An efficient nodal Jacobian method for 3D electrical capacitance tomography image reconstruction

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1. Introduction

Electrical Capacitance Tomography (ECT) is a non-invasive and non-destructive imaging technique that uses electrical capacitance measurement at the periphery of an object\(^1\). Electrodes at the surface of the volume apply electrical potential into the surrounding area and emergent charge is measured at the boundary. The measured data is then used in an electrostatic model to determine the dielectric permittivity properties of the object. Three-dimensional ECT imaging will become an important tool for industrial imaging for process monitoring in the future\(^2\). Much work has gone into the production of accurate forward models to describe the electrostatic forward model\(^3\) and the inverse model to efficiently recover images of the dielectric parameters\(^4\). For the inverse problem, several image reconstruction techniques have been developed\(^5\). The problem of ECT image reconstruction is two-fold: the model to describe the electrical field distribution within the region must be accurate and the inverse problem must be reliable and computationally efficient at estimating the electrical properties within the imaging area. Numerical algorithms based on the Finite Element Method (FEM) for the forward model rely on the accurate definition of the volume and the mesh discretisation must be adequate for the calculation. It has been found that, for an accurate forward model, the FEM mesh resolution must be of a high quality\(^6\). In this paper we are using a large FE mesh composed of 22,640 nodes and 114,139 elements to discretise the forward model and improve the efficiency of the inverse solution by using a nodal Jacobian. Traditionally, an element-based Jacobian is used\(^7\), the use of a nodal Jacobian will reduce the size of the inverse problem by around five times, providing an important improvement in the application of 3D ECT.

2. Forward problem in 3D ECT

The forward problem is the simulation of measurement data for given values of excitation and material permittivity distribution and the inverse problem is the imaging result for a given set of measurement data. To solve the inverse problem we need to solve the forward problem. At this stage we assume there is no wave effect and use a low frequency approximation to Maxwell’s equations. In future, a forward model that takes into account the wave effect could be developed but with high computational costs.

With high frequency a more complicated model is needed. In a simplified mathematical model, the electrostatic approximation \(\nabla \times E = 0\) is taken, effectively ignoring the effect of wave propagation. Let’s take \(E = -V\) and assume no internal charges, then the following equation holds:

\[
\nabla \cdot \varepsilon \nabla u = 0 \quad \text{in } \Omega \label{eq:forward}
\]

where \(u\) is the electric potential, \(\varepsilon\) is dielectric permittivity and \(\Omega\) is the region containing the field. The electric potential on each electrode is known as:

\[
u = V_i \quad \text{on electrode } e_i \label{eq:boundary}
\]

where \(e_i\) is the \(k\)-th electrode held at the potential \(V_i\). For each excitation pattern one of the electrodes (in turn) is set to a given voltage \(u=V\) and the rest are grounded \(u=0\). Using the FEM\(^8\) method we obtain:

\[
Ku = B \label{eq:system}
\]

where the matrix \(K\) is the discrete representation of the operator \(\nabla \cdot \varepsilon \nabla\), the vector \(B\) is the boundary condition term and \(U\) is the vector of electric potential solution. The total charges on the \(k\)-th electrode is given by:

\[
I_k = \int e \frac{\partial u}{\partial n} d\Omega \label{eq:charge}
\]

where \(n\) is the inward normal on the \(k\)-th electrode.

3. A nodal Jacobian approach for 3D ECT

A nodal Jacobian approach has been successfully applied to both 3D electrical impedance tomography\(^9\) and 2D ECT\(^10\). In this paper a nodal Jacobian approach for 3D ECT has been introduced for the first time. Calculation of the nodal sensitivity matrix is the inverse problem for a given set of excitation and material permittivity distribution \(\varepsilon\). As both equations describe the same energy they can be combined as follows:

\[
\begin{align*}
\frac{1}{2} U^T \varepsilon \cdot \nabla u = & \frac{1}{2} \int e(x,y,z) \hat{E}(x,y,z) \cdot \hat{E}(x,y,z) d\Omega \label{eq:energy1}\,
\frac{1}{2} U^T C = & \frac{1}{2} \int e(x,y,z) \hat{E}(x,y,z) \cdot \hat{E}(x,y,z) d\Omega \label{eq:energy2}\,
\end{align*}
\]

where \(W_c\) is energy accumulated in the capacitor with capacity \(C\) and applied voltage \(U\), \(W_e\) is energy of the electric field \(E\) with electric permittivity distribution \(\varepsilon\). As both equations describe the same energy they can be combined as follows:

\[
\begin{align*}
\frac{1}{2} U^T \varepsilon \cdot \nabla u = & \frac{1}{2} \int e(x,y,z) \hat{E}(x,y,z) \cdot \hat{E}(x,y,z) d\Omega \label{eq:energy1}\,
\frac{1}{2} U^T C = & \frac{1}{2} \int e(x,y,z) \hat{E}(x,y,z) \cdot \hat{E}(x,y,z) d\Omega \label{eq:energy2}\,
\end{align*}
\]
and consequently:

\[ C = \frac{1}{U^2} \int \varepsilon(x,y,z) \hat{E}(x,y,z) d\Omega \]  

(8)

The sensitivity of any point \( j \) describes the connection between a change in permittivity distribution in that point and the resulting change of the capacity \( i \), as can be expressed by the following equation:

\[ S_{ij} = \frac{\partial C_i}{\partial \varepsilon_j} \]  

(9)

Combining both equations (8 and 9) and omitting the index \( i \), which is connected with the measurement sequence, we can obtain:

\[ S_j = \frac{1}{U^2} \int \varepsilon(x,y,z) \hat{E}^2(x,y,z) d\Omega \]  

(10)

It remains to define \( \varepsilon \), and how it is connected with the function \( \varepsilon(x,y,z) \). Typically, when we use finite element modelling for ECT, the function \( \varepsilon(x,y,z) \) is defined that \( \varepsilon(x,y,z) \) is equal to some constant value inside each element, so equation (10) can be written as:

\[ S_j = \frac{1}{U^2} \int \varepsilon \hat{E}^2 d\Omega \]  

(11)

This can be expanded as:

\[ S_{j}(x,y,z) = \frac{1}{V_{i1}, V_{i2}} \frac{\partial}{\partial \varepsilon_j} \int e \hat{E}_{i1}(x,y,z) \hat{E}_{i1}(x,y,z) d\Omega \]  

(12)

where: \( S_{j}(x,y,z) \) – sensitivity value for the \( j \)-th voxel described as \( (x,y,z) \) while the \( i \)-th capacitance measurement \( C_i \) between electrodes \( e_i \) and \( e_j \) was carried out, \( V_{i1} \) and \( V_{i2} \) are potentials applied to electrodes \( e_i \) and \( e_j \) respectively, \( E_{i1}(x,y,z) \) and \( E_{i2}(x,y,z) \) are electric field vectors and \( \varepsilon_j \) is a permittivity value for the \( j \)-th voxel.

Let us now define for each element of the FEM mesh the function \( \varepsilon(x,y,z) \), which does not have a constant value but is defined as \( \varepsilon(x,y,z) \) as can be expressed by the following equation (14). The total sensitivity value for a single node can be calculated as:

\[ s_{j}(node) = \sum_{j=1}^{n} s_{j}(voxel) \]  

(17)

Figure 1 shows the sensitivity map for the 32-electrode ECT sensor developed at University of Lodz, Poland. In this 3D system there are four planes of eight electrodes and the sensitivity maps are shown for measurement between various electrodes in different planes. The sensitivity map here shows the equal sensitivity regions between the excitation and measuring electrodes.

In order to verify the new sensitivity formula the Jacobian from this sensitivity formula has been tested using experimental data. Figure 2 shows the image reconstruction result using a linear back projection (LBP) algorithm using this new nodal Jacobian matrix.

5. Conclusions

In this work we have shown that an efficient Jacobian reduction technique can reduce the computation time and memory requirement of the inversion of the Hessian matrix in the inverse problem without impinging on the qualitative and quantitative accuracy of the reconstructed images. This is particularly important for the reconstruction of images of a large complicated volume with many unknowns where using a fine mesh would usually be computationally inefficient.

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7. References


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