HIERARCHICAL FRACTURE PROCESS IN BRITTLE ROCKS BY MEANS OF HIGH-SPEED MONITORING OF AE HYPOCENTER

XINGLIN LEI*, OSAMU NISHIZAWA, ANDRE MOURA and TAKASHI SATOH

Geological Survey of Japan (GSJ), National Institute of Advanced Industrial Science and Technology (AIST), Higashi 1-1-1, Tsukuba, Ibaraki, 305-8567, Japan; *Also at Institute of Geology & Laboratory of Tectonophysics, China Seismological Administration, Beijing, China.

Abstract

Damage evolution and fault formation processes in brittle rocks under stress were studied based on data collected with a high-speed recording system of acoustic emission (AE) waveforms. Experimental results show that the faulting process is hierarchical. Quasi-static nucleation of faults represents dynamic fractures of the asperities on the fault plane; likewise, a quasi-static nucleation process characterized by dynamic microcracking precedes the fracture of an asperity. The progressive fracturing of multiple and coupled asperities results in short-term precursory fluctuations in both the $b$-value and event rate of AE. For proposing more general models, three granites of different grain size distributions were used for fracture tests under triaxial compression with two different loading rates of 27.5 and 2.5 MPa/min. The damage creation in these intact brittle rocks shows a typical process characterized by three phases of microcracking activity: named as primary, secondary, and nucleation phases. For applying the experimental results to real problems, the damage evolution was quantitatively modeled from two approaches: 1) the corrosion-aided sub-critical crack growth theory for crack population with fractal size distribution; 2) renormalization-group theory applied to second-order phase transition with the time of the global failure assimilated to a critical time. The event rates and $b$-values in the secondary and nucleation phases can be modeled well by the first model. Energy release calculated from the event rates and magnitude can be modeled well by the second model with two domains corresponding to the secondary and nucleation phases. These results confirm that 1) the larger the grain size, the stronger the crack-crack interaction; 2) the faster the loading rate (stress-rate), the stronger the interaction; 3) the secondary domain, i.e., the nucleation phase, shows normally longer interaction than the first domain, indicating the increase of interaction distance.

Keywords: Damage evolution, Rock fracture, Microcrack, Renormalization-group

1. Introduction

The study of damage evolution in rock and other industrial materials like cement under stress is a subject of widespread interest, with relevance to both artificial applications such as optimization of geothermal recovery, oil recovery, safe design of nuclear waste repositories, rock bursts, and natural processes such as volcano eruptions and earthquakes. It is important to predict the time of catastrophic failure that corresponds to the lifetime of industrial material or the occurrence of rock burst and earthquake. For this goal, challenges on modeling the damage evolution have been made through both experimental and theoretical approaches.

In laboratory, damage evolution in stressed materials has been studied extensively by a number of methods including 1) the direct observation of sample surface by scanning electron
microscopy [e.g., 1] or optical microscopy [e.g., 2] operated during or after a fracture test; and 2) monitoring the hypocenter distribution of acoustic emission (AE) events caused by microcracking activity [e.g., 3, 4]), which is the so-called AE technique. AE technique is advanced in the analysis of the microcracking activity inside the sample volume and can be performed under confining pressure, which is very important in the simulation of underground conditions.

The recently developed high-speed multi-channel waveform recording technology has made it possible to monitor the hypocenters of AEs associated with spontaneously/unstably fracturing processes in stressed samples with high precision. The system is capable of recording AE waveforms in 32 channels at sampling rates up to 5000 events per second, and has been successfully used to study the final stage (quasi-static) of fracturing nucleation in intact brittle rocks. It has been applied to the analysis of the fracture process of hornblende schist [5], granitic rocks [6], and mudstone containing quartz veins, which play a role as strong asperities [7]. Lei et al. [5] observed that once a shear fault is initiated or nucleated in fine-grained homogeneous rock, the shear fault grows quasi-statically with a process zone at the fault front. It is found that the process zone is governed by progressive triggering of tensile microcracking, in agreement with the microscopic examination on a torque test revealed by Cox and Scholz [2].

More recently, it has been observed that the microcracking activity associated with the fracture of inhomogeneous fault consists of 3 stages or phases with a spatial hierarchical structure [8]. These long-term microcracking phases referred to as primary, secondary and nucleation can be identified from the changes in event rates, $b$-values, and AE hypocenter distribution. It was suggested that the predictability of catastrophic fracture is strongly dependent on the heterogeneity on the potential fault plane. A homogeneously healed fault is notably unpredictable, whereas a fault of non-homogeneous healing strength or asperity distribution undergoes a predictable fracturing process with a remarkably clear nucleation phase that can be observed as a precursory anomaly of the $b$-values or other statistical parameters of AE activity. The appearance of a nucleation phase with rapidly decreasing $b$-values, a non-linearly increasing event rates and spatio-temporal clustering can be considered to be a signal of the initiation of catastrophic fracture.

The above described experiments agree with the general knowledge that global failure comes from spatially distributed micro-separations including microcracking, fiber rupture, interfacial debonding, etc., which is in contrast to the case of perfect crystal where global failure occurs from a few or even from only one unstable crack. The multi-step character of arising damages, reflecting the discrete nature of the fracturing process, makes it possible to compare the diffusion of damages in brittle materials with the diffusion on random lattices [9, 10]. This analogy has been the starting idea for extensive numerical simulations applied to the lattice models of fracture, which predicted the existence of scaling laws in the vicinity of fracture in brittle materials [11]. The fracturing of stressed materials is considered then as analogous to a critical phenomenon happening at a second-order phase transition in analogy to percolation phenomenon [12, 13]. The moment of rupture is similar to a critical point, so that the fracturing process can be described by a renormalization-group scheme [14]. In the vicinity of the critical point of rupture, there exists a critical region [15, 16], where the variations of the free energy, which is the energy release in the case of rock fracture, can be characterized by a power law of time normalized by the critical time decorated by log-periodic oscillations [17]. Such oscillations, which are useful for prediction purpose, are related to complex exponents that appear in renormalization-group solutions for critical phenomena [18].
However, above model based on critical phenomena is somewhat mathematical. The physical bases behind the model parameters are still unclear. Hence, further studies are required for linking mathematical models with physical mechanisms in order to elaborate reasonable prediction schemes. Another approach for modeling the damage evolution with some physical bases has been proposed by considering the sub-critical crack growth under stress as the result of stress-aided corrosion at the crack tip. Some models based on the experimental results of a single macroscopic extensional crack have been proposed [e.g., 19, 20] and later extended to crack population with fractal size distribution [21]. The models can fit not only energy release (here events rate) as the model from the renormalization-group solutions but also the $b$-value in the magnitude-frequency relation. Hence these models could explain some precursory anomalies such as quiescence [22] and $b$-value decrease [23] associated with large earthquakes, although they are unable to take account for interaction phenomena.

In this paper, we will at first introduce the most recent experimental results associated with the hierarchical fracturing process. Then we will model the damage evolution based on detailed AE measurement by two different models come from firstly, the stress corrosion constitutive laws and secondly, the renormalization-group scheme. We will discuss next on some possible physics behind the model parameters and the relation between these models.

2. Experiment and Data Processing

It is normally observed that the AE rates may reach several thousands per second prior to the catastrophic failure [e.g., 8]. For clarifying the damage evolution though the monitoring of the spatio-temporal distribution of AE events, a high-speed waveform recording system has been developed. The system has the following advantages.

i) It has a rapid waveform recording facility on 32 channels with a sampling rate up to 25 MHz and dynamic range of 12 bits.

ii) The mask time of the system is less than 200 $\mu$s, thus it is possible to record AE waveforms without any important loss, particularly when the AE activity is very high - on the order of several thousands per second.

iii) The system has a 2-channel detector to capture the values of peak amplitudes with an effective dynamic range of 55 dB, which corresponds to a magnitude range of about 2.75. Further, since the peak detector shares a common base-clock with the waveform recording system, it is possible to obtain a complete set of temporal and spatial distribution of AE events.

iv) The piezoelectric transducers (PZT) function not only as receivers of acoustic signals, but also as acoustic sources for measuring the P- and S- velocity during experiment. Being a receiver or transmitter is controlled by an automatic switching system.

v) It can record stress and strain data on 16 channels with 16-bit resolution and 100 kHz sampling rate.

vi) AE hypocenters can be determined automatically by using the first arrival time data of P-waves and measured P velocities. The location error of an AE hypocenter is generally less than 1–2 mm and 2–3 mm for fine-grained and coarse-grained rocks, respectively.

Test samples were normally shaped as cylinders of 125 mm (100 mm for some earlier tests) in length and 50 mm in diameter. All samples were dried under normal room conditions for more than one month and then compressed with constant stress rate including creep (sample was loaded with constant stress rate to a stress level of ~95% fracture strength and then held constant) at room temperature. During the deformation confining pressure was kept constant at 40-60
MPa. Under these conditions, a rock sample normally fractures with a shear fault of ~30° with the maximum stress axis. Table 1 is a list of rocks and their dominant grain sizes referred to in this paper. These granitic rocks contain mainly quartz, K-feldspar, and biotite. Granitic Porphyry has a very low density of pre-existing microcracks due to recrystallization. Other rocks contain relatively higher density of pre-existing microcracks. The size and spatial distribution of the pre-existing microcracks are controlled by the grain size distribution. Hence, from the microscopic (from μm to mm) point of view, the larger the grain size is, the more heterogeneous the rock.

### Table 1 Dominant grain size in test samples.

<table>
<thead>
<tr>
<th>Rock</th>
<th>Westerly Granite, WG</th>
<th>Oshima Granite, OG</th>
<th>Inada Granite, IG</th>
<th>Granitic Porphyry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major grain size (mm)</td>
<td>&lt;1-2</td>
<td>1-5</td>
<td>1-10</td>
<td>1-10</td>
</tr>
</tbody>
</table>

A total of 32 PZT sensors (5 mm in diameter, resonant frequency of 2 MHz) were mounted on the surface of each test sample to detect AE signals from microcracking events. A series of 6-8 cross-type strain gauges (12-16 channels) were mounted on the surface of the sample to measure local axial and circumferential strains. The local volumetric strain ($\varepsilon_v$) was calculated from the axial ($\varepsilon_a$) and circumferential ($\varepsilon_c$) strains according to the equation $\varepsilon_v = \varepsilon_a + 2\varepsilon_c$.

The mechanics of rock deformation and crack growth can be inferred from AE statistics, because the number of AE events is proportional to the number of growing cracks and AE amplitudes are proportional to the length of crack growth increments [23, 21, 25, 26]. The well-known Gutenberg and Richter [27] relationship,

$$\log_{10} N = a - bM$$

where $N$ is the number of events of magnitude $M$ or greater, holds for not only earthquakes but also AE events in laboratory [e.g., 28, 29]. In equation (1), $a$ is a constant and $b$ is the seismic $b$-value. An estimate of the $b$-value in the Gutenberg-Richter relation can be obtained by using either the least-squares method or the maximum likelihood method [30].

### 3. Hierarchical Fracturing Process

The final stage of the damage evolution prior to the catastrophic fracture is the most important subject because it is associated with the predictability based on either precursory phenomena or damage constitutive law. Some recent experimental results have improved our understanding on the fracture behavior leading to the catastrophic failure. For instance, the fracture of a shear fault containing several unbroken asperities in a granitic porphyry is examined in detail by Lei et al. [24, 32]. A detailed description of the test sample, identification of asperities, and experimental setup can be found from the cited references. Since this result has provided us some physical bases for understanding the mathematical models presented in the following sections, it is convenient to briefly summarize the results here. As shown in Fig. 1(a), AEs caused by the fracture of individual asperities exhibit similar characteristics to the sequence for natural earthquakes, including foreshock, mainshock, and aftershock events. Foreshocks, initiated at the edge of the asperity, occur with an event rate that increases according to a power law of the temporal distance to the mainshock, and with a decreasing $b$-value (from ~1.1 to ~0.5). One or a few mainshocks then initiate at the edge of the asperity or the front of the foreshocks. The aftershock period is characterized by a remarkable increase and subsequent gradual decrease in
the $b$-value and decreasing event rates obeying the modified Omori law, which has been well established for earthquakes. The fracture of neighboring asperities is then initiated after the mainshock of a particular asperity, presumably due to redistribution of the strain energy accumulated within an asperity, which is released by the mainshock, resulting in enhancement of the stress concentration around the nearest neighboring intact asperities. The progressive fracturing of multiple, coupled asperities results in short-term precursory fluctuations in both the $b$-value and event rate (Fig. 1(b)), which may prove useful in the prediction of catastrophic failure. For applying the experimental results to real problems, a constitutive model must be established firstly based on experimental data and then examined again and again based on measurable data from real applications.

Fig. 1 AE data associated with the fracture of an individual asperity (a) and coupled asperities (b) on the fracture plane. The $b$-values are calculated for sets of 100 (in a) or 500 (in b) events with a running step of 25/125 events by the maximum likelihood method. The standard error for the estimated $b$-value is $\sim 0.1b$ or $0.05b$. Dashed line denotes the power law of temporal distance from the main shock.
3. Damage Model Based on Sub-Critical Crack Growth Theory

Under stress, sub-critical crack growth can occur as a result of stress-aided corrosion at the crack tip though the details are still unclear. The quasi-static rupture velocity \( V \) is found experimentally to be related to the stress intensity factor \( K \) by Charles’ law [33]:

\[
V = \frac{dc}{dt} = V_0 (K / K_0)^n
\]  

(2)

where \( c \) is the half-crack length and \( n \) is referred to as the stress corrosion index that has a typical value in the range 20-60 for polycrystalline rocks [19]. In the classic Griffith crack loading conditions, \( K = Y\sigma c^4 \) with \( q = 1/2, Y = \sqrt{\pi} \). The solution of the differential equation (2) depends on the loading history \( \sigma(t) \). Under static loading, the crack length predicted from the above equation of \( V \) and definition of stress intensity \( K \) has the form,

\[
c = c_0 \left(1 - t / t_f\right)^{2/(2-n)}
\]  

(3)

where \( c_0 \) is the initial crack length at an arbitrary initial time \( t = 0 \), and \( t_f \) is the failure time [19]. For a time-varying stress, a similar equation is a good approximation if the stress-drop is small compared to the ambient remote stress [34]. For stress-aided corrosion crack, Meredith and Atkinson [20] showed that the event rate \( N \) has also a non-linear relationship to the stress intensity in the same manner as the above Charles’ power law as:

\[
N = N_0 (K / K_0)^n
\]  

(4)

where \( n' \) is referred to as an ‘effective’ stress corrosion index and found to be equal to \( n \) within a few percent in brittle rocks [22].

The above model is somewhat based on the experimental results of microcracking during a single extensional macroscopic crack growth. However, it can be modified for a crack population of fractal size distribution by introducing only an effective mean value for each parameter [21]. Put the definition of \( K \) with \( q = 1/2 \) into equation (2) then the event rate can be expressed as

\[
\log N = \frac{2n'}{2-n} \log \left(1 - t / t_f\right) + n' \log \left(\frac{\sigma}{\sigma_0}\right) + \log N_0
\]  

(5)

For examining the fracture model, three granites WG, OG and IG are selected for systematic fracture tests. As listed in Table 1, the major difference of these granites is their grain size distribution. Two runs have been operated with constant stress rate at 27.5 MPa/m and 2.5 MPa/m, respectively. The confining pressure was kept constant at 50 MPa for all tests. Consistent with the previous study [8], the experimental data demonstrate that the damage creation, or in other words the microcracking activity, in these brittle rocks is characterized explicitly by three typical long-term phases referred to as primary, secondary and nucleation, respectively (Fig. 2). The above theoretical model is applicable to the secondary and nucleation phases but not to the behavior of the primary phase. Main features and fitting results are summarized as follows:

**Primary phase:** The event rate is low and increases slightly with stress or time. The \( b \)-value increases with increasing stress or time slightly from an initial value of 0.5–1.0 to 1.0–1.4. This initial value and increment depends strongly on grain size and density of pre-existing microcracks. In the fine-grained WG, the primary phase has a small number of events, the \( b \)-value is initially high and shows only a small increase (from ~1.3 to ~1.4). On the other hand, in the
Fig. 2 The $b$-values and event rates of Westerly, Oshima, and Inada granite samples under constant stress-rate loading at 2.5 MPa/s. Dashed lines show the fitting results of equation (5). Lower plots are zoom up views of the last 1000 seconds, corresponding to the secondary and nucleation phases. “P | S” and “S | N” mark the transitions from the primary to secondary and secondary to nucleation, respectively.

Coarse-grained IG, the primary phase has a larger number of events, the $b$-value is initially low and shows a large increase (from 0.5 to 1.2). The medium-grained OG shows a primary phase similar to IG, but with a smaller event number, higher initial $b$-value, and smaller $b$-value increase. In an earlier work, it has been found that the coarse-grained granitic porphyry (GP) sample of a very low pre-existing microcrack density demonstrates a primary phase of a small number of events [8]. Therefore, we conclude that the primary phase reflects the density and size distribution of pre-existing microcracks, which are somehow controlled by grain size distribution. Since the major mechanism of microcracking in the primary phase is an initial opening (tensile) or rupturing (shear) of pre-existing microcracks rather than crack-growth, the theoretical model based on sub-critical crack-growth is presumably inapplicable.

**Secondary phase:** The secondary phase corresponds to the microcracking activity following the primary phase. This phase shows increasing event rates and slightly decreasing $b$-values with increasing stress or time. The event rate data can be fitted very well with the theoretical model given that this phase is associated with the growth of pre-existing microcracks. For WG, OG, and IG samples, event rate $N$ can be represented well by equation (5) with parameters $n’ = 12$ and $n = 35$. Interaction between cracks and linkage of neighbor cracks become higher and higher as both the density and mean length increase together. That probably justifies that $n’ = \sim 0.3n$.

**Nucleation phase:** The nucleation corresponds to the period where event rate rapidly increases and the $b$-value decreases to the global minimum of 0.5-0.8.

Experimental data show also short-term fluctuations on both the event rate and $b$-value on the three long-term backgrounds. The amplitude of the fluctuations is function of the grain size.
in a positive relation. The larger the grain size, the larger the amplitude. The fluctuations change both in amplitude and period following the fracture evolution, and therefore they may be useful for failure prediction. However, this kind of fluctuation cannot be modeled from the above fracture constitutive laws but fortunately recent models based on renormalization-group theory are able to describe this phenomenon.

4. Damage Model Based on Renormalization-Group Theory

In this section, we describe a scheme for modeling the damage evolution based on the renormalization-group theory. The key to this approach is to consider the stressed materials or rocks as a thermodynamic system with the supplied energy coming from the applied external load and, its transformation up to rupture, as a second-order phase transition. This transformation is highly irreversible and the system state is never in equilibrium. Micro-separations are the dominant irreversible processes, and the energy release, which has not been consumed by irreversible transformations such as crack advances and plastic deformation around the crack tip, is converted into the decrease of the free energy of the system.

Concerning the catastrophic regime, we are currently far from establishing a behavior law from the statistical approach on a purely thermodynamic point of view. This is because fracturing is both a non-linear and unstable process where the long-distance interactions play a major role and the material properties are every time modified. However, based on the fact that the moment of rupture is similar to a critical point, the fracturing process can be described by a renormalization-group scheme [14]. In the vicinity of the critical point of rupture, there exists a critical region, where the variations of free energy, here referred to as the energy release rate, can be characterized by a power law of the normalized time \( x = (t - t_c)/t_c \), where \( t_c \) is the critical time, i.e., time of rupture, decorated by log-periodic oscillations [18]. For monophasic medium, the self-similar extrapolated form of the energy release far from the neighborhood of the critical point is written as [35]

\[
\frac{E(x = (t_c - t)/t_c)}{E(x = 0)} = \cos(c x^\alpha \sin g(x) \exp h(x)) \exp(c x^\alpha \cos g(x) \exp h(x)), x \in [0,1] \tag{6}
\]

where \( g(x) = \rho \sin(\omega \ln x + \varphi) \), \( h(x) = \rho \cos(\omega \ln x + \varphi) \), \( E(0) = E_c \), \( \rho \), \( \varphi \), \( c \), parameters, \( \alpha \) and \( \omega \) the real and imaginary part of the complex critical exponent \( z = \alpha + i \omega \). There is no theoretical limit for \( \omega \) whereas \( \alpha \) belongs to the interval \( [0, 1] \), as we are concerned with second-order phase transitions. The value of \( \alpha \) is correlated to both the heterogeneity and the disorder of the structure of materials. For pure crystals, the damage is not diffuse and the rupture is unpredictable, so one has \( \alpha \rightarrow 0^+ \). The value of \( \omega \) is somehow related with the interaction of events. Longer interaction distance (both spatial and temporal) results in larger \( \omega \).

The scheme from single domain can be extended to multi-domain in order to take account for successive dominant damage mechanisms in different stages or phases as presented in the previous section. Since the primary phase has a very low energy release rate, it is not considered here.

The cumulative energy release for above-mentioned experiments is shown in Fig. 3 together with the corresponding 2-domain fits based on the least-squares method. We focus on the first domains where damage randomly occurs over the whole sample, which confirm that 1) the higher the grain size, the longer the interaction, and the higher the \( \omega \); 2) the faster the loading
Fig. 3 Dimensionless cumulative energy release versus dimensionless time. a), b) and c) correspond to Inada, Oshima, Westerly granites, respectively. Upper and lower rows correspond to the results of fast and slow loading conditions, respectively. The black circles and the red lines represent the measurement data and the fits, respectively. Vertical dotted line indicates the boundary between the domains 1 and 2, respectively.

rate (stress-rate), the higher the $\omega$, and therefore the stronger the interaction; 3) the secondary domain, i.e., the nucleation phase, shows normally a higher $\omega$ than the first domain indicating the increase of interaction distance.

5. Conclusion

In our experiment, the nucleation of shear fault was found to correspond to the fracture of coupled asperities on the fault plane. The fracture of individual asperities has similar characteristics to the sequence of natural earthquakes, consisting of foreshocks, mainshock, and aftershocks. The progressive fracturing of multiple, coupled asperities during the nucleation of shear faulting results in short-term precursory fluctuations in both the $b$-value and event rate. For proposing more common models, three granites of different grain size distribution were used for fracture test under triaxial compression at two different loading rates of 27.5 and 2.5 MPa/min, respectively. The damage creation in these intact brittle rocks showed a typical damage creation process characterized by three phases of microcracking activity named as: primary, secondary, and nucleation phases, respectively. The primary phase reflects the initial opening or rupture of pre-existing microcracks, showing increases of event rate and $b$-value with increasing stress. The secondary phase corresponds to the sub-critical growth of the microcrack population, showing an increase of event rate and a decrease of $b$-value with increasing stress. The nucleation phase corresponds to the initiation and accelerated growth of the final fracturing along one or a few major fracture planes. In the nucleation phase, the $b$-value decreased rapidly to the minimum value. Both the event rate and $b$-value in the secondary and nucleation phases can be modeled very well by the model based on the sub-critical crack growth theory. Energy release data calculated from the event rate and magnitude can be modeled quite
well by the theoretical model considering materials failure as a second-order phase transition. These results confirm that the higher the grain size, the longer the interaction, and the higher the \( \omega \). Experimental results also indicate that the faster the loading rate (stress-rate), the higher the \( \omega \), and therefore the stronger the interaction.

Our results indicate that the precursor-based predictability of the catastrophic failure strongly depends on the pre-existing heterogeneity and loading condition. The three-phase model is meaningful for transforming the experimental results to real problems associated with rock bursts, volcanic eruptions as well as tectonic earthquakes.

Acknowledgments

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