Crack Diagnosis in Beams Using Propagated Waves and Hilbert Huang Transformation

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Abstract - This paper is a numerical simulation and verification of health monitoring of beam structures using propagating piezo-actuated lamb waves. The goal of this research is to detect the location of a linear crack in a beam using a piezoelectric actuator/impact hammer and a piezoelectric sensor based on the time-of-flight of propagating waves. The actuation signal is first determined based on the propagating lamb wave modes in a thin structure and group dispersion curve of an aluminum plate. The commercial finite element code (ABAQUS) has been employed to model a beam with an actuator-sensor pair and a tiny groove representing the crack. After a transient dynamic analysis, the sensor response has been acquired. Using Hilbert Huang Transform (HHT) method, from the time-energy spectrum of sensor respons and propagating wave time-of-flight, location of the crack has been detected.

Keywords: Piezoelectric, Lamb wave, Hilbert Huang Transformation, Time of Flight

1 -Introduction

The use of ultrasonic guided waves for non-destructive evaluation (NDE) of structures is effective due to their long propagation range, as stated in much published literature such as Park *et al* 1996 [1], Quek *et al* 2001 [2], Quek *et al* 2003 [3]Tua *et al* 2004 [4] and Tua *et al* 2005 [5]. However the generation of the guided wave must be selective to effectively locate and quantify the defect (Shin and Rose 1999 [6], Tua *et al* 2004 and 2005[4,5]).

Piezoelectric materials, according to their capability to convert mechanical load to electrical response and vice versa, and also their high bandwidth, are good candidates to be used as both actuators and sensors for producing and gathering guided lamb waves.

The object of this paper is to numerically simulate and extend the experimental work done by Quek *et al* 2003 [3]. In this study a crack of sub-millimeter width is investigated for the possibility of being detected and localized using PZT transducers and also impact as actuation generator, adopting the fundamental concept that a propagating wave will be reflected and/or partly transmitted when it encounters a defect. First the actuation pulse and its frequency has been selected and then commercial finite element (FE) code, ABAQUS FE software has been employed to model the problem. After performing a transient dynamic analysis, sensor response has been acquired and processed using Hilbert-Huang Transformation. From energy-time spectrum of the sensor response which has been obtained from HHT analysis, one could find the crack location from time of flight analysis. The first peak in the energy-time spectrum is the result of the incident wave which could be used for calculation of the wave speed in the medium. Having calculated the wave speed, from the second peak in the spectrum, the flight distance of the wave could be calculated and the location of the crack could be found. Figure-1 shows the geometry of the model which exprimentaly investigated by Quek *et al* (2003)[3].

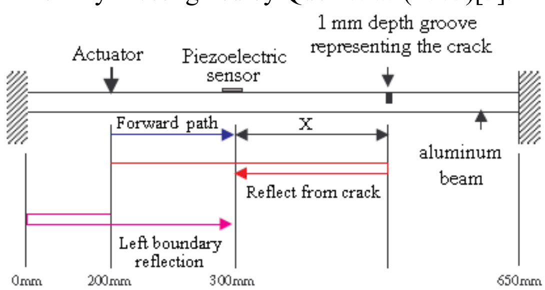

Figure 1- Geometry of the Problem

This model has been investigated for detection of crack or anomaly in beams based on the Hilbert Huang Transformation. This method is based on finding a change in the amplitude-time response of the sensor which could be considered as a result of a crack. Figure 2 shows the results obtained by Quek *et al* (2003). It was necessary to compare the sesor response of the danaged beam with the healthy one as shown in figure 2.

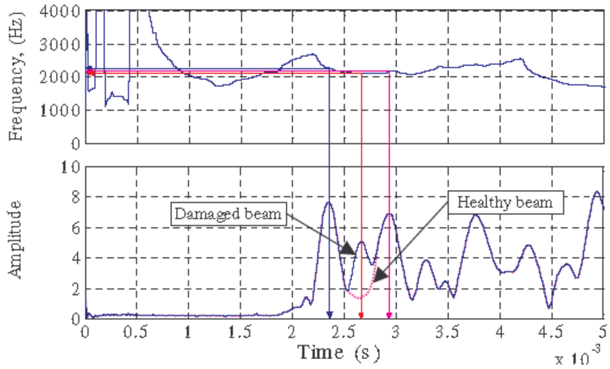

Figure 2- Amplitude- time plot of the sensor response [3]

2-Choice of actuation wave

In a semi-finite medium, the three basic waves are compressive, shear and Rayleigh waves. For plates, the existence of upper and lower surfaces leads to guided waves. Lamb waves in plates result from the superposition of guided longitudinal and transverse shear waves within an elastic layer. Lamb waves contain distinct symmetric and anti-symmetric wave modes, as shown in Figure 3, and the number of Lamb wave modes that can be propagated increases with frequency. Each mode has a particular cut-off frequency below which the mode will not be present. This cut-off frequency is dependent on the velocities of the longitudinal and transverse waves.[4],[7]

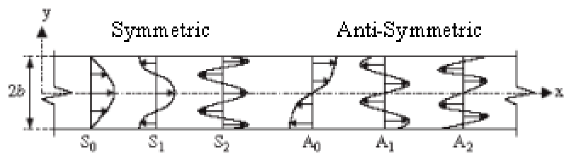

Figure 3-The first few fundamental lamb wave modes for propagation within a plate of thickness 2b [4],[7]

The Rayleigh-Lamb frequency equation for propagation of symmetric waves in a plate is given as

$$\frac{\tan \beta b}{\tan \alpha b} = -\frac{4\alpha\beta\xi^2}{(\xi^2-\beta^2)^2} \qquad (1)$$

where b is the half-thickness of the plate, ξ (=2π/λ) is the wavenumber , α and β are given as follows:[4]

$$\alpha^2 = \frac{\omega^2}{c_1^2} - \xi^2 \qquad (2a)$$

$$\beta^2 = \frac{\omega^2}{c_2^2} - \xi^2 \qquad (2b)$$

in which ω is the angular frequency, c_1 and c_2 are the longitudinal and transverse wave velocity respectively given by [4]

$$c_1 = \sqrt{\left(\frac{1-\nu}{1-\nu-2\nu^2}\right)\frac{E}{\rho}}, \quad c_2 = \sqrt{\frac{G}{\rho}} \qquad (3)$$

where E and G are the Young's and shear moduli of the plate material respectively, ρ is the density and υ is the Poisson ratio. Likewise, the Rayleigh-Lamb frequency equation for the propagation of anti-symmetric waves in a plate is given as :

$$\frac{\tan \beta b}{\tan \alpha b} = -\frac{(\xi^2-\beta^2)^2}{4\alpha\beta\xi^2} \qquad (4)$$

The group velocity, c_g, of the symmetric and anti-symmetric wave propagation in the plate is given by:

$$c_g = \frac{d\omega}{d\xi} \qquad (5)$$

Figure 4 shows the group velocity dispersion curves for aluminum.

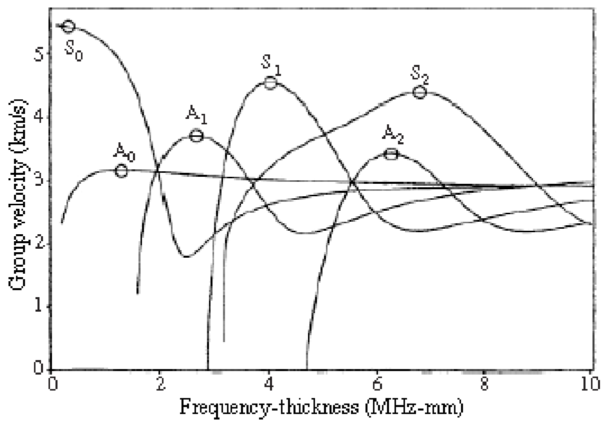

Figure 4- The group dispersion curve for aluminum plate

As it is seen in Figure-4, there are multi modes of Lamb waves and to avoid complications, selective generation of modes is necessary. To achieve this, a Lamb wave with a narrow band actuation pulse, Figure 5, is applied to the actuator based on the following equation:

$$X(t) = \cos(2\pi\omega t)e^{-a(t-t_0)^2} \qquad (6)$$

where ω is the actuation frequency (Hz), and a and t_0 are constants.

Also a triangular impact has been used as actuation signal to investigate Tua's experiment. It is expected that applying triangular impact causes more complicated results.

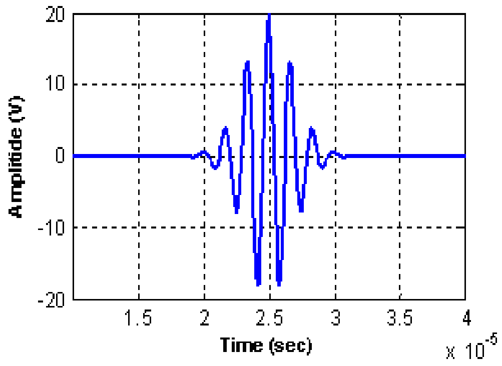

Figure 5- The actuation pulse [4]

There are several ways for lamb wave generation. Figure 6 shows this methods. The first one which is shown in figure 6-a is based on using Y-cut piezoelectric transducers. In this case applying voltage to the piezo element cuases horizental deformation of the piezo element and consequently the host structure. This method is used in this paper. The second one (6-b) is based on using X-cut piezoelements while placing them with a predefined distance (which is the wavelength of peopagated wave). Firing the transducer causes repeated pulses act in the structure and this yields the wave to propagate. The third method is using inclined probe to produce lamb wave.[8]

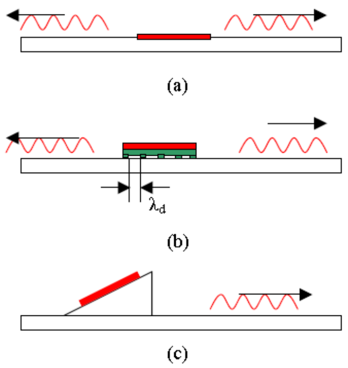

Figure 6- Methods for lamb wave generation. a) using Y-cut piezoelements.b) using X-cut piezoelements with predefined spacing c) using inclined piezoelectric probe.[8]

The frequency of the actuation pulse plays a very vital role here. It should be selected in a way that the least possible modes will be excited. For example actuation frequencies less then 0.8 MHZ for an aluminum plate of thickness 2 mm limit the analysis to only the fundamental Lamb modes S_0 and A_0. By estimating the flight path due to either S_0 or A_0 propagation, the distance traveled by the wave from the actuator to the sensor can be computed as

$$l_i = \Delta t_i c_g \qquad (7)$$

where Δt_i is the time of flight referenced from the actuation time, and c_g is the group velocity of either the S_0 or A_0 mode.[4]

3-Hilbert Huang Transform

One of the main concepts in detecting damage in structures is to focus on a sudden change in system response. A sudden change in the output signal of a system could be resulted from a sudden change in the structure that could be a crack, a hole or etc.
But instantaneous changes in signals are equivalent to a high frequency appearance in the signal. In HHT the concept is to find instantaneous frequency of a non-stationary signal. Simply a non stationary signal is a signal that its frequency varies from time to time.
Historically Fourier analysis has provided a general method for examining the global energy-frequency distributions. Although the Fourier transform is valid under extremely general conditions, there are crucial restrictions of the Fourier spectral analysis: the system must be linear; and the data strictly periodic or stationary; otherwise, the resulting spectrum will make little physical sense.[9]

One could employ Hilbert transform to define instantaneous frequency as follow:
For an arbitrary time series, *X(t)* we can always have its Hilbert transform , *Y(t)*, as[9]

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt', \qquad (8)$$

where P indicates the Cauchy principal value. This transform exists for all functions of class L^p. With this definition, *X(t)* and *Y(t)* form the complex conjugate pair, so we can have an analytic signal, *Z(t)*, as

$$Z(t) = X(t) + iY(t) = a(t) e^{i\theta(t)}, \qquad (9)$$

in witch

$$a(t) = \left[X^2(t) + Y^2(t)\right]^{1/2}, \qquad \theta(t) = \tan^{-1}\left(\frac{Y(t)}{X(t)}\right) \qquad (10)$$

Essentially equation (8) defined the Hilbert transform as the convolution of *X(t)* with *1/t*, therefore it emphasizes the local properties of *X(t)*.[9]
The instantaneous frequency then could be defined as

$$\omega = \frac{d\theta}{dt} \qquad (11)$$

But there is still considerable controversy in defining the instantaneous frequency as equation (11). The instantaneous frequency given by equation (11) is a single valued function of time. This lead to introduction of 'monocotmponent function'. At any given time there is only on frequency value. Unfortunately, no clear definition of monocomponent signal was given in literature to judge whether a signal is or is not monocomponent. So 'narrow band' was adapted as a limitation on a data for the instantaneous frequency to make sense. According to *Huang et al(1998)* two necessary conditions for a signal to have meaningful instantaneous frequency is that 1) The signal should be symmetric with respect to the local zero mean and 2) It should has the same number of zero crossings and extreme.[9]

Based on the stated conditions, *Huang et al (1998)* introduced the Intrinsic Mode Functions (IMFs) for a signal. The main characteristic of IMFs are that (1) in whole data set the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local minima and envelope defined by the local maxima is zero.[9]

To define the instantaneous frequency one should first decompose the signal into its IMFs using a procedure called 'sifting process" which was introduced by Huang *et al (1998).* Having obtained the intrinsic mode functions components, there would be no difficulty in applying the Hilbert transform to each component and computing the instantaneous frequency according to equation (11). After performing the Hilbert transform on each IMF component, the data could be expressed in the following form[9]

$$X(t) = \sum_{j=1}^{n} a_j(t) \exp\left(i \int \omega_j(t) dt\right) \qquad (12)$$

As it seen, the IMF represents a generalized Fourier expansion in witch both magnitude and frequency could be functions of time.[9]

Equation (12) enables us to represent the amplitude and the instantaneous frequency as functions of time in three dimensional plot, in which the amplitude can be contoured in frequency-time plane. This frequency-time distribution of the amplitude is designated as the Hilbert amplitude spectrum, *H(ω,t),* or simply Hilbert spectrum.
With Hilbert spectrum defined, one can also define the marginal spectrum *h(ω),* as [9]

$$h(\omega) = \int_0^T H(\omega, t) dt. \qquad (13)$$

In addition to marginal spectrum, it is also possible to define the instantaneous energy density level, IE, as[9]

$$IE(t) = \int_{\omega} H^2(\omega, t) d\omega \qquad (14)$$

Energy density level, IE, will be used to find the incident and reflection peaks.

4-Finite Element Model

ABAQUS FE software has been used to model the system shown in figure-1. The value X in figure 1 is set to *150mm* here.

There are two basic FE methods for modeling dynamic prablems; Implicit dynamic procedure and Explicit dynamic. It was shown in [10] that implicit dynamic procedure is not suitable for wave propagation modeling and causes wrong results. On the other hand, explicit dynamic is able to model the wave propagation problem accurately. But the explicit dynamic procedure in ABAQUS has a big shortcoming which is lack of piezoelectric elements. One choice of actuation pulse is applying displacement to related nodes in contact with piezoelements. Since displacement based finite element method has been used here, applying voltage to the piezoelectric transducer causes displacement in slave nodes on the aluminum beam. Thus the related displacement caused by piezoelement can be applied instead of voltage. In order to model the wave generated by a piezoelectric transducer a suitable model of piezoelectric actuator and sensor is required. According to [10] and [11] in this research the pin-force model which has been used for the modeling of piezoelectric actuators on beams is employed.for the piezoelectric actuation modeling in the explicit dynamic procedure in this model, a concentrated force represents the actuation.
Figure-7 shows the impact applied to the beam as actuation pulse.

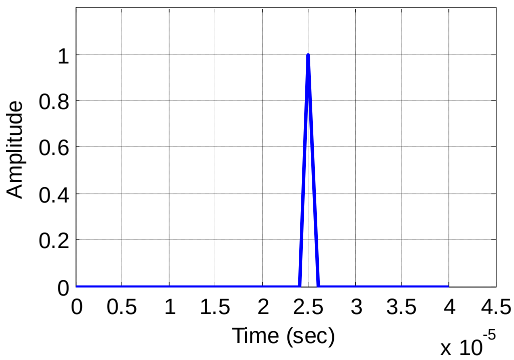

Figure 7-Impulse actuation signal

Both verical and horizontal forces have been applied as impact to the beam which results are shown in figures 8 and 9. Because of unavailability of piezoelectric elemnets in ABAQUS/Explicit, there should be another replacemnent for the piezo transducer, but this time for the sensor. To overcome this problem, a smal transducer with PZT Mechanical properties (Only mechanical properties including modulas of elesticity, poission ratio and density) have been placed as the sensor. Notice that electrical properties of PZT have not been included in this model beacause explicit elements do not have voltage degree pf freedom. So to read the sensor voltage, ε_{11} has been read from the part representing PZT. As we know, in piezolecetric materials, the voltage caused by strain has linear relationship with it, so the strain itself could be used as sensor output. The ε_{11} has been read because of the Y-cut piezoelectric transducer.

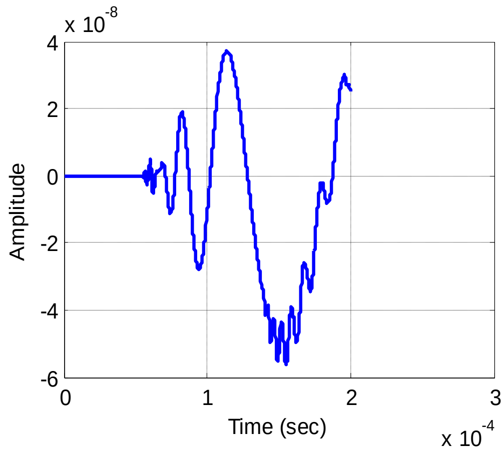

Figure 8- sensor response caused by vertical impact actuation

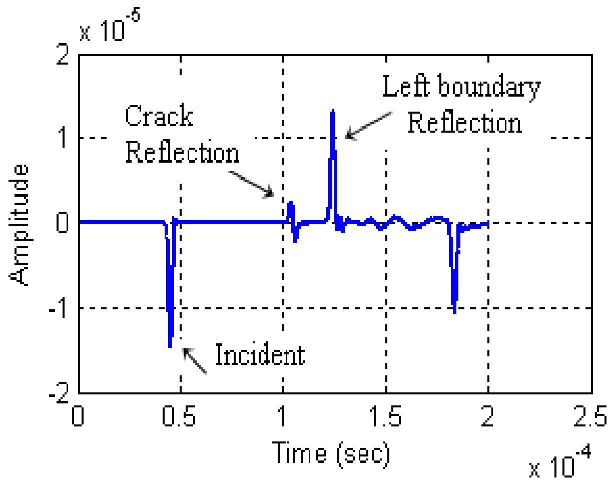

Figure 9- Sensor response caused by horizontal impact actuation

As it is shown, applying vertical impact to the beam for actuation, causes complicated response which is difficult to analyse. On the other hand, horizontal impact causes better results and several peaks are clearly seen in that. From the first peak it is possible to find the wave speed and by multiplying the wave speed to the time span between two peaks, the flight distance of wave after passing the sensor could be computed. The flight distance of wave according to figure 9 is 297 mm which closely matches experimental results.

Also the signal shown in figure 5 has been used for actuation. Again because of using Y-cut piezolements, the equivalent force has been applied horizontaly. Figure 10 shows the response resulted from actuation pulse shown in figure 5.

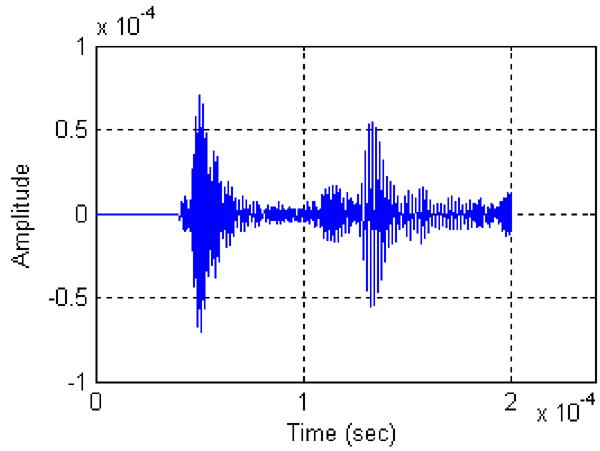

Figure 10- response of sensor caused by actuation pulse shown in figure 5

This signal have been processed using HHT which results are stated below.

5-Signal processing with HHT

Having obtained the sensor response, HHT transform has been applied to the signal using Matlab 7.0. The signal has 11 IMFs. The first four are shown in Figure 11.

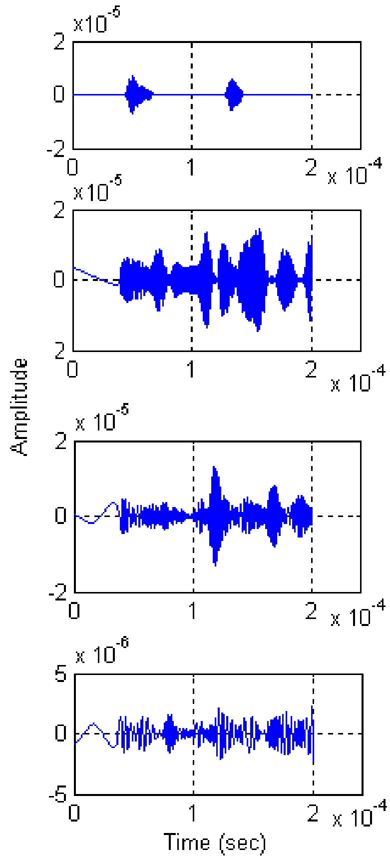

Figure 11- First four IMFs for response signal of horizontal actuation force.

Having done the empirical mode decomposition, the Hilbert transform has been applied to each IMF and instantaneous frequencies for them was extracted. There are 11 instantaneous frequencies at each time since the signal has 11 IMFs.

By applying equation (14) to the spectrum, energy level of the signal versus time could be obtained. Figure 12 shows the energy- time spectrum.

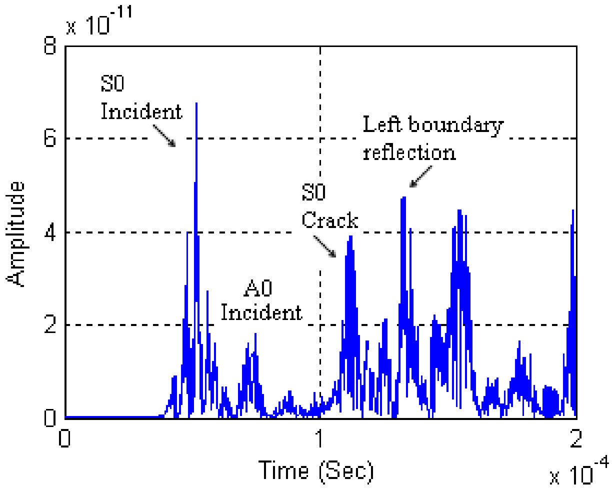

Figure 12-Energy density level versus time plot of sensor response obtained from HHT analysis

From energy-time spectrum shown in Figure 12, it is possible to calculate wave speed by dividing the actuator-sensor distance by arrival time. Crack location then could be found by multiplying the wave speed to time span between two peaks. It is also better to look at the first IMF's hilbert transform and especially the magnitude of its complex signal. Figure 13 shows the amplitude of the complex signal resulted from Hilbert transform of the first IMF.

The flight distance of wave obtained above is 315mm, so the crack is located 157.5mm from sensor. As it is seen, the error of the method is about 5% that could be an acceptable amount of error.

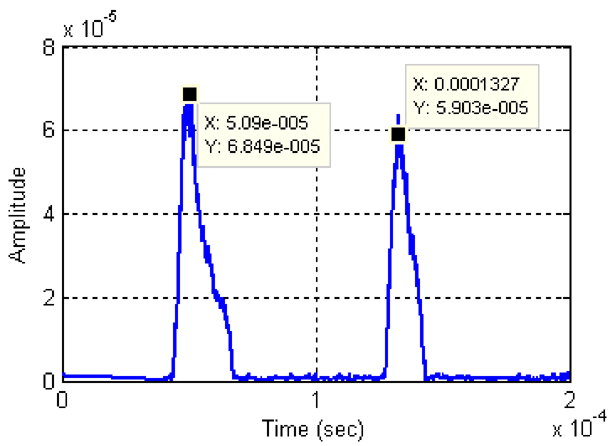

Figure 13- The Ampliude of the complex signal obtained by hilbert transform of the first IMF

6-Conclusions

The experimental work done by *Tua et al(2003)* was numerically simulated, extended and verified. It was shown that the triangular vertical impact as actuation pulse in the FE modeling of the wave propagation, caused inaccurate results. An Explicit dynamic procedure demonstrated a significant improvement with high accuracy results. In the Explicit dynamic model of the beam a Pin-Force model of the piezoelectric actuator was employed. HHT method was employed to find the instantaneous frequencies of the sensor signal. Using energy density level, the incident and reflected waves were detected. The location of the crack was accurately detected using time-of-flight of waves. Simulation results of the explicit FE matched closely the experimental results.

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