which just propagated through the specimen of length  $L$ , undergoes a reflection giving rise to a wave with the form  $T_{12} R_{23} u(L, t - L/c)$ , where the term  $L/c$  is the time delay along the specimen. This reflected signal will give rise to the first backwall echo after being transmitted to medium 1 (see figure 1) with the form  $T_{12} R_{23} T_{21} u(2L, t - 2L/c)$ .

Of course the wave also suffers some energy loss as it propagates through the medium. This energy loss is characterised by an exponentially decaying ( $e^{-\alpha x}$ ) where  $\alpha$  is the decay coefficient or attenuation. It is easily verified that there is a multitude of reflections inside the specimen with a corresponding number of transmitted signals to medium 1. Thus, the second backwall echo will appear as  $T_{12} R_{23}^2 T_{21} u(4L, t - 4L/c)$ .

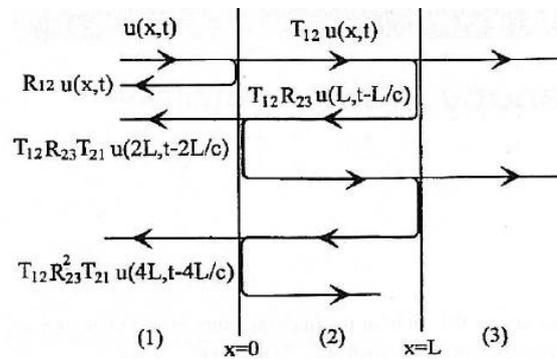


Figure 1. Reflected and transmitted signals from a layer between two media.

The development of the expressions for the reflection and transmission coefficients is made applying the superposition principle to determine the response of a layered medium to a continuous wave train. According to the behaviour of waves in the interfaces 1 and 2, the total of all transmitted waves is given by

$$T(t) = T_{12} T_{23} u(t - L/c) + T_{12} T_{23} R_{21} R_{23} u(t - 3L/c) + \dots,$$

$$T(t) = T_{12} T_{23} \sum_{n=0}^{\infty} (R_{21} R_{23})^n u(t - ((2n + 1)L/c)) \quad (5)$$

Similarly, the total wave reflected back into medium 1 is given by

$$R(t) = R_{12} u(t) + T_{12} T_{21} R_{21}^{-1} \sum_{n=1}^{\infty} (R_{21} R_{23})^n u(t - 2nL/c) . \quad (6)$$



For sake of simplicity, the attenuation was not considered in this analysis. Applying the Fourier transform to equations (5) and (6),

$$T(\omega) = \left[ T_{12}T_{23} e^{-j\omega L/c} \sum_{n=0}^{\infty} (R_{21}R_{23})^n e^{-j2\omega nL/c} \right] u(\omega) \quad (7)$$

$$R(\omega) = \left[ R_{12} + T_{12}T_{21}R_{23} e^{-j2\omega L/c} \sum_{n=0}^{\infty} (R_{21}R_{23})^n e^{-j2\omega nL/c} \right] u(\omega) \quad (8)$$

When  $n \rightarrow \infty$ , the generalized frequency-dependent reflection and transmission coefficients  $r(\omega)$  and  $t(\omega)$ , can be defined as follow:

$$t(\omega) = \frac{T(\omega)}{u(\omega)} = \frac{T_{12}T_{23} e^{-j\omega L/c}}{1 - (R_{21}R_{23})^n e^{-j2\omega L/c}} \quad (9)$$

$$r(\omega) = \frac{R(\omega)}{u(\omega)} = R_{12} + \frac{T_{12}T_{21}R_{23} e^{-j2\omega L/c}}{1 - (R_{21}R_{23})^n e^{-j2\omega L/c}} \quad (10)$$

## Results of simulation

In the simulation process the coefficients can be represented against frequency only or against all important parameters like frequency, velocity  $c$ , and thickness  $L$ . The expression relating these three parameters is

$$\Delta f = \frac{c}{2L} \quad (11)$$

where,  $\Delta f$  is the frequency value among two maxima (minima) of the reflection (transmission) coefficients.

The frequency representation of the coefficients would require the knowledge of the layer thickness. However, that is precisely what we want to know. A more useful representation makes use of an abstract variable, which can be defined by,

$$\text{var} = \frac{2fL}{c} \quad (12)$$

This kind of representation allows us to define the frequency range to use in the inspection process as well as to calculate the thickness and velocity of layers with precision. As an example, if  $\text{var} = 0$ ,  $f_1=0$ ; for  $\text{var} = x$ , then  $f_2= xc/(2L)$ . Thus, the frequency sweep should be made for  $[0, f_2]$ . It is important to mention that  $\Delta f$  is best defined between two maxima or minima with very sharp peaks or valleys.

A Graphical User Interface (GUI) using matlab was implemented to provide an easy way to represent the reflection and transmission coefficients. The operator is able to select some parameters like; amplitudes of  $r(\omega)$  and  $t(\omega)$ , considering the superposition of some or an infinity reflections and transmissions in the layer being studied. Also, the operator can select a representation against frequency only or consider the variable  $\text{var}$ . Figure 3 shows the developed application as well as the above mentioned fields. The illustrated reflection coefficient refers to a coating over an iron substrate, identified by  $Z_2$  and  $Z_3$ , respectively, for an infinite number of pulses. Water was considered as propagation medium ( $Z_1$ ). Figure 4 outlines the same setup, now for the superposition of a finite number of pulses (2 and 3), ignoring the first one which presents no importance for the subject. Figure

5, shows the superposition of the first three pulses of the reflection coefficient for a coating over an aluminium substratum.

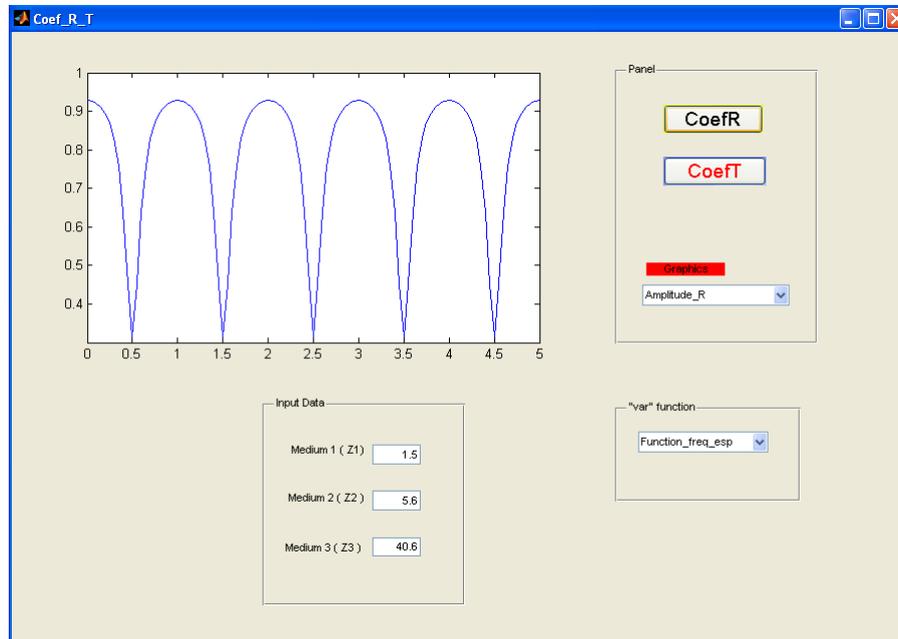


Figure 3 – Graphical User Interface. Coefficient of reflection  $r(\omega)$  for a coating over iron.

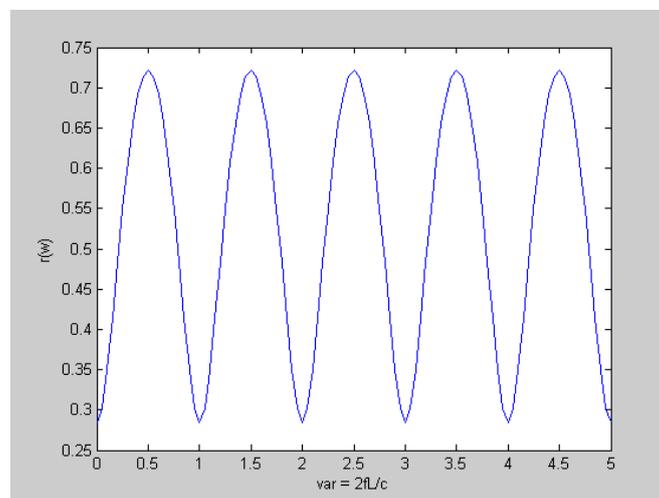


Figura 4 – Reflection coefficient  $r(\omega)$ ; superposition of a finite number of pulses (2 and 3).

### Frequency selection

As shown before, the frequency is related to the coating thickness. The interferometric procedure shows that for a given layer of thickness  $L$  and ultrasonic velocity  $c$ , the frequency variation between two minima (maxima) is evaluated by equation 11. It is easy to conclude that this frequency variation ( $\Delta f$ ) should allow the detection of two minima (maxima) for the largest range of thicknesses ( $L = L_{\min}$



and  $L=\infty$ ), at least. Assuming that  $\Delta f_{\max}$  is the maximum variation between minima (maxima), it is clear that the first and second pulses occur for the frequencies  $\Delta f_{\max}/2$  e  $3/2\Delta f_{\max}$ , respectively.

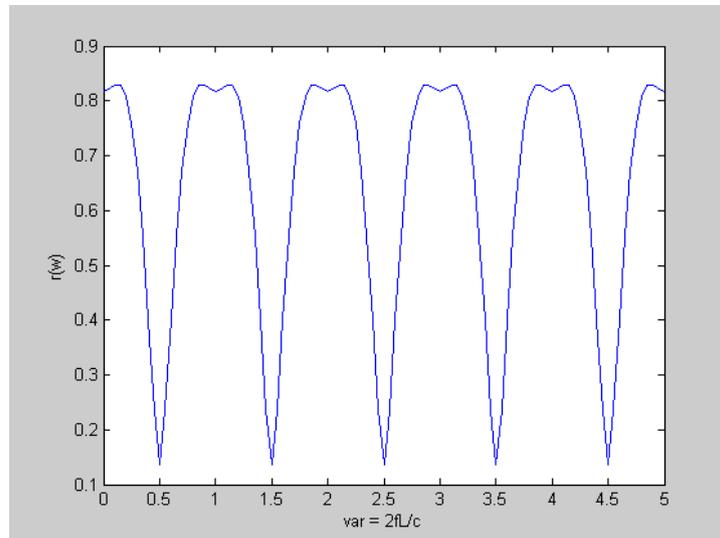


Figure 5- Coefficient of reflection  $r(\omega)$ ; coating over aluminium. Superposition of the first three pulses.

### Discussion and conclusions

A simulation tool has been developed to describe the interactions of ultrasonic waves with layers over any substrate. The Graphical User Interface, implemented in matlab, makes possible the estimation of the frequency or an abstract variable-dependent reflection and transmission coefficients. Their values can be accurately calculated considering the sharp resonance peaks.

To calculate the thickness of micrometric layers, it is of fundamental importance to define adequately the frequency sweep range in order to have, in any circumstances, two minima (maxima) well characterised and with sharp peaks.

This simulator is very important in the experimental process. The theoretical knowledge of the behaviour of frequency-dependent coefficients  $r(\omega)$  and  $t(\omega)$ , appears as a good help in the implementation of the setup. Also, transducers must have very good performance for a large range of frequencies.

### References

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