

# An Experimental Research on Three-Dimensional Waves in a Concrete Panel

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**Abstract** - An experimental verification of three-dimensional waves is performed in a concrete panel. Frequency-controlled method is applied to guided wave experiments to induce the central frequency as high as 25 kHz in the concrete panel. The measured acceleration waveforms are transformed into spectrograms with the joint time-frequency analysis. A mode identification method is developed to find the induced wave modes. Several wave modes on the lower branches are identified on a group velocity-frequency plot by comparing to the predictions of theoretical dispersive group velocity curves. The wave modes on the lower branches with simple displacement profiles are easily induced and detected in the guided wave experiments.

**Keywords:** Three-dimensional wave, frequency-controlled method, mode identification, joint time-frequency analysis

## 1. Introduction

The as-built condition of diaphragm walls or pile foundations can affect the performance of a structure supported by these elements. The quality evaluation of foundations of infrastructures or buildings is most commonly performed by the non-destructive testing (NDT) techniques based on the use of stress waves. These NDT techniques are more economical and more conveniently performed than the traditional semi-destructive *in-situ* testing techniques, such as pile loading capacity tests [1].

Waves traveling in diaphragm walls or piles include different clusters of waves, usually named guided waves. These three-dimensional (3-D) wave clusters arise from the incidences and reflections of a variety of compression, shear, and surface waves along the boundaries of foundation structures. The wave velocity is a function of frequency, and the displacement magnitudes vary along the wall or pile cross-section. The assessment of existence and types of guided waves propagating in prototype piles has been verified recently [2-4]. Presumably, the applicability of guided waves can be extended to the non-destructive evaluations of diaphragm walls by inducing stress waves with high frequencies.

This research provides an experimental verification of existence of three-dimensional (guided) waves in an intact concrete panel. To simulate guided waves propagating in diaphragm walls, the dimension characteristics of a concrete panel is determined by referring to relevant information. Frequency-controlled method is performed in guided wave experiments to induce guided waves in a panel. The measured acceleration signals in the time domain are transformed into spectrograms with the joint time-frequency analysis. Using a group velocity-frequency plot is an efficient method to identify the induced wave mode attribute.

## 2. Characteristics of 3-D Waves Propagating in a Panel

A three-dimensional wave propagating in a traction-free plate typically is called Rayleigh-Lamb waves (or Lamb waves). A panel extends infinitely in the  $y$  and  $z$  directions and has thickness  $2b$  in the  $x$  direction, as shown in Figure 1.

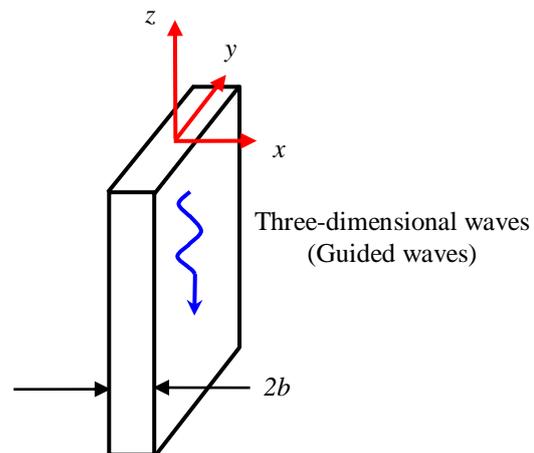


Figure 1: Schematic representation for three-dimensional (guided) waves propagating in a panel

The key physical property of three-dimensional waves is group velocity from a measurement perspective for the velocity of energy transportation and important physical quality [2, 5-8]. The solutions of dispersive group velocities for the first four branches of Rayleigh-Lamb waves in a panel are illustrated in Figure 2. Non-dimensional group,  $\underline{C}_g$  is defined as



$$\underline{C}_g = \frac{C_g}{C_s} \quad (1)$$

where  $C_g$ , is the group velocity and  $C_s$  is the shear wave velocity. The definition of non-dimensional frequency,  $\Omega$ , is

$$\Omega = \frac{\omega 2b}{\pi C_s} \quad (2)$$

where  $\omega$  is the angular frequency. Rayleigh-Lamb waves are labeled as aRL and sRL, where “a” refers to an anti-symmetric mode and “s” refers to a symmetric mode, and followed by order number.

All branches of group velocity curves are dispersive (see Figure 2). The lowest anti-symmetric mode, aRL0, starts from zero, but reaches a narrow plateau around the shear wave velocity and then gradually decreases to the Rayleigh wave velocity as frequency increases. The lowest symmetric mode, sRL0, starts from the plate wave velocity and remains constant in a narrow range at low frequencies. Its group velocity falls to a minimum value

and then gradually increases and approaches the Rayleigh wave velocity at higher frequencies.

The group velocities of branches, aRL1 and sRL1, start from zero, but reach a narrow plateau below their plate wave velocity. At frequencies higher than the plateau, group velocity steeply drop to a minimum value, then gradually increases and approaches the shear wave velocity.

The position,  $x$ , to set accelerometers on the panel top is normalized by

$$T = \frac{x}{b} \quad (3)$$

where  $T$  is the non-dimensional position to the central line on the panel top. The range  $-b \leq x \leq b$  may be replaced by  $-1 \leq T \leq 1$ . Relevant guided wave experiments show that the induced wave modes, such as the modes on the lower branches, with simple displacement profiles are easily excited and detected during testing [2-4].

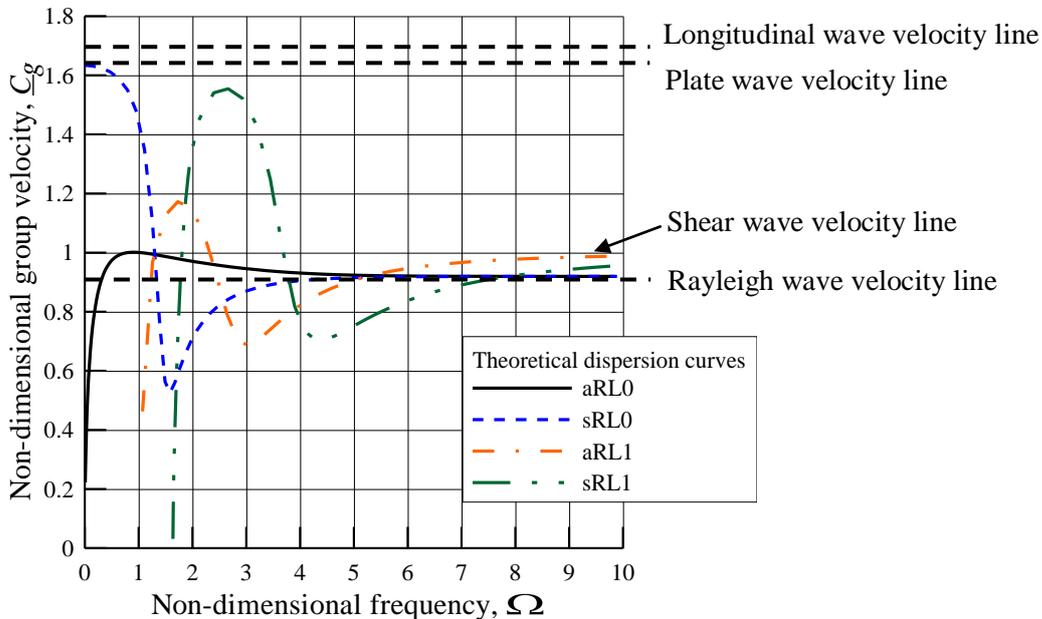
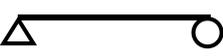


Figure 2: Variations of non-dimensional group velocity with non-dimensional frequency for first four branches of three-dimensional waves in a panel

Table 1: Minimum thickness of one-way construction slabs in the ACI 318-95

| Boundary condition | Simply supported  | Cantilever  | One end continuous   | Both ends continuous  |
|--------------------|---|---|--|---|
| Diagram            |  |  |  |  |
| Minimum thickness  | $L/20$  | $L/10$  | $L/24$   | $L/28$  |

### 3. Determination of Panel Dimension

To simulate waves propagating in structural diaphragm walls, the dimension characteristics of a concrete panel are referred to the industrial concrete design codes, design

cases of diaphragm walls, and concrete slabs/walls used in laboratory non-destructive testing.

The minimum thickness of one-way construction slabs in the ACI 318-95 of building code requirements for structural concrete is listed in Table 1 [9]. The code gives the minimum thicknesses for a concrete slab span,  $L$ , for four boundary conditions, simply supported, cantilever,



one end continuous, and two ends continuous. The minimum value, 10, of length-to-thickness ratio in the cantilever boundary condition can be regarded as the lower bound for the concrete panel in this research.

Several design types of constructed diaphragm walls, in practice, have been developed for several factors, such as site foundation situation, excavation size, construction feasibility, and financial considerations [10]. The dimension characteristics of diaphragm walls related to this research are listed in Table 2 [11]. To avoid unwanted wave reflections from the lateral support system during testing, a diaphragm wall without lateral support system is chosen as the simulation object. The dimension characteristics, such as length-to-thickness and width-to-thickness ratios, range from 11 to 23 and from 2.67 to 7.44, respectively.

A number of concrete slabs/walls have been performed for laboratory non-destructive testing, as listed in Table 3

[11]. A slab/wall with thickness less than 0.2 m is generally used for thickness measurement. The value of the length-to-thickness ratio ranges from 5 to 7. The width-to-thickness ratio has a wide range from 5 to 12. A slab/wall with thickness more than 0.2 m is used to simulate intended flaw detection. The length-to-thickness ratio is various from 5 to 12.5. Its width-to-thickness ratio ranges from 3.75 to 5.

For offering a proper space to set a shaker and two triaxial accelerometers at the panel top, the minimum thickness of a panel is selected as 76.2 mm (3 in.). The length-to-thickness ratio is selected as 15 for the casting feasibility in laboratory and the general design of diaphragm walls. The designed width-to-thickness ratio is selected as 6 for reducing signal noises reflected from the two edges of the panel. The geometric dimension of the concrete panel is 1148 mm (45.2 in.) in length, 45.7 mm (1.8 in.) in width, and 76.2 mm (3 in.) in thickness.

Table 2: Dimension characteristics for precast and *in-situ* diaphragm walls [11]

| Diaphragm wall type   | Precast diaphragm wall    |                          | <i>In-situ</i> diaphragm wall |
|---|---------------------------|--------------------------|-------------------------------|
| Support system/excavation                                   | Length-to-thickness ratio | Width-to-thickness ratio | Length-to-thickness ratio     |
| No lateral support system and shallow excavation            | 11-23                     | 2.67-7.44                | 10-25                         |
| Tied-back or internal support system and typical excavation | 18-42                     | 3-9.13                   | 25-40                         |
| Double support system or/and large-size excavation          | Not available             | Not available            | >40                           |

Table 3: Dimension characteristics of concrete slabs/walls used in laboratory [11]

| Slab/wall type    | Length-to-thickness ratio | Width-to-thickness ratio |
|-------------------|---------------------------|--------------------------|
| Thickness < 0.2 m | 5-7                       | 5-7                      |
| Thickness > 0.2 m | 5-12.5                    | 3.75-5                   |

## 4. Design of Guided Wave Experiments

### 4.1 Instrumentation

The equipment arrangement for guide wave experiments on a concrete panel is shown in Figure 3. The portable computer controls the system and processes the guide wave experiments in a LabVIEW<sup>®</sup> environment. A shaker vertically mounted on the central point of the concrete panel top generates stress waves. The vibration responses of the concrete panel are measured by two triaxial accelerometers, T1 and T2, set at two selected positions  $T=0$  and 0.71, respectively, on the panel top close to the shaker position. All input and vibration response signals are recorded, displayed, and analyzed in the portable computer.

A vibration shaker, Model F7, manufactured by Wilcoxon Research Inc. performs the pulse source for the high frequency guide wave experiments. The shaker is driven by high voltage power supply system, including power amplifier, Model PAC8C, and matching network, Model N8L, manufactured by Wilcoxon Research Inc. A transducer base, Model Z7, is embedded in the shaker

head. The transducer base contains a force gage and an accelerometer to measure the force applied to the concrete panel and the resulting acceleration.

The two triaxial accelerometers, PCB Piezotronics IMI Model 629A11, are affixed on bases mounted to the concrete surface by using epoxy for higher frequency measurement [6]. The triaxial accelerometers measure the vibration responses in the three mutually perpendicular directions individually. All signal sources, including the shaker and triaxial accelerometers, are connected to a signal conditioner. A signal conditioner, PCB-482 A20, is used as a signal amplifier and power source for the accelerometers. A connection block, BNC 2080 board, provides analog/digital signals conversion for later processing. Through a general purpose interface bus (GRIB) cable, all measured signals are transported, in a high speed dataflow, into the data acquisition (DAQ) card, National Instruments NI-PCI-MID-16E-U, set in the portable computer.

The PC-based program performs 8-channel triggered data acquisition when the shaker is used as the vibration source. The sampling rate is set as 0.1 MS/sec and the recorded period automatically lasts 40.96 msec. All

measured average time domain waveforms are displayed on virtual oscilloscopes and stored in a text-formatted file.

## 4.2 Preparation and Physical Properties of a Concrete Panel

The operation of the guided wave experiment is demonstrated on a selected concrete panel. The panel was vertically cast with a water-cement ratio of 0.52, a fine aggregate-cement ratio of 1.74, and a coarse aggregate-cement ratio of 2.25 by following ASTM C192-90a [12]. Concrete parameters are identified experimentally with traditional non-destructive testing. The density of concrete is around  $2400 \text{ kg/m}^3$ . Computed from the impact echo testing results, the longitudinal wave velocity in concrete is  $4382 \text{ m/sec}$ . Based on a curve fitting process on a phase-velocity-versus-frequency plot, the suitable Poisson's ratio value of concrete is 0.25 [6]. Consequently, the shear wave velocity,  $C_s$ , is  $2530 \text{ m/sec}$ .

## 4.3 Frequency-Controlled Method

The potential of guided wave theory has been experimentally verified to apply to prototype concrete piles with traction-free and embedded conditions [2-4]. The excited wave modes are identified at certain frequency ranges. To induce waves with a specific frequency into a concrete panel, the central frequency should be forecast reliably. The measurement positions of vibration responses at a panel surface should be selected at locations with a simple displacement pattern and a higher relative displacement magnitude.

In Figure 3, the input waveforms are waves with central frequencies of  $f_c=14$  and  $25 \text{ kHz}$ . The vibration responses of the panel are measured by two triaxial accelerometers set at positions of  $T=0$  and  $0.71$ , with the distances of  $0$  and  $27.05 \text{ mm}$  from the central line, respectively, on the panel top. The measurement positions of two triaxial accelerometers are chosen near the shaker for evaluating the reflections from the two edges of the panel. The input force and resulting acceleration signals measured by the shaker and the responding acceleration signals measured by the two triaxial accelerometers are transmitted and stored in a computer.

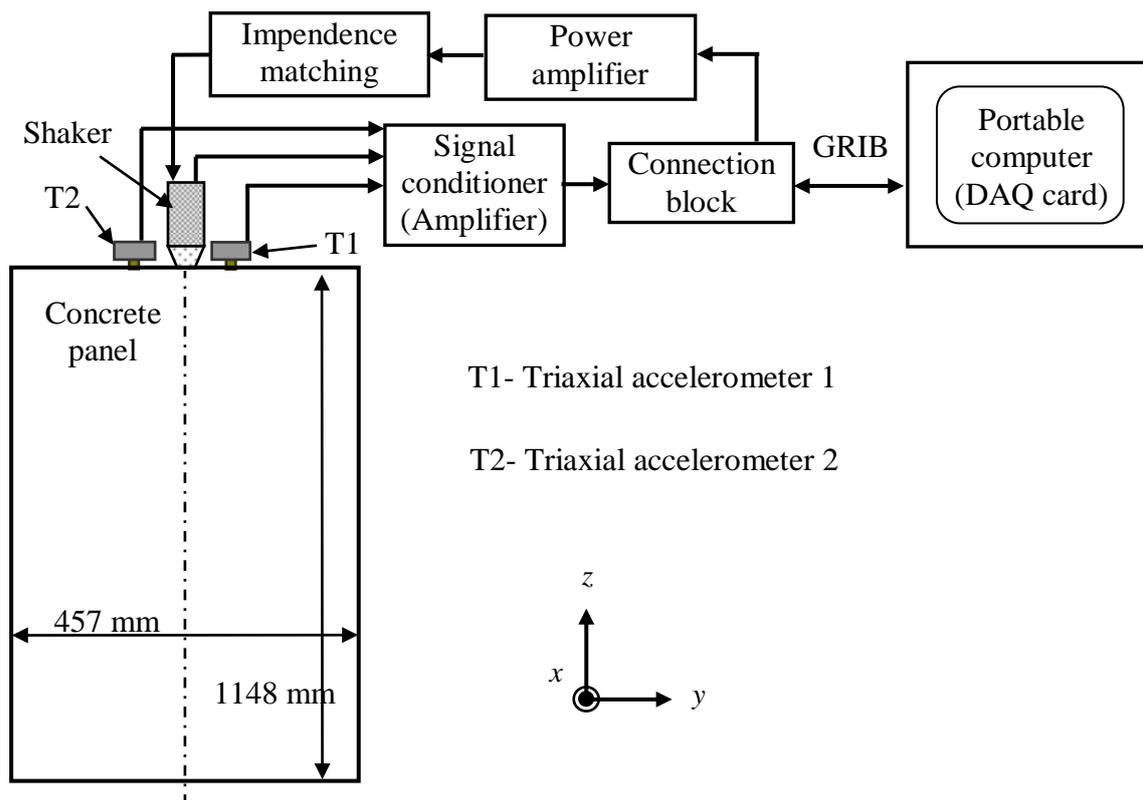


Figure 3: Equipment arrangement of guided wave experiments on a concrete panel (not in scale)

## 5. Results of Guided Wave Experiments

### 5.1 Joint Time-Frequency Analysis

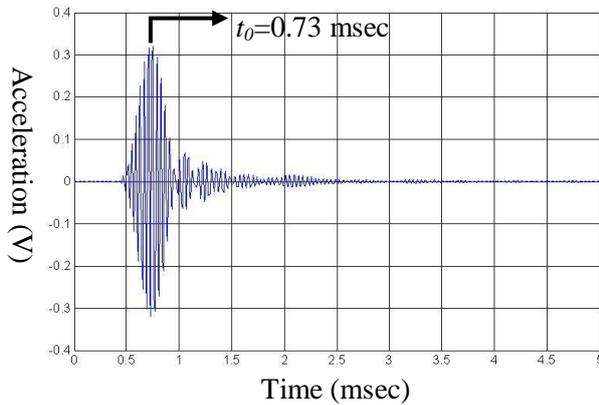
Figure 4 shows the results of the guided wave experiment at central frequency of  $f_c=25 \text{ kHz}$  ( $\Omega=1.51$ ) with frequency-controlled method. The input acceleration waveform in Figure 4 (a) is the waveform generated by a shaker. The surface vibration responses in the  $z$  and  $x$  directions shown in Figure 4 (c) and (e), respectively, are measured at  $T=0.71$  by a triaxial accelerometer. The  $z$ -



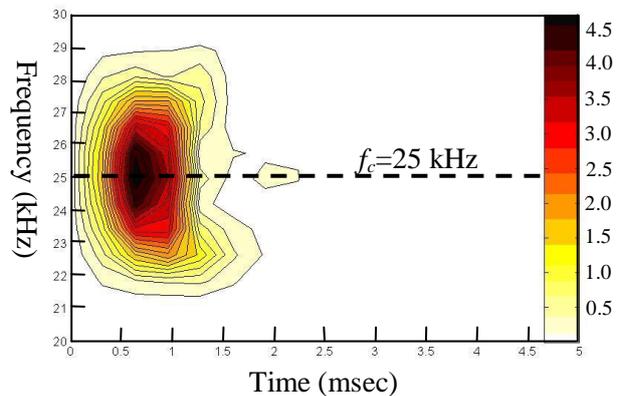
direction vibration response represents the responses parallel to the excitation direction. The x-direction vibration response represents the out-of-panel responses. These time-domain waveforms are subsequently transformed into the joint time-frequency domain. The spectrograms shown in Figure 4 (b), (d), and (f) are the results of waveform transformation for the input, z-direction, and x-direction accelerations, respectively. The spectrograms presenting as intensity distributions in the joint time-frequency domain are the frequency content of the input and responding vibrations with time. The gray-scaled intensity distribution is described for the magnitude of the response—the darker the shading, the larger the magnitude of the response.

The input acceleration wave packet in Figure 4 (a) is the waveform averaged from 20 repetitive waveform excitations each composed of 8-cycle square waves with central frequency of  $f_c = 25$  kHz. The highest intensity of the frequency content is identified as around 25 kHz in the joint time-frequency domain in Figure 4 (b).

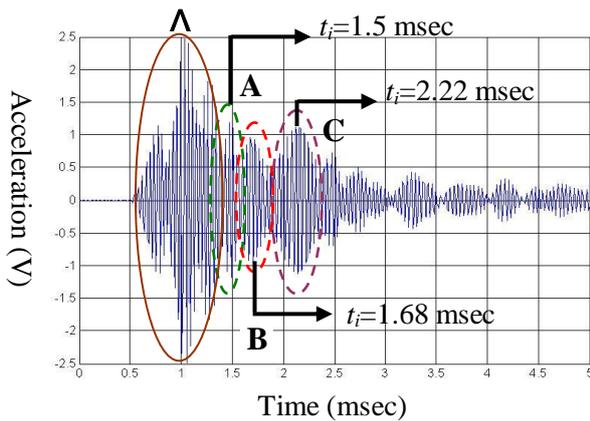
The first arrival wave packets  $\Lambda$  observed in the surface vibration responses shown in Figure 4 (c) and (e) are usually the waveforms superimposed by the input waves, surface waves, and the waves reflected from the panel edges. The wave packets following the first arrival wave packets  $\Lambda$  are defined as the primary reflection wave packets, such as wave packets A, B, C, D, and E. The primary reflection wave packets are composed of the 3-D waves reflected from the panel bottom and the second reflection waves from the panel edges. Because of the major excitation direction parallel to the z direction, the measured maximum magnitude of the responding vibration in the z direction is apparently higher than that in the x direction. The intensity distributions of the frequency content in the joint time-frequency domain are found to be around 25 kHz in the spectrograms shown in Figure 4 (d) and (f). The values of the central frequency of the primary reflection wave packets may be slightly different from the central frequency of the excitation.



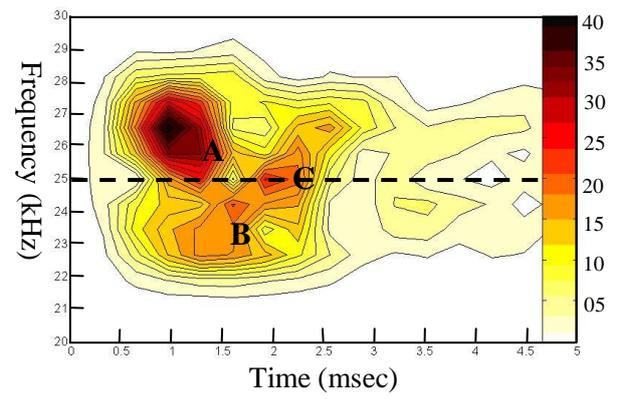
(a)



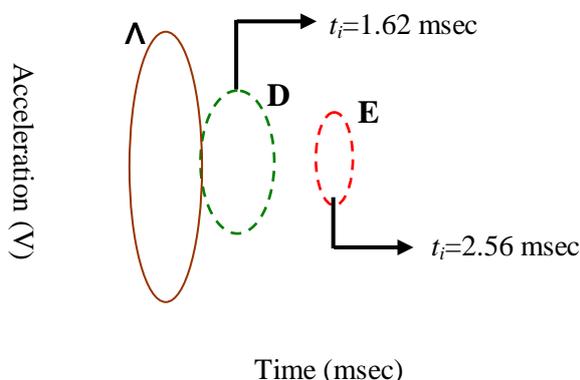
(b)



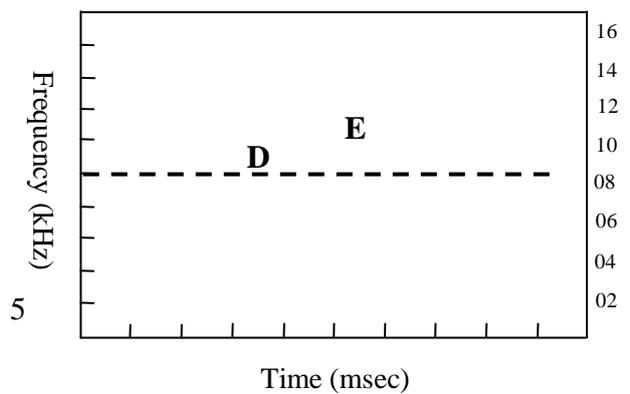
(c)

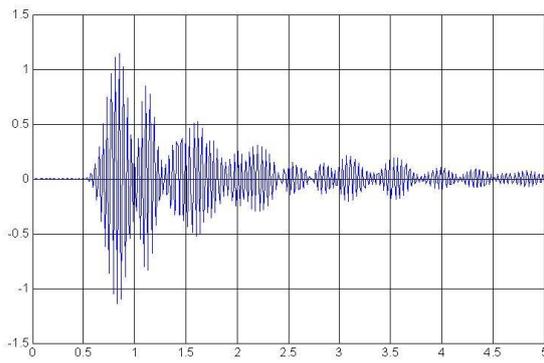


(d)

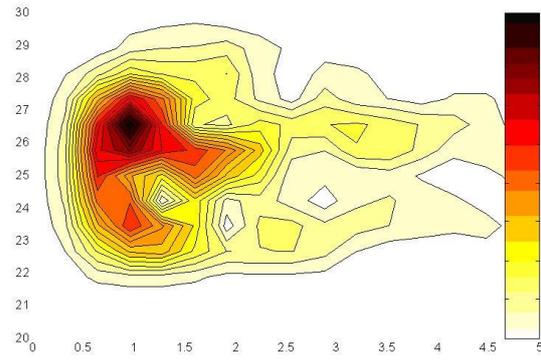


(e)





(e)



(f)

Figure 4: Results of guided wave experiment at central frequency  $f_c=25$  kHz ( $\Omega=1.51$ ) in a concrete panel

## 5.2 Mode Identification

In Figure 4 (c) and (e), the primary reflection wave pockets are consisted of the guided wave, a wave guide confined along the concrete panel and traveling between the panel top and bottom, and the second reflection waves from the panel edges. It is not easily to identify the wave attribute by completely separating the primary reflection wave pocket into several individual wave pockets. A mode identification method is developed to circumvent the problem by using the theoretical group velocity-frequency plot.

The procedures of the mode identification method are described in details to find the induced guided wave modes. The reference time,  $t_0$ , of excitation is first found in the input acceleration wave packet by taking the time corresponding to the peak value of the wave packet. The individual arrival time,  $t_i$ , of an induced guided wave mode is found by taking the time corresponding to the peak value of the individual wave packet. The computed group velocity,  $C_i$ , of an individual guided wave mode is

$$C_i = \frac{2 \times l}{t_i - t_0} \quad (4)$$

where  $l$  is the panel length. The corresponding central frequency is also identified from the spectrogram. Knowing central frequency and individual group velocity, the induced wave mode is marked on a group velocity-frequency plot. By comparing its coordinate position to theoretical dispersive group velocity curves, the individual wave mode is identified.

In Figure 4 (a), the reference time,  $t_0$ , of excitation is 0.73 msec. In Figure 4 (c), the arrival times of induced wave modes measured in the z direction are 1.5, 1.68, and 2.22 msec for wave pockets A, B, and C, respectively. The computed group velocities are 2982, 2417, and 1541 m/sec, respectively, by using equation (4). Their corresponding central frequencies are around 25.7, 24.2, and 25 kHz, respectively, as shown in Figure 4 (d). In Figure 4 (e), the arrival times of induced wave modes measured in the x direction are 1.62 and 2.56 msec for wave pockets D and E, respectively. The computed group velocities are 2580 and 1255 m/sec, respectively. Their corresponding central frequencies are around 25.7 and 26.5 kHz, respectively, as shown in Figure 4 (f).

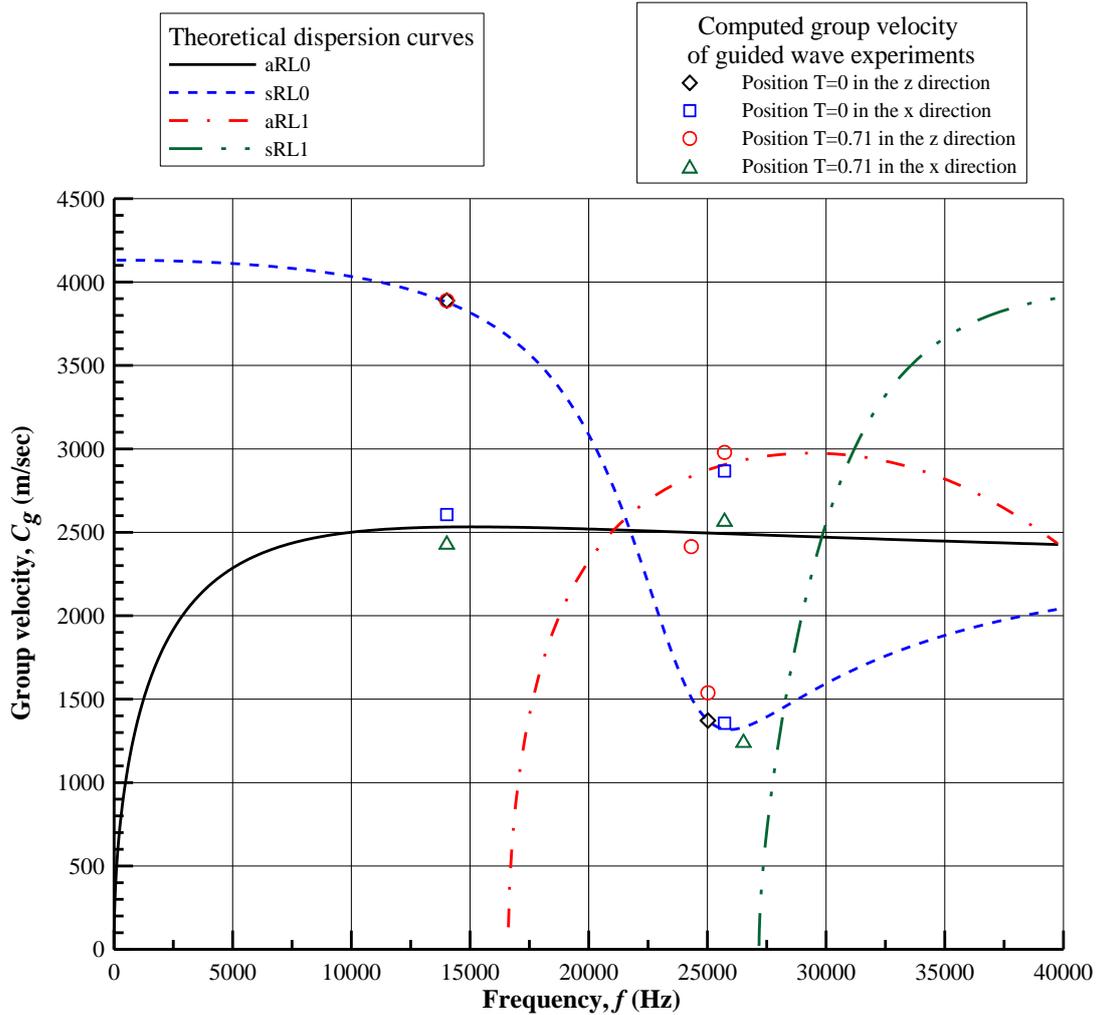


Figure 5: Guided wave experimental results and theoretical group velocity curves on a group velocity-frequency plot

Figure 5 shows the all computed group velocities at measurement positions  $T=0$  and  $0.71$  with excited central frequencies  $f_c=14$  and  $25$  kHz and corresponding guided wave modes superimposed on a theoretical group velocity-frequency plot. The guided wave modes for wave pockets A, B, and C represent the modes on branches aRL1, aRL0, and sRL0, respectively. The guided wave modes for wave pockets D and E represent the modes on branches aRL0 and sRL0, respectively. The induced wave modes in a concrete panel are identified on the lower wave branches, such as aRL0, sRL0, and aRL1. The experimental results meet the conclusions of relevant experimental results and theoretical evaluations [2-4, 8]. Induced wave modes on the lower branches, with simple displacement profiles, are easily excited and detected in guided wave experiments.

## 6. Conclusions

This research provides an experimental verification of three-dimensional (guided) waves propagating in a concrete panel. To simulate waves propagating in structural diaphragm walls, the dimension characteristics

of a concrete panel are referred to the relevant information. Frequency-controlled method is performed in the guided wave experiments to induce guided waves at central frequencies  $f_c=14$  and  $25$  kHz in a concrete panel. The vibration responses of the panel are measured with two triaxial accelerometers at positions of  $T=0$  and  $0.71$  of the panel top. The group velocities of guide waves at different central frequencies are computed from the measured acceleration signals. The time-domain waveforms are transformed into the intensity-time-frequency spectrograms with the joint time-frequency analysis. A mode identification method is developed to identify the induced wave mode attribute. The guided wave modes on the lower branches are identified on a group velocity-frequency plot by comparing to the theoretical dispersive group velocity curves. The guided wave modes with simple displacement profiles are easily excited and detected in the guided wave experiments.

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