MOMENT TENSORS OF IN-PLANE WAVES ANALYZED BY SIGMA-2D

MASAYASU OHTSU\textsuperscript{1}, KENTARO OHNO\textsuperscript{1} and MARVIN A. HAMSTAD\textsuperscript{2}

\textsuperscript{1) Graduate School of Science and Technology, Kumamoto University, Kumamoto, Japan.  
\textsuperscript{2) Department of Engineering, University of Denver, Denver, CO 80208 USA.}

Abstract

For quantitative waveform analysis of AE signals, SiGMA (Simplified Green’s functions for Moment tensor Analysis) procedure has been developed. Kinematics on AE source, such as crack location, crack type and crack orientation, can be determined from recorded AE waveforms. Since the SiGMA code was originally implemented for three-dimensional (3D) source characterization, the applicability of SiGMA to AE signals detected in a plate-like sample was studied here. Employing a finite-element modeling code, theoretical AE waves of in-plane motion were computed. AE sources were modeled by assuming three types of moment tensors. These tensor elements are recovered by two-dimensional SiGMA code (SiGMA-2D). Comparing results between assumed and recovered sources, the applicability of SiGMA-2D to in-plane AE waves was confirmed.

As a practical application, AE waves due to water leakage from a pipe were analyzed. In laboratory, water leakage from a slit-like defect was generated in a pipe model. It was found that dominant cracking motions at the slit are associated with AE waves due to shear cracks, with orientations approximately parallel to the slit plane. For tensile cracks of AE sources, crack-opening directions are perpendicular to the slit plane. These results show a great promise for clarifying AE generating mechanisms due to water leakage by SiGMA-2D analysis.

1. Introduction

Moment tensor analysis of acoustic emission (AE) waves can identify the kinematics of AE sources, such as location, crack-type classification and crack orientation. Taking into account only the first motions of waveforms, an analytical procedure was implemented as the SiGMA (simplified Green’s functions for moment tensor analysis) procedure (Ohtsu, 1991). In three-dimensional (3D) massive body of concrete, the applicability of SiGMA-3D has been confirmed (Ohtsu et al., 1998). In the case of plate-like samples, such as metals and composites, however, the SiGMA-3D procedure is difficult to use. This is because out-of-plane motions of AE waves are basically taken into consideration in the SiGMA-3D analysis. For AE detection in a plate, Lamb waves are predominantly observed as out-of-plane motions, but unfortunately they are not due to the kinematics of AE sources.

Previously, we developed SiGMA-2D for a plate-like sample (Shigeishi and Ohtsu, 1999). Taking into account in-plane motions of AE waves, crack kinematics of AE sources were determined in PMMA and mortar plates with a slit under compressive loading. To confirm the feasibility, results were compared visually with crack traces. One example of SiGMA-2D results in a mortar plate is shown in Fig. 1. An arrow symbol shows a tensile crack with crack-opening orientation, and a cross symbol denotes a shear crack with two orientations of crack-motion and crack-normal vectors. Reasonable agreement was observed between SiGMA results and
observed crack traces of the broken lines in Fig. 1. So far, the accuracy and applicability of SiGMA-2D to other types have not yet been fully confirmed.

In order to synthesize AE waves in the plates, a dynamic finite-element modeling code has been developed (Hamstad et al., 2001). Computing theoretical AE waves due to sources of different types, source identification was attempted by applying a wavelet transform (Hamstad et al., 2002).

In the work reported here, the accuracy of the SiGMA-2D code was examined with theoretical in-plane wave motions. By employing the finite-element modeling code with certain moment tensor elements, theoretical AE waves with in-plane motions were synthesized. The moment tensors were recovered by SiGMA-2D analysis of the waves. In addition, as one practical application, AE waves due to water leakage from a slit-like defect in a pipeline model were detected and analyzed by SiGMA-2D.

2. SiGMA Analysis

2.1 Theory of Moment Tensor

As formulated in the generalized theory (Ohtsu and Ono, 1984), AE waves are elastic waves generated by dynamic-crack (dislocation) motions inside a solid. By introducing moment tensor, $M_{pq}$, the integral representation of the elastic wave $u_k(x,t)$ is represented,

$$ u_k(x,t) = G_{kp,q}(x,y,t) M_{pq} * S(t). \quad (1) $$

Here, $G_{kp,q}(x,y,t)$ are spatial derivatives of Green's functions and $S(t)$ represents the source kinetics (often called the source-time function). Inverse solutions of equation 1 are two-fold. Source kinetics are determined from the source-time function $S(t)$ by a deconvolution procedure. Source kinematics are represented by the moment tensor, $M_{pq}$. In order to perform the deconvolution and to determine the moment tensor, the spatial derivatives of Green's functions or the displacement fields of Green's functions due to the equivalent force models are inherently
required. So far, numerical solutions of the displacement fields have been obtained by finite difference method (Enoki et al., 1986) and by FEM (Finite Element Method) (Hamstad et al., 1999). These solutions, however, need a vector processor for computation and are not readily applicable to processing a large amount of AE waves. Consequently, based on the far-field term of the P-wave, a simplified procedure was developed (Ohtsu, 1991). This is suitable for a PC-based processor and is robust in computation. The procedure was implemented as the SiGMA (Simplified Green's functions for Moment tensor Analysis) code.

Mathematically, the moment tensor in equation 1 is defined by the tensor product of the elastic constants, the normal vector \( \mathbf{n} \) to the crack surface and the crack-motion (dislocation or Burgers) vector \( \mathbf{l} \),

\[
M_{pq} = C_{pqkl} l_k n_l \Delta V
\]  

(2)

The elastic constants \( C_{pqkl} \) have a physical unit of \([\text{N/m}^2]\) and the crack volume \( \Delta V \) has a unit of \([\text{m}^3]\). The moment tensor has the physical unit of a moment, \([\text{Nm}]\). This is the reason why the tensor \( M_{pq} \) was named the moment tensor. The moment tensor is a symmetric second-rank tensor and is comparable to the elastic stress in elasticity as,

\[
[M_{pq}] = \begin{bmatrix}
m_{11} & m_{12} & m_{13} \\
m_{12} & m_{22} & m_{23} \\
m_{13} & m_{23} & m_{33}
\end{bmatrix} \Delta V
\]  

(3)

All elements of the moment tensor are illustrated in Fig. 2. In a similar manner to stress, diagonal elements represent normal components and off-diagonal elements are shown as tangential or shear components.

![Fig. 2 Elements of the moment tensor.](image)

2.2 Equivalent Force Models

AE sources can be represented by equivalent force models, such as a monopole force, a dipole force and a couple force. Relations among crack (dislocation) models, equivalent force models and moment tensors are straightforward. From equation 2, in an isotropic material we have,
\[ M_{pq} = (\lambda k n_k \delta_{pq} + \mu l_p n_q + \mu l_q n_p). \] (4)

Here, \( \lambda \) and \( \mu \) are Lamé constants.

In the case that a tensile crack occurs on a crack surface parallel to the x-y plane and opens in the z-direction as shown in Fig. 3, the normal vector \( \mathbf{n} = (0, 0, 1) \) and the crack vector \( \mathbf{l} = (0, 0, 1) \). Substituting these into equation 4, the moment tensor becomes,

\[
[M_{pq}] = \begin{bmatrix}
\lambda & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda + 2\mu
\end{bmatrix} \Delta V
\] (5)

Only diagonal elements are obtained, which are shown in Fig. 3. Replacing these diagonal elements as dipole forces, three dipole-forces are illustrated in Fig. 3(c). This implies that combination of three dipoles is necessary and sufficient to model a tensile crack.

\[ \text{Crack motion vector } b \]

\[ \lambda + 2\mu \]

\[ \lambda \]

\[ \lambda \]

\[ \text{Double force couple model} \]

\[ \text{related moment tensor elements} \]

\[ \text{Three-dipole forces} \]

Fig. 3 (a) Tensile dislocation model, (b) related moment tensor elements, and (c) three-dipole forces.

In Fig. 4, the case of a shear crack parallel to the x-y plane is shown with the normal vector \( \mathbf{n} = (0, 0, 1) \). Shear motion occurs in the y-direction with the crack vector \( \mathbf{l} = (0, 1, 0) \). From equation 4, we have,

\[
[M_{pq}] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & \mu \\
0 & \mu & 0
\end{bmatrix} \Delta V
\] (6)

As seen in Fig. 4(c), the double force couple model is comparable to off-diagonal elements of the moment tensor in equation 6.
2.3 SiGMA Code

Taking into account only P-wave motion of the far field (1/R term) of Green’s function in an infinite space and considering the effect of reflection at the surface, the amplitude of the first motion is derived from equation 1. The reflection coefficient $\text{Ref}(t, r)$ is obtained as $t$ is the direction of sensor sensitivity and $r$ is the direction vector of distance $R$ from the source to the observation point, and $r = (r_1, r_2, r_3)$. The time function is neglected in equation 1, and the amplitude of the first motion $A(x)$ is represented,

$$A(x) = C_s \cdot \frac{\text{Ref}(t, r)}{R} \cdot (r_1, r_2, r_3) \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

where $C_s$ is the calibration coefficient of the sensor sensitivity and material constants. Since the moment tensor is a symmetric tensor of 2nd rank, the number of independent elements is six. These are represented in equation 7 as $m_{11}$, $m_{12}$, $m_{13}$, $m_{22}$, $m_{23}$, and $m_{33}$.

These can be determined from the observation of AE waves at more than six sensor locations. In SiGMA procedure, the two parameters, the arrival time ($P_1$) and the amplitude of the first motion ($P_2$), are visually determined from AE waveform as shown in Fig. 5. In the location procedure, the source (crack) location $y$ in equation 1 is determined from the arrival time differences $t_i$ between the observation $x_i$ and $x_{i+1}$, solving equations,

$$R_i - R_{i+1} = \| x_i - y \| - \| x_{i+1} - y \| = v_p t_i.$$  

Here $v_p$ is the velocity of P wave.
After solving equation 8, the reflection coefficient $\text{Ref}(t,r)$, the distance $R$, and direction vector $r$ are readily obtained to solve equation 7. The amplitudes of the first motions $P_2$ in Fig. 5 at more than 6 channels are substituted into equation 7, and all the elements of the moment tensor are determined. Since the SiGMA code requires only relative values of the moment tensor elements, the relative coefficients $C_s$ are sufficient.

![Fig. 5 Detected AE waveform.](image)

2.4 Two-Dimensional Treatment

Special care must be taken for the case of a plate-like sample for two-dimensional (2-D) problems. When cracks occur in a plate as shown in Fig. 6, wave motions detected are classified into two types. One is in-plane motion, which can be detected at the edge of the plate. The other is out-of-plane motion. AE waves are often detected as the out-of-plane motions even in the

![Fig. 6 Detection of AE waves of out-of-plane and in-plane motions.](image)
plate-like sample. In this case, the reflection coefficient in equation 7 is almost equal to zero, and thus the equation is neither well-posed nor solvable.

Consequently, only AE waves of the in-plane motion can be analyzed by SiGMA procedure. Assuming that the z-components of vectors $l$ and $n$ are equal to zero, the moment tensor in equation 4 is represented by,

$$
[M_{pq}] = \begin{bmatrix}
\lambda l_k n_k + 2\mu l_i n_i & \mu(l_i n_2 + l_2 n_i) & 0 \\
\mu(l_i n_2 + l_2 n_i) & \lambda l_k n_k + 2\mu l_i n_2 & 0 \\
0 & 0 & \lambda l_i n_k
\end{bmatrix}
$$

In the case that AE sensors are attached only on the edge of the plate, no motions are detected in the out-of-plane direction. As a result, elements of the tensor in equation 9 could be estimated except for the $m_{33}$ element. Here, $m_{11} + m_{22} = 2(\lambda + \mu) l_k n_k$ and,

$$
2(\lambda + \mu) = 2\lambda + \lambda(1 - 2\nu)/\nu = \lambda/\nu.
$$

Accordingly, the $m_{33}$ component is readily determined from,

$$
m_{33} = \lambda l_k n_k = \nu (m_{11} + m_{22})
$$

where $\nu$ is the Poisson’s ratio.

---

Fig. 7 Unified decomposition of eigenvalues of the moment tensor.
2.5 Unified Decomposition

In order to classify a crack into the tensile or shear type, a unified decomposition of the eigenvalues of the moment tensor was developed (Ohtsu, 1991). In general, crack motion on the crack surface consists of slip motion (shear components) and crack-opening motion (tensile components), as illustrated in Fig. 7. Thus, it is assumed that the eigenvalues of the moment tensor are the combination of those of a shear crack and those of a tensile crack, as the principal axes are identical. Then, the eigenvalues are decomposed uniquely into those of a shear crack, the deviatoric components of a tensile crack and the isotropic (hydrostatic mean) components of a tensile crack. In Fig. 7, the ratio X represents the contribution of a shear crack. In that case, three eigenvalues of a shear crack become X, 0, –X. Setting the ratio of the maximum deviatoric tensile component as Y and the isotropic tensile component as Z, three eigenvalues of a tensile crack are denoted as Y + Z, –Y/2 + Z, and –Y/2 + Z. Eventually the decomposition leads to relations,

\[ 1.0 = X + Y + Z, \]
\[ \text{the intermediate eigenvalue/the maximum eigenvalue} = 0 - Y/2 + Z, \]
\[ \text{the minimum eigenvalue/the maximum eigenvalue} = -X - Y/2 + Z. \] (11)

It should be pointed out that the ratio X becomes larger than 1.0 in the case that both the ratios Y and Z are negative (Suaris and van Mier, 1995). The case happens only if the scalar product \( l \cdot n \) is negative, because the eigenvalues are determined from relative tensor components. Making the scalar product positive and re-computing equation 11, the three ratios are reasonably determined. Hereinafter, the ratio X is called the shear ratio.

Classification of cracks was proposed elsewhere on the basis of angles between two vectors \( l \) and \( n \) (Ouyang et al., 1992). But, the crack classification based on the angle is not reasonable, because the relationship between the angle and the shear ratio X is nonlinear (Ohtsu, 1995).

In the present SiGMA code, AE sources with shear ratios less than 40%, are classified as tensile cracks. The sources with X > 60% are classified as shear cracks. In between 40% and 60%, the cracks are referred to as mixed-mode.

From the eigenvalue analysis, three eigenvectors \( e_1 \), \( e_2 \), \( e_3 \) are also obtained. Theoretically, these are derived as,

\[ e_1 = l + n \]
\[ e_2 = l \times n \]
\[ e_3 = l - n. \] (12)

Here \( \times \) denotes the vector product, and the vectors \( l \) and \( n \) are interchangeable. In the case of a tensile crack, the vector \( l \) is parallel to the vector \( n \). Thus, the vector \( e_1 \) could give the direction of crack-opening, while the sum \( e_1 + e_3 \) and the difference \( e_1 - e_3 \) give the two vectors \( l \) and \( n \) for a shear crack.

To locate AE sources, at least 5-channel system is necessary for 3-D analysis. Since 6-channel system is the minimum requirement for the moment tensor, 6-channel system is required for SiGMA-3D analysis. In the 2-D problem, a 3-channel system is applicable to determine the moment tensor elements, but generally a 4-channel system is necessary for 2-D location analysis. Consequently, a 4-channel system is the minimum recommended for SiGMA-2D analysis.
3. IN-PLANE THEORETICAL WAVEFORMS

3.1 Source Models

In order to study the accuracy of SiGMA-2D, in-plane theoretical waves are analyzed. These were computed by employing the FEM-based AE signal database (Downs et al., 2003). The configuration of the detection is illustrated in Fig. 8. As a propagating medium, we selected aluminum with P-wave velocity of 6320 m/s and Poisson’s ratio of 0.34. The sources were located at the coordinate origin at 1.41 mm depth. In-plane waves were calculated at five locations 180 mm from the source. The radiation angles were set to 12°, 22.5°, 45°, 67.5° and 78°. In-plane displacements (waves) in both the x-direction and the y-direction were computed with 0.1-μs increment of 2000 words at each observation point.

At the origin, three moment tensors were considered as source models. One model consists of three dipole forces, corresponding to the nucleation of a tensile crack. The crack surface is perpendicular to the x-axis and parallel to the y-axis. The second model was one dipole force applied in the x-direction, and the third model was a shear crack with out-of-plane orientation.

![Diagram of wave angles and source depth]

**Source depth**

= 1.41 mm

Fig. 8 Five observation points for in-plane theoretical waveforms.

(1) Tensile crack

Configuration of the source is illustrated in Fig. 9. An associated dipole-force model is also given. The moment tensor is represented by,

\[
[M_{pq}] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0.52 & 0 \\
0 & 0 & 0.52
\end{bmatrix}.
\]  

(13)

(2) One dipole-force model

The model is given in Fig. 10. The moment tensor is obtained as:

\[
[M_{pq}] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]  

(14)
It is noted that no crack is equivalent to this model, because three dipole-forces are associated with a tensile crack and a shear crack is modeled by double couple-forces.

(3) Shear crack inclined 45° to x-axis.

A source model and the double-couple force model are illustrated in Fig. 11. The eigenvalues of the corresponding moment tensor are represented by,

\[
[M_{pq}] = \begin{bmatrix}
-1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 
\end{bmatrix}.
\]

(15)
3.2 Results of SiGMA-2D Analysis

Examples of waveforms computed are shown in Fig. 12. Applying SiGMA-2D to the first arrivals of these waves, moment tensor elements are recovered by equation 7. For the tensile crack, tensor elements in equation 13 are recovered,

\[
[M_{pq}] = \begin{bmatrix}
1 & -0.01 & 0 \\
-0.01 & 0.25 & 0 \\
0 & 0 & 0.43
\end{bmatrix}
\]

Here, the tensor elements are normalized by the maximum value, and the element \( m_{33} \) is calculated by equation 10. It is noted that the elements \( m_{22} \) and \( m_{33} \) are recovered as smaller values than those in equation 13.
For the case of one dipole-force model in equation 14, results are obtained as,

\[
[M_{pq}] = \begin{bmatrix}
1 & -0.02 & 0 \\
-0.02 & -0.06 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Since the model does not correspond to a crack, the element \(m_{33}\) is not computed. Comparing with equation 14, a good agreement is found.

In the case of the shear crack in equation 15, tensor elements are recovered as,

\[
[M_{pq}] = \begin{bmatrix}
-1 & 0.01 & 0 \\
0.01 & 0.26 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Because this source corresponds to a shear crack, the scalar product \(l_kn_k\) in equation 10 is equal to zero. Accordingly, the element \(m_{33}\) is again not computed. As a result, agreement with equation 15 is not so clear that element \(m_{33}\) is not comparable to that in equation 15 and fairly large error is observed in the element \(m_{22}\). This is because dominant motions are generated in the out-of-plane direction due to shear motion of out-of-plane orientation, and no information from out-of-plane motions was obtained.

These results imply that the elements of the moment tensors are recovered reasonably well by SiGMA-2D in the case that such dominant source motions are generated in the in-plane direction as one dipole-force model. For the general cases of cracking, crack types are classified correctly, but the loss of information on out-of-plane motions can result in large errors.

4. LEAK DETECTION

4.1 Experiment

As one practical application, SiGMA-2D was applied to AE events due to water leakage. A slit-like defect is modeled, through which water leakage occurs. A defect model of PMMA and AE sensor locations are shown in Fig. 13. The defect is a slit of 1 mm width and 5 mm length located at the center of the 10-mm thick plate. AE events were detected by a 4-channel system, which consists of AE sensors of 150 kHz resonance (R15, PAC) and the sampling frequency for recording waveforms is 1 MHz.

The slit model was attached at the edge of a water chamber in a pipe model as shown in Fig. 14. By employing a large water chamber, 0.3 MPa pressure was constantly applied to the model in laboratory. AE events due to leakage were detected. Detected AE waves for one event are shown in Fig. 15. Although mostly continuous-type AE events were observed, in some cases the first arrivals could be determined as illustrated in the figure.

4.2 Analysis

During the leakage test, 109 AE events with detectable first arrivals were analyzed. The results are classified by the shear ratio \(X\) as plotted in Fig. 16. It is found that almost 40 % of the events have the shear ratio of over 80 %. This implies that dominant source motions at the slit-like defect due to leakage are shear-type motions. This result might be associated with the fact
that only large AE events were identified as the burst-type and readable for the analysis. That is, slip motions on the slit surface were mostly detected as AE events due to leakage.

Next, crack orientations were estimated. Typical events having the shear ratios smaller than 10% are plotted in Fig. 17. Because these are classified as the tensile cracks, crack-opening directions obtained are indicated. It is found the source opening directions are almost vertical to the slit surface, suggesting that water flows due to leakage open the slit. Some events of the shear ratios larger than 80% are plotted in Fig. 18. It is clearly observed that orientations of the shear events are parallel to the slit surface. This implies that the dominant source motions detected as AE events are in-plane shear motions at the slit-like defect. This may result from dynamic water outflow through the slit.
5. CONCLUSION

The applicability of SiGMA-2D analysis to in-plane motions of AE waves was studied. Employing an FEM code, theoretical waves were computed by assuming moment tensor elements, and the in-plane displacements were analyzed by SiGMA-2D. The accuracy of SiGMA-2D was numerically examined.

The elements of the source moment tensors are recovered reasonably well by SiGMA-2D in such case that dominant source motions are generated in the in-plane direction as one dipole-force model. For the general cases of cracking, loss of information on out-of-plane motions can result in large errors, while crack types are classified correctly.

As a practical application, AE waves due to water leakage from a slit were detected and analyzed. Dominant motions at the slit-like defect are found to be AE waves of shear type for
the large events, of which first arrival motions can be read. For the tensile events, crack-opening
directions were vertical to the slit surface, suggesting that water flows due to leakage open the
slit. Orientations of the shear events are parallel to the slit surface. This implies that dominant
motions detected as AE events are in-plane shear motions at the slit-like defect, which may result
from dynamic water outflow through the slit. These results show promise for clarifying AE gen-
erating mechanisms due to leakage by the moment tensor analysis.

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Fig. 17 Crack orientations for tensile events.
Fig. 18 Crack orientations for shear events.