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Novel optimisation of bicoherence estimation for fatigue monitoring

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The bicoherence method is widely used for nonlinearity and non-Gaussianity monitoring in many engineering applications, including damage monitoring in aerospace applications, for example fatigue monitoring in aircraft structures and gear teeth checking in helicopter transmissions. A size optimisation of the time segment for bicoherence estimation for fatigue monitoring is investigated here for the first time, it is claimed. The existence of optimal ranges of segment sizes is shown. These findings are in contrast to traditional bicoherence estimation in which the unique optimal value of segment size is employed.

Keywords: Damage monitoring, bi-linear systems, bicoherence, fatigue.

Introduction

The higher order spectra of order 3, *ie* the bispectrum and the normalised bispectrum, the bicoherence, are widely used for many engineering applications for nonlinearity and non-Gaussianity monitoring[1-9]. In particular, the bicoherence method has been applied for detection of nonlinearity and the quadratic phase coupling in waves[1], speech signals[2], control loops[3], fluids[4] and for condition monitoring of different types of machinery and structures[5-9] including damage monitoring in many aerospace applications, for example fatigue monitoring in aircraft structures and gear teeth analysis in helicopter transmissions[10, 11].

The most important advantages of the bicoherence are high sensitivity to the presence of system nonlinearity and non-Gaussianity and capability to suppress the Gaussian noise. One of the main disadvantages of bicoherence is a relatively high variance of bicoherence estimate; it is much higher than the variance of estimate of the power spectral density[12].

A common way to reduce the variance of bicoherence estimate is splitting signal duration into the segments, followed by averaging of the bicoherence over segments. For finite signal duration segment size defines the bias and the variance of bicoherence estimate. Following the literature[13], the usual practice for defining the optimal segment size is the square root of signal length. Usually, this rule is used for bicoherence estimation in damage monitoring tasks but some authors use rather empiric approaches to segment size selection[1, 6, 8, 9]. However, this rule is based on the optimal trade-off between the bias and the variance of bicoherence estimate and has been developed for the linearity test[13], which involves only one class of signals.

For damage monitoring, optimisation of segment size should be performed for two or multi-class diagnostics and should be based on optimisation criterion for monitoring effectiveness rather than on a trade-off between the bias and the variance of bicoherence estimate. Optimisation of the main bicoherence parameters for damage monitoring involving two or multi-class signals has been investigated in literature only to a limited extent. In particular, it is believed that no-one has optimised the segment size of bicoherence estimate for fatigue monitoring. In addition, no-one has investigated the influence of damage level on optimal segment size for fatigue monitoring.

The purpose of this paper is to investigate optimisation of segment size for bicoherence estimation for fatigue monitoring.

It is important to investigate this problem for fatigue monitoring for many engineering applications including aerospace applications.

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Optimisation of segment size for bicoherence estimation

For estimating the normalised higher order spectra of order 3, the bicoherence, the discrete time domain signal $y(l)$ should be divided into segments $y_i(l)$ by the time window $w_i(l)$, $l = 1,2,\ldots, l$ is the discrete sample number; $i = 1,\ldots,M$, M defines the total number of segments in the signal, $M = \left\lceil \frac{N}{n} \right\rceil$; N is a signal length, n is a segment length, $\lceil\ \rceil$ is a symbol of integer part of value.

The direct bicoherence estimate by segment averaging could be written as follows[14]:

$$b(k,l) = \frac{\sum_{i=1}^{M} X_i(f_1) X_i(f_2) X_i^*(f_1+f_2)}{\sqrt{\sum_{i=1}^{M} \left| X_i(f_1)\, X_i(f_2) \right|^2 \sum_{i=1}^{M} \left| X_i(f_1+f_2) \right|^2}} \qquad (1)$$

where $X_i(f)$ is the discrete Fourier transform of a segment of the time domain signal, f_1 and f_2 are discrete frequencies.

The frequency resolution of the Fourier transform is defined as follows:

$$\Delta f = \frac{f_s}{N} M \qquad (2)$$

where f_s is the sampling frequency.

Variance of bicoherence estimate (1) can be presented as follows[14]:

$$\operatorname{var}\{ b(f_1,f_2) \} \approx \frac{1}{M}\left[1 - b^2(f_1,f_2) \right] \qquad (3)$$

Normally, fatigue damage creates an additional level of nonlinearity in monitored systems[15]. Thus, a two-class diagnostics of system nonlinearity is considered for evaluation of the optimal segment size which maximises diagnostic effectiveness.

There are a number of complicated stiffness variations reported in the literature for modelling of fatigue damage in systems and structures. However, in applied engineering, owing to insufficient knowledge about the nature of the true stiffness variation due to the damage, the bilinear system with bilinear stiffness is widely used[15, 16] for investigation of fatigue damage.

Assuming that a monitored system is a single degree of freedom bilinear oscillator under the narrowband random excitation, equations of motion can be written as follows[10, 17, 18]:

$$\begin{cases} \ddot{X} + 2h\dot{X} + \omega_S X = A(t)\cos(\omega_f t + \varphi) & x \geq 0 \\ \ddot{X} + 2h\dot{X} + \omega_C X = A(t)\cos(\omega_f t + \varphi) & x \geq 0 \end{cases} \quad \text{..........(4)}$$

where $X = \frac{x}{m}$, x is displacement; $h = \frac{c}{2m}$, h is the damping; $\omega_S = \sqrt{\frac{k_S}{m}}$, $\omega_C = \sqrt{\frac{k_C}{m}}$, m and c are the mass and damping coefficient respectively; k_S and k_C are the stiffness for positive and negative displacements, $A(t)$ is the Rayleigh envelope of the narrowband Gaussian process, ω_f and φ are constant excitation frequency and random initial phase[18] uniformly distributed in the interval $[0; 2\pi]$.

The bilinear system (4) is widely used in aerospace applications, for example aircraft structures, engine blades[10, 16], helicopter gearboxes[11], etc. It is also widely used for investigation of sub-harmonic resonances of offshore structures: free-hanging risers, tension leg platforms and suspended loads[19-21], articulated loading towers, constrained by a connection to a massive tanker or vessels moored against fenders[22], oscillating parts with clearances and motion limiting stops[23].

Parameters used in simulation of the input signal and system response are as follows: the sampling frequency f_s is 5 kHz; the stiffness ratios $k_r = \frac{k_C - k_S}{k_C}$ are 0 (the linear system), 0.05 and 0.1 (the bilinear system); the resonance frequencies of the system are 500 Hz, 493.6 Hz and 486.8 Hz at the stiffness ratios 0, 0.05 and 0.1 respectively; damping h is 16 s^{-1}. The excitation frequency is equal to the resonance frequency of the system for both the linear and the bilinear systems.

The output signal at the resonance consists of the fundamental resonance harmonic (for the stiffness ratio 0, the linear system) and of the fundamental and the higher resonance harmonics (for the stiffness ratios 0.05 and 0.1, the bilinear system). Gaussian white noise was added to the output signal to hinder the monitoring of the nonlinearity and so to more closely mimic data from the early stage of damage development; the signal/noise ratio is 30 dB.

It is considered a two-class diagnostics of the stiffness ratio which characterise the level of the system bi-linearity (the relative damage level): $k_r = k_{rj}$ for class H_j, $j = 0, 1$. This two-class diagnostics is generic because it is independent of any particular correlation between the stiffness ratio and relative damage level.

Parameters and functions used for bicoherence estimation are as follows: segment size is changed in the range [50-1.5·10^5] (*ie* frequency resolution is changed in the range [100-0.033] Hz respectively); the time window is the Hamming window; the signal length is 5·10^5.

The bicoherence was estimated at the system resonance.

For segment optimisation, 100 simulation tests were performed for each value of the system bi-linearity and segment size. The Fisher criterion[24] of the monitoring effectiveness is employed for optimisation; it is known[24] that higher values of the Fisher criterion provide better monitoring effectiveness.

Changes in the Fisher criterion against relative segment size for two levels of the system bi-linearity are shown at Figure 1, where the relative segment size is a ratio of the segment length n to the signal length N. Obviously, the relative segment size is inversely proportional to the number M of segments.

It can be seen from Figure 1 that in both cases diagnostic effectiveness is aggravated at low and high values of the relative segment size. For low values of the relative segment size, one has a relatively large number of averaged segments and therefore relatively low values of the bicoherence variance according to equation (3). However, because of poor frequency resolution due to

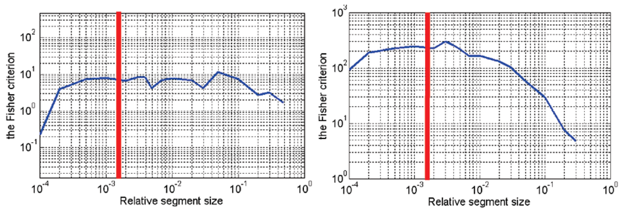

Figure 1. Values of the Fisher criterion *vs* the relative segment size for for different stiffness ratios: 0.05 (left) and 0.1 (right)

the large number of segments (see equation (2)), there are essential errors in amplitude estimation of the harmonic components and therefore essential errors in bicoherence estimation; these errors aggravate diagnostic effectiveness.

For high values of the relative segment size, one has a relatively small number of averaged segments and therefore the frequency resolution gets better (see equation (2)); however, there are relatively high values of the bicoherence variance due to the small number of averaged segments (see equation (3)); high values of the bicoherence variance also aggravate diagnostic effectiveness.

Thus, there are optimal ranges of segment sizes in Figure 1, which maximise the effectiveness criterion due to the above-mentioned tendencies. It can be also seen from Figure 1 that the optimal range of segment sizes depends on level of the system bi-linearity (relative damage size). There is a decrement in the optimal range due to increment of level of the system bi-linearity.

Let us compare the obtained new optimisation results with the known optimisation results. Following[13] the optimal segment size is $N^{1/2}$ and the optimal relative segment size is $1/\sqrt{N}$; the relative segment size is 1.4·10^{-3} (shown by red lines in Figure 1) for the signal length used in the simulation. It can be seen from Figure 1 that the optimal relative segment size[13] is within the obtained optimal ranges for both levels of the bi-linearity.

The obtained optimal ranges allow flexibility in bicoherence estimation. It is possible to perform additional optimisation of the relative segment size within these ranges based on computational requirements, for example an optimal trade-off between the number M of averaging and the computational complexity of the Fourier transform, etc.

Conclusions

- An optimisation of a segment size for bicoherence estimation for two class diagnostics of fatigue damage (the system bi-linearity) has been considered for the first time.
- The existence of the optimal ranges of segment sizes is shown. These optimal ranges are in contrast to most published applications in which the unique optimal value of segment size is used.
- It is obtained that increment of relative damage level decreases the optimal range of segment sizes.
- The results obtained in this study do not counteract with traditional optimisation results[13]: the traditional optimal value of segment size is within the obtained optimal ranges.

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