Crack Diagnosis in Plates Using Propagated Waves and Hilbert Huang Transformation

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Abstract - This paper is a numerical simulation and verification of health monitoring of plate structures using propagating piezo-actuated lamb waves. The goal is to detect the location of a linear crack in a plate using piezoelectric actuator and sensor based on the time-of-flight of propagating waves. The actuation signal is first determined based on the lamb wave modes for propagation in a thin structure and the group dispersion curve of an aluminum plate. The commercial finite element code (ABAQUS) has been employed to model a plate with three piezoelectric elements as both actuator and sensor. Also equivalent forces have been used to substitute the piezoelectric elements in explicit dynamic procedure. Elliptical loci of possible crack positions are constructed based on the flight time of crack reflected waves estimated using energy spectra from the Hilbert Huang Transform of the sensor signals.

Keywords: Piezoelectric, Lamb wave, Hilbert Huang Transformation, Time of Flight

1-Introduction

The use of ultrasonic guided waves for non-destructive evaluation (NDE) of structures is effective due to their long propagation range, as stated in much published literature such as Park *et al* 1996 [1], Quek *et al* 2001 [2], Quek *et al* 2003 [3]Tua *et al* 2004 [4] and Tua *et al* 2005 [5]. However the generation of the guided wave must be selective to effectively locate and quantify the defect (Shin and Rose 1999 [6], Tua *et al* 2004 and 2005[4, 5]).
Piezoelectric materials, according to their capability to convert mechanical load to electrical response and vice versa, and also their high bandwidth, are good candidates to be used as both actuators and sensors for producing and gathering guided lamb waves.
The object of this paper is to numerically simulate and extend the experimental work done by Tua *et al* 2004 [4]. In this study a crack of sub-millimeter width is investigated for the possibility of being detected and localized using PZT transducers and also concentrated force as actuation generator, adopting the fundamental concept that a propagating wave will be reflected and/or partly transmitted when it encounters a defect. First the actuation pulse and its frequency has been selected and then commercial finite element (FE) code, ABAQUS FE software has been employed to model the problem. After performing a transient dynamic analysis, sensors responses have been acquired and processed using Hilbert-Huang Transformation. From energy-time spectrum of the sensors responses which has been obtained from HHT analysis, one could find the elliptical crack loci from time of flight analysis. The first peak in the energy-time spectrum is the result of the incident wave which could be used for calculation of the wave speed in the medium. Having calculated the wave speed, from the second peak in the spectrum, the flight distance of the wave could be calculated and the location of the crack could be found. Figure-1 shows the geometry of the model which experimentally investigated by Tua *et al* (2004) [4].

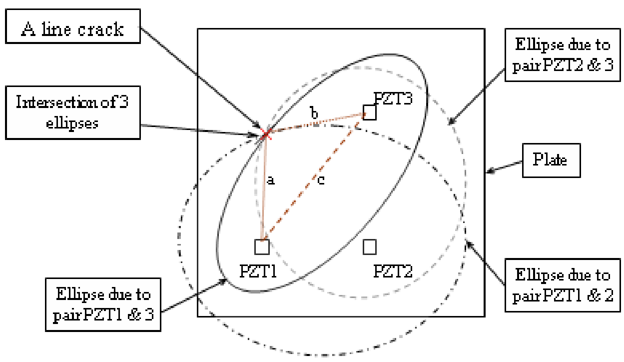

Figure 1- Geometry of the Model

This model has been investigated for detection of crack or anomaly in plates based on the time-of-flight analysis of lamb wave propagation and reflection. Consider the actuator PZT1 attached on a plate shown in figure1, firing waves with the sensor PZT3 receiving signals. If a crack is present which intercepts and reflects the wave from PZT1, then PZT3 will receive packets of signals, the first one corresponding to the direct wave traveling along line joining PZT1 to PZT3 and the second one corresponding to the wave along the lines from PZT1 to the crack and then reflected to PZT3, assuming that other boundaries are sufficiently far away. On the basis of the recorded flight times and assuming constant wave propagation velocity, a value, say *k*, for *a*+*b* can be estimated, where *a* is the linear distance from the actuator to the crack and *b* is the linear distance from the crack to the sensor. Because the position of the crack is unknown there are infinitely many combinations of *a* and *b* for a given *k*. The locus of the possible crack locations is an ellipse with PZT1 and PZT3 as the foci. In order to identify the exact location of the crack from the infinite solutions provided by one ellipse, signals from actuator/sensor pairs at different positions need to be used. This will allow more ellipses to be constructed and their intersection will then provide an estimated location of the crack. A minimum of three ellipses provides an unambiguous estimate of the location, as illustrated in figure 1.

2-Choice of actuation wave

In a semi-finite medium, the three basic waves are compressive, shear and Raleigh waves. For plates, the existence of upper and lower surfaces leads to guided waves. Lamb waves in plates result from the superposition of guided longitudinal and transverse shear waves within an elastic layer. Lamb waves contain distinct symmetric and anti-symmetric wave modes, as shown in Figure 2, and the number of Lamb wave modes that can be propagated increases with frequency. Each mode has a particular cut-off frequency below which the mode will not be present. This cut-off frequency is dependent on the velocities of the longitudinal and transverse waves. [4], [7]

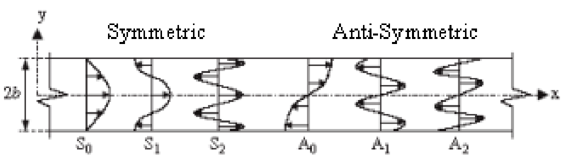

Figure 2-The first few fundamental lamb wave modes for propagation within a plate of thickness 2b [4],[7]

The Raleigh-Lamb frequency equation for propagation of symmetric waves in a plate is given as:

$$\frac{\tan \beta b}{\tan \alpha b} = -\frac{4\alpha\beta\xi^2}{(\xi^2-\beta^2)^2} \qquad (1)$$

where b is the half-thickness of the plate, ξ (=2π/λ) is the wave number , α and β are given as follows:[4]

$$\alpha^2 = \frac{\omega^2}{c_1^2} - \xi^2 \qquad (2a)$$

$$\beta^2 = \frac{\omega^2}{c_2^2} - \xi^2 \qquad (2b)$$

in which ω is the angular frequency, c_1 and c_2 are the longitudinal and transverse wave velocities respectively given by [4]

$$c_1 = \sqrt{\left(\frac{1-\nu}{1-\nu-2\nu^2}\right)\frac{E}{\rho}}, \qquad c_2 = \sqrt{\frac{G}{\rho}} \qquad (3)$$

where E and G are the Young's and shear module of the plate material respectively, ρ is the density and ν is the Poisson ratio. Likewise, the Raleigh-Lamb frequency equation for the propagation of anti-symmetric waves in a plate is given as:

$$\frac{\tan \beta b}{\tan \alpha b} = -\frac{(\xi^2-\beta^2)^2}{4\alpha\beta\xi^2} \qquad (4)$$

The group velocity, c_g of the symmetric and anti-symmetric wave propagation in the plate is given by:

$$c_g = \frac{d\omega}{d\xi} \qquad (5)$$

Figure 3 shows the group velocity dispersion curves for aluminum.

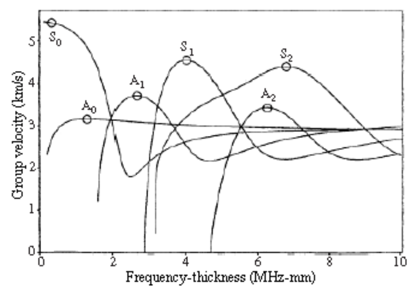

Figure 3- The group dispersion curve for aluminum plate

As it is seen in Figure-3, there are multi modes of Lamb waves and to avoid complications, selective generation of modes is necessary. To achieve this, a Lamb wave with a narrow band actuation pulse, Figure 4, is applied to the actuator based on the following equation:

$$X(t) = \cos(2\pi\omega t)e^{-a(t-t_0)^2} \qquad (6)$$

where ω is the actuation frequency (Hz), and a and t_0 are constants.

There are several ways for lamb wave generation. Figure 5 shows these methods. The first one which is shown in figure *5-a* is based on using Y-cut piezoelectric transducers. In this case applying voltage to the piezoelement causes horizontal deformation of the piezoelement and consequently the host structure. The second one *(5-b)* is based on using X-cut piezoelements while placing them with a predefined distance (which is the wavelength of propagated wave). Firing the transducer causes repeated pulses act in the structure and this yield the wave to propagate. The third method is using inclined probe to produce lamb wave. [8]

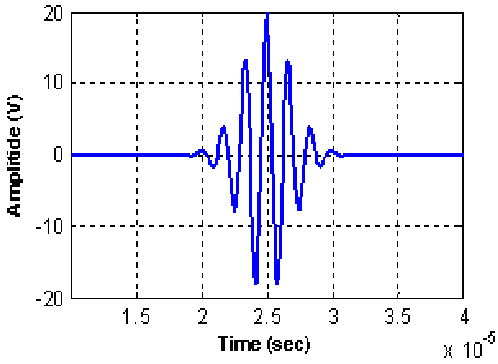

Figure 4- The actuation pulse [4]

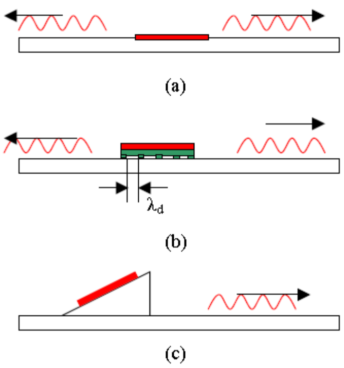

Figure 5- Methods for lamb wave generation. a) using Y-cut piezoelements.b) using X-cut piezoelements with predefined spacing c) using inclined piezoelectric probe.[8]

The frequency of the actuation pulse plays a very vital role here. It should be selected in a way that the least possible modes will be excited. For example actuation frequencies less then 0.8 MHz for an aluminum plate of thickness 2 mm limit the analysis to only the fundamental Lamb modes S_0 and A_0. By estimating the flight path due to either S_0 or A_0 propagation, the distance traveled by the wave from the actuator to the sensor can be computed as

$$l_i = \Delta t_i c_g \tag{7}$$

where Δt_i is the time of flight referenced from the actuation time, and c_g is the group velocity of either the S_0 or A_0 mode.[4]

3-Hilbert Huang Transform

One of the main concepts in detecting damage in structures is to focus on a sudden change in system response. A sudden change in the output signal of a system could be resulted from a sudden change in the structure that could be a crack, a hole or etc.

But instantaneous changes in signals are equivalent to a high frequency appearance in the signal. In HHT the concept is to find instantaneous frequency of a non-stationary signal. Simply a non-stationary signal is a signal that its frequency varies from time to time.

Historically Fourier analysis has provided a general method for examining the global energy-frequency distributions. Although the Fourier transform is valid under extremely general conditions, there are crucial restrictions of the Fourier spectral analysis: the system must be linear; and the data strictly periodic or stationary; otherwise, the resulting spectrum will make little physical sense. [9]

One could employ Hilbert transform to define instantaneous frequency as follow:

For an arbitrary time series, X(t) we can always have its Hilbert transform , Y(t), as[9]

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt', \tag{8}$$

where P indicates the Cauchy principal value. This transform exists for all functions of class L^p. With this definition, X(t) and Y(t) form the complex conjugate pair, so we can have an analytic signal, Z(t), as

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)}, \tag{9}$$

in which

$$a(t) = \left[X^2(t) + Y^2(t)\right]^{1/2},$$
$$\theta(t) = \tan^{-1}\left(\frac{Y(t)}{X(t)}\right) \tag{10}$$

Essentially equation (8) defined the Hilbert transform as the convolution of X (t) with 1/t; therefore it emphasizes the local properties of X (t). [9]

The instantaneous frequency then could be defined as

$$\omega = \frac{d\theta}{dt} \tag{11}$$

But there is still considerable controversy in defining the instantaneous frequency as equation (11). The instantaneous frequency given by equation (11) is a single valued function of time. This leads to introduction of *'monocotmponent function'*. At any given time there is only on frequency value. Unfortunately, no clear definition of monocomponent signal was given in literature to judge whether a signal is or is not monocomponent. So '*narrow band*' was adapted as a limitation on a data for the instantaneous frequency to make sense. According to Huang *et al* (1998) two necessary conditions for a signal to have meaningful instantaneous frequency is that 1) The signal should be symmetric with respect to the local zero mean and 2) It should has the same number of zero crossings and extreme. [9]

Based on the stated conditions, Huang et al (1998) introduced the Intrinsic Mode Functions (IMFs) for a signal. The main characteristic of IMFs are that (1) in whole data set the number of extreme and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local minima and envelope defined by the local maxima is zero.[9]

To define the instantaneous frequency one should first decompose the signal into its IMFs using a procedure called 'sifting process" which was introduced by Huang et al (1998). Having obtained the intrinsic mode functions components, there would be no difficulty in applying the Hilbert transform to each component and computing the instantaneous frequency according to equation (11). After performing the Hilbert transform on each IMF component, the data could be expressed in the following form [9]

$$X(t) = \sum_{j=1}^{n} a_j(t) \exp\left(i \int \omega_j(t) dt\right) \qquad (12)$$

As it seen, the IMF represents a generalized Fourier expansion in witch both magnitude and frequency could be functions of time. [9]

Equation (12) enables us to represent the amplitude and the instantaneous frequency as functions of time in three dimensional plots, in which the amplitude can be contoured in frequency-time plane. This frequency-time distribution of the amplitude is designated as the Hilbert amplitude spectrum, *H (ω,t)*, or simply Hilbert spectrum.

With Hilbert spectrum defined, one can also define the marginal spectrum h (ω), as [9]

$$h(\omega) = \int_0^T H(\omega,t)dt. \qquad (13)$$

In addition to marginal spectrum, it is also possible to define the instantaneous energy density level, IE, as [9]

$$IE(t) = \int_{\omega} H^2(\omega,t)d\omega \qquad (14)$$

Energy density level, IE, will be used to find the incident and reflection peaks.

4-Finite Element Model

ABAQUS FE software has been used to model the system shown in figure-1. A spacing of 100-141.4 mm between the actuator and sensor was assumed. Figure 6 shows the FE model of the system in which the crack and PZTs are positioned in a square with 100 *mm* spacing.

There are two basic FE methods for modeling dynamic problems; implicit dynamic procedure and explicit dynamic. Piezoelectric elements are just available in implicit dynamic procedure in ABAQUS but it has been shown in [10] that implicit dynamic procedure is not suitable for wave propagation modeling and causes inaccurate results.

On the other hand, explicit dynamic is able to model the wave propagation problem accurately. But the explicit dynamic procedure in ABAQUS has a big shortcoming which is lack of piezoelectric elements. In order to model the wave generated by a piezoelectric transducer a suitable model of piezoelectric actuator and sensor is required.

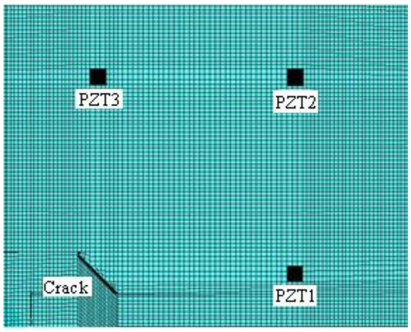

Figure 6- FE model of the system

One choice of actuation pulse in explicit dynamic procedure is applying displacement to related nodes in contact with piezoelements. Since displacement based finite element method has been used here, applying voltage to the piezoelectric transducer causes displacement in slave nodes on the aluminum plate. Thus the related displacement caused by piezoelement can be applied instead of voltage. According to [10] and, one of the other suitable substitutions for piezoelectric elements is the pin-force model which has been used for the modeling of piezoelectric actuators on beams [11]. This actuation force has been used in [10] and for crack detection using lamb wave method. But in those cases the lamb wave has been modeled in one direction and there were no two dimensional lamb wave modeling. Having fired the actuator with the horizontal force, there was no suitable sensor response and wave did not reflect in two dimensions. To model two-dimensional lamb wave propagation, a vertical force has been used. Based on the expected lamb mode, symmetric or anti-symmetric, the forces could be applied symmetrically or anti-symmetrically. Figure 7 shows two basic lamb modes, S_0 and A_0.

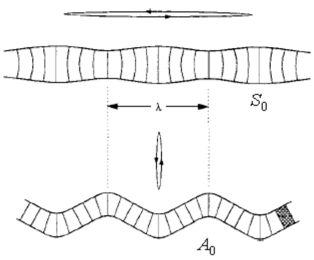

Figure 7- Two basic lamb modes, S_0 and A_0

To excite the A_0 mode, two vertical unidirectional concentrated forces have been applied on both sides of plate. For excitation of the S_0 mode, forces were used in the opposite direction with each other. Figure 8 shows the way S_0 and A_0 modes have been excited. Both of the forces had the magnitude of signal shown in figure 4.

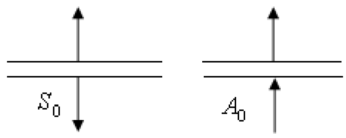

Figure 8- Excitation of S_0 and A_0 modes

Actuation signal shown in figure 4 have been employed. Figure 9 shows response of PZT2 while firing PZT 1. The vertical displacement has been read as the sensor response. Also response of PZT3 while firing PZT1 and PZT2 is shown in figures 10 and 11, respectively.

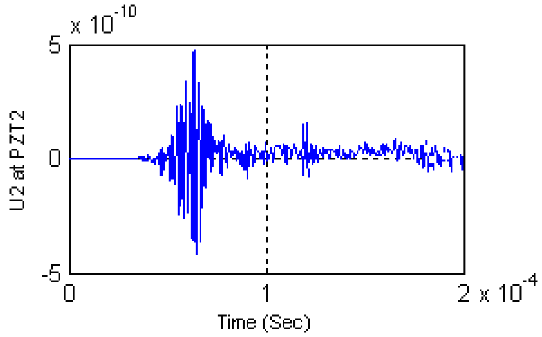

Figure 9- Response of PZT2 caused by PZT1 firing

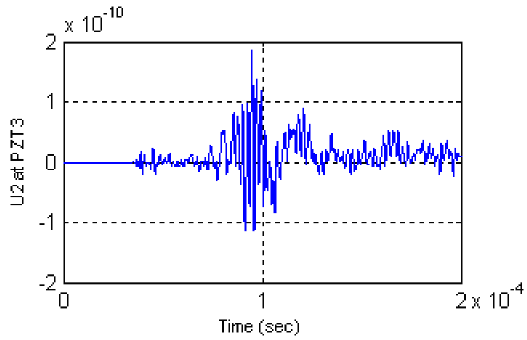

Figure 10- Response of PZT3 caused by PZT1 firing

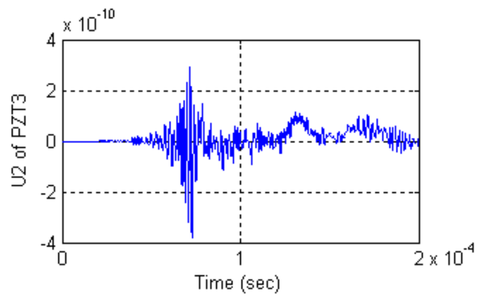

Figure 11- PZT3 response caused by PZT2 firing

These three signals have been processed using HHT; and their Energy density level versus time has been obtained. Figures 12 through 14 show the related energy density level versus time.

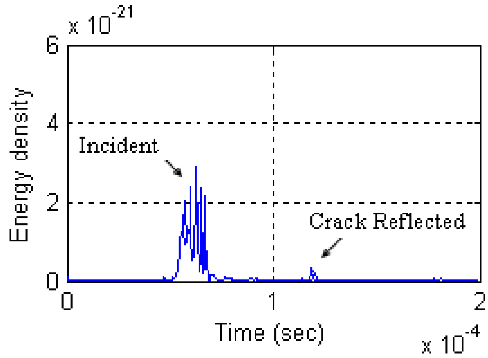

Figure 12- Energy density of PZT2 response while PZT1 fired.

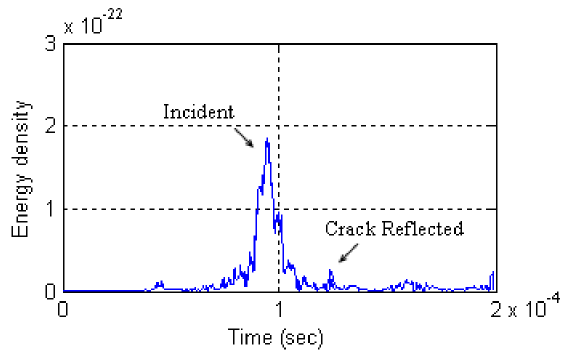

Figure 13- Energy density of PZT3 response to PZT1 actuation

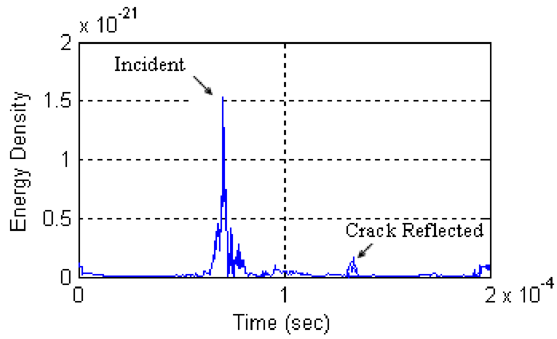

Figure 14- Energy density of PZT3 response to PZT1 actuation

According to figures 9 to 14 the wave speed which was obtained from FE modeling was less than theoretical value. The theoretical values of S_0 and A_0 lamb modes are 4916 and 3143 ms^{-1} respectively. But the wave speeds obtained from numerical modeling of the problem with ABAQUS were 4637 and 2673 ms^{-1}. The S_0 modes velocity is close to theoretical value with 6% error but the A_0 mode velocity has more than 10% error. On the other hand the A_0 mode is reflected from discontinuities better than S_0 mode. So the excitation forces have been selected in order to A_0 mode generation. Although the wave speed did not completely match the theoretical value, the crack location was found accurately. The value $a+b$ shown in figure 1, have been obtained 238, 203 and 242 mm for pairs PZT1-PZT2, PZT1-PZT3 and PZT2-PZT3 respectively.

6-Conclusions

The experimental work done by Tua *et al (2004)* was numerically simulated, extended and verified. It has been shown that implicit dynamic procedure is not suitable for wave propagation modeling. An Explicit dynamic procedure demonstrated a significant improvement with high accuracy results. In the Explicit dynamic model of the beam, vertical concentrated forces have been used to excite S_0 and A_0 lamb modes. Also it has been shown that the S_0 mode did not reflect from the crack as much as A_0 mode. The obtained wave velocity for S_0 mode matches theoretical and experimental work closely but the A_0 one has some error. HHT method was employed to find the instantaneous frequencies of the signal. Using energy density level, the incident and reflected waves were detected. Although the wave velocity of propagated wave has some error, the location of the crack was accurately detected using time-of-flight of waves. Simulation results of the explicit FE matched closely the experimental results.

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