DISTINCT ELEMENT ANALYSIS FOR ROCK FAILURE
CONSIDERING AE EVENTS GENERATED BY THE SLIP AT CRACK SURFACES

HIROYUKI SHIMIZU, SUMIHIKO MURATA and TSUYOSHI ISHIDA
Dept. of Civil and Earth Resources Engineering, Kyoto University
Katsura, Nishikyo, Kyoto, 615-8540, Japan

Abstract

As the fundamental research of rock fracture, we have simulated the uniaxial compression test of rock using the distinct element method (DEM) and discussed the influence of the slip at crack surface to a relative number of AE events. Simulation results agree well with the AE activities observed in an actual experiment and provide new findings to resolve the disagreement; the conventional theories and microscopic observations suggest that tensile cracks cause AE events, whereas an abundance of shear AE events is observed in experiments. Our simulation results indicate that the energy released from a tensile microcrack is very small and is most likely buried in noise compared with that from a shear crack, which should be observed predominantly, due to much smaller tensile strength compared to compressive strength. Further, AE is mainly generated from new tensile microcracks when the stress level is low, while the main sources of AE shift to the slip at the existing crack surface as the macroscopic failure approaches. That is, the burst of AE events during the formation of macroscopic fracture is from the slip occurrence at the existing crack surface. The results indicate that DEM is an effective numerical analysis technique for studying the dynamics of microcracking in brittle materials like rock.

Keywords: Distinct element method (DEM), Uniaxial compression, Crack, Slip, Rock

1. Introduction

Microcracking in rock is a very important issue in rock engineering, because macroscopic behaviors, such as fracture and failure, are strongly controlled by the generation and interaction of microcracks [1]. Actual rock specimen contains many pre-existing flaws such as pores, microcracks and grain boundaries. The fracture process of rock is complicated and sometimes shows probabilistic aspects because such microstructures in a rock specimen cause the heterogeneous transmission, orientation and magnitude of microscopic forces and moments.

In order to understand the mechanism of microcracking in brittle rock samples, a considerable amount of experiment has been conducted by various methods in the past few decades. Among them, one approach is monitoring acoustic emission (AE) events caused by microcracking activity. By using the recently developed high-speed, multichannel waveform recording device, we can record many waveforms of AE events associated with fracture process in a stressed rock specimens with high resolution. Thus, the measurement of the AE is an effective technique for studying the dynamics of microcracks [2-5].

However, even at present, it is still difficult to record the waveform of all AE events generated in an experiment due to the limitation of storage capacity and recording speed of a measuring device, and the influence of noise. In particular, when the catastrophic fracture is formed in a rock specimen, there is a burst of AE events in a short time. Therefore, sufficient AE waveform
data cannot be recorded by most experimental systems. In addition, though the generation of new cracks and propagation of existing cracks seems to be the dominant mechanisms of AE events, slipping at the crack surface should also cause AE events. However, it is difficult to distinguish AE events caused by slip at pre-existing crack surfaces from those by new cracks.

Hazzard et al. [6, 7] have reported the distinct element method (DEM) modeling for AE activity. They presented a technique to simulate AE behavior in brittle rock under uniaxial compression using the commercially available DEM code (particle flow code: PFC) by considering the kinetic energy released when the bonds break. The DEM can represent grain-scale microstructural features directly by considering each grain in actual rock as a DEM particle. The grain-scale discontinuities in the DEM model induce complex macroscopic behaviors without complicated constitutive laws [8, 9]. However, one of inaccuracies with the AE produced by their PFC model is the narrow range in observed magnitudes and consequently low b-values. According to their results, the magnitude of smallest AE events produced by their model is about an order larger than the corresponding actual AE monitoring. One possible solution for this problem is to somehow consider re-activation of cracks such that seismicity could occur on the contacts where bonds had already broken [6, 7].

Therefore, we have newly programmed our own DEM code that can model the AE events generated by the slip at pre-existing crack surfaces, and have simulated the uniaxial compression test of rock by using our DEM model. The mechanical behavior in a brittle rock including not only generation of microcracks but also slip occurrence at existing crack surfaces can be discussed in detail. The simulation results are compared with the fracture process deduced from the laboratory AE measurements conducted by previous researchers in order to discuss the process, in which microcracks are induced inside a rock and result in a macroscopic fracture.

2. Simulation methodology

2.1 Formulation of mechanics of bonded particles

In this study, two-dimensional distinct element method (2D-DEM) was employed. The DEM for granular materials was originally developed by Cundall and Strack [8]. In this section, only a summary of formulation for the mechanical behavior of bonded particles will be given.

In 2D-DEM, the intact rock is modeled as a dense packing of small rigid circular particles. Neighboring particles are bonded together at their contact points with a set of three kinds of springs as shown in Fig. 1 and interact with each other. The increments of normal force \( f_n \), the tangential force \( f_s \), and the moment \( f_\theta \) can be calculated from the relative motion of the bonded particles, and are given as

\[
f_n = k_n (dn_j - dn_i)
\]

\[
f_s = k_s \left\{ ds_j - ds_i - \frac{L}{2} (d\theta_j + d\theta_i) \right\}
\]

\[
f_\theta = k_\theta (d\theta_j - d\theta_i)
\]

where, \( k_n \), \( k_s \), and \( k_\theta \) are the stiffness of normal, shear, and rotational springs, respectively; \( dn \), \( ds \) and \( d\theta \) are normal and shear displacements and rotation of particles; \( r_i \) and \( r_j \) are the radii of the bonded particles. A bond between the particles is presented schematically as a
gray rectangle in Fig. 2, where, \( L \) and \( D \) are the bond length and the bond diameter, respectively. \( D \) is obtained from harmonic mean of the radius of two particles. \( L \) and \( D \) are given by

\[
L = r_i + r_j \tag{4}
\]

\[
D = 2 \cdot \frac{2r_i r_j}{r_i + r_j} \tag{5}
\]

The stiffness of the normal and rotational springs, \( k_n \) and \( k_\theta \) are calculated using beam theory, and the stiffness of shear springs \( k_s \) is calculated by multiplying the stiffness of the normal spring \( k_n \) and stiffness ratio \( \alpha \) [9]. Thus, the stiffness of the springs is given by the following equations:

\[
k_n = \frac{E_p A}{L} \tag{6}
\]

\[
k_s = \alpha \cdot k_n \tag{7}
\]

\[
k_\theta = \frac{E_p I}{L} \tag{8}
\]
where $A$ and $I$ are the area and moment of inertia of the bonds, and $E_p$ is the Young’s modulus of particle and bonds. The moment of inertia $I$ depends on the shape of the cross-section, and rectangular cross-section is assumed in this study.

The normal stress $\sigma$ and shear stress $\tau$ acting on the cross-section of the bond are calculated using the following equations. The stress and the strain are positive in compression.

\[
\sigma = \frac{f_n}{D} \quad \text{(9)}
\]

\[
\tau = \frac{f_s}{D} \quad \text{(10)}
\]

2.2 Microcrack generation and slip occurrence

When $\sigma$ exceeds the strength of normal spring $\sigma_c$ or $\tau$ exceeds the strength of shear spring $\tau_c$, then the bond breaks and three springs are removed from the model altogether. Each bond breakage represents generated microcracks. A microcrack is generated at the contact point between two particles, and the direction of it is perpendicular to the line joining the two centers.

- **(Bond break criterion 1)** $|\sigma| \geq \sigma_c$ and $\sigma < 0$ (tensile stress)
- **(Bond break criterion 2)** $|\tau| \geq \tau_c$

In the parallel-bond model developed by Potyondy and Cundall [9], the moment acting on the parallel-bond (which is expressed as elastic beam) contributes the normal stress acting on the particles. This means that the bond breakage is judged by the maximum tensile stress acting on the cross section of the assumed elastic beam. On the other hand, in this study, since the spring is introduced to restrict the rotation of the particles and used only to calculate the moment acting on the particles, the normal stress calculated by equation (9) does not include the moment of the elastic beam. This means that the bond breakage in our model is judged by the average normal stress acting on the cross section of the assumed elastic beam. This is the difference in the mechanism of particle bondage between the parallel-bond model proposed by Potyondy and Cundall and our model presented in this paper.

When the unbonded particles or particles with bond breakage are in contact with each other, springs and dashpots are introduced into the contact points in both normal and tangential directions, and compressive normal force $f_n$ and tangential (frictional) force $f_s$ act at the contact points. The no-tension constraint condition should be satisfied for the springs in the normal direction. If the frictional force $f_s$ exceeds the critical value $f_{s,\text{max}}$, the slip occurs at the contact points between the particles and the frictional force $f_s$ will be replaced. According to the Coulomb’s frictional law, the critical value $f_{s,\text{max}}$ is calculated by the following equation.

\[
f_{s,\text{max}} = \tan \phi_f \cdot f_n \quad \text{(11)}
\]

where $\tan \phi_f$ is a coefficient of friction.

2.3 Correlation with AE

In actual AE measurement, the AE hypocenter can be calculated by the arrival time of the P-wave first motion and focal mechanisms of AE events are determined from the spatial distribution of P-wave first-motion polarities [10]. For tensile AE, all sensors detect the P-wave first motion as compression wave. On the other hand, for shear AE, both compressional and dilatational
P-wave first motions are detected. This suggests that the mode of cracking (tensile or shear) depends on the stress state at the crack generation because the polarity of the P-wave first motion will depend on the stress state. Therefore, in our previous works [11, 12], the crack modes in the DEM simulation are classified by shear-tensile stress ratio $|$τ/σ$|$ regardless of broken spring type (normal spring or shear spring) as follows.

(Crack classification criterion 1) $|$τ/σ$|$ ≤ 1 and $σ < 0$ (tensile stress) Tensile Crack
(Crack classification criterion 2) $|$τ/σ$|$ > 1 and $σ < 0$ (tensile stress) Shear Crack
(Crack classification criterion 3) $σ > 0$ (compressive stress) Shear Crack

In this research, in addition to the shear AE and the tensile AE, we have introduced the classification and the failure criterion for the slip AE in our own code by expanding conventional concept of the DEM [13]. When the frictional force acting at the contact points exceeds the critical value, the slip occurs as mentioned in previous section. It is thought that such a slip occurring at the crack surface should also generate AE events. Thus, the slip at crack surfaces is added to the bond breakage as a possible mechanism of AE event occurrence. Consequently, AE events in the DEM simulation are classified by their source mechanisms as follows.

- Generation of new tensile cracks $→$ Tensile AE
- Generation of new shear cracks $→$ Shear AE
- Slip occurrence at the crack surface $→$ Slip AE

When a new microcrack is generated, the strain energy stored in both normal and shear springs at the contact point is released. The strain energy $E_k$ calculated using following equation is assumed to be the energy corresponding to the magnitude of tensile and shear AE event.

$$E_k = \frac{f_n^2}{2k_n} + \frac{f_s^2}{2k_s} \quad \text{(12)}$$

On the other hand, when a slip occurs, frictional force will be replaced by the critical value calculated by the equation (11). During this process, the strain energy stored in springs at the contact point is partly released. The released strain energy $E_{slip}$ is given by

$$E_{slip} = E_{kaf ter} - E_{kbef ore} \quad \text{(13)}$$

where $E_{kaf ter}$ and $E_{kbef ore}$ are the strain energy calculated by equation (12) at the time step before and after slip occurrence, respectively. The released strain energy $E_{slip}$ is assumed to be the energy corresponding to the magnitude of slip AE.

3. Rock Specimen Model and the Loading Condition for the Simulation

As shown in Fig. 3, the rock model, which was 10 cm in width and 20 cm in height, was used to simulate the uniaxial compression test. The rock model is expressed by the assembly of particles bonded to each other. The particle radius was chosen to have a uniform distribution between maximum radius and minimum radius. The number of particles was 9319. The particles were irregularly arranged in positions by using a random number.

The platen under the rock model was fixed and the upper loading platen was moved downward slowly to reproduce the uniaxial compression test. At this time, frictional force was acting between the rock model and the platens.
Fig. 3 Loading condition for the simulation of uniaxial compression tests. The monitored particles for the axial and radial strain were located slightly inside from the edge of the rock model. The distance between two measuring points is 90% of the rock model width or height.

The axial stress applied to the rock model during the uniaxial compression test was calculated from total force acting on the upper loading platen from particles and model width. The strain is calculated by displacements of the monitored particles. As shown in Fig. 3, four monitored particles for the axial and radial strain were located slightly inside from the edge of the rock model. The distance between two measuring points is 90% of the rock model width or height. Axial strain $\varepsilon_1$ and radial strain $\varepsilon_2$ can be calculated using the following equations.

$$
\varepsilon_1 = \frac{(y'_4 - y'_2) - (y'_4 - y'_2)}{y'_4 - y'_2} \\
\varepsilon_2 = \frac{(x'_3 - x'_1) - (x'_3 - x'_1)}{x'_3 - x'_1}
$$

where superscript 0 and $t$ means initial and measuring time, respectively. Plane strain condition is assumed to calculate elastic macroscopic parameters, and Young’s modulus and Poisson’s ratio were calculated according to the ISRM (International Society for Rock Mechanics) Suggested Method [14, 15]. For proper simulation using DEM, appropriate microscopic parameters are required. Therefore, preliminary simulations of the uniaxial compression test and the Brazilian test were repeated beforehand, and the microscopic parameters should be adjusted to represent a certain macroscopic mechanical properties. In this study, macroscopic mechanical properties of Kurokamijima granite are used to calibrate the microscopic parameters. The microscopic parameters and calibration results are shown in Table 1.
Table 1. Rock model properties and calibration results.

<table>
<thead>
<tr>
<th>ROCK MODEL DATA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of the Rock model:</td>
<td>100mm</td>
</tr>
<tr>
<td>Height of the Rock model:</td>
<td>200mm</td>
</tr>
<tr>
<td>Number of particles:</td>
<td>9319</td>
</tr>
<tr>
<td>Maximum particle radius:</td>
<td>1.0mm</td>
</tr>
<tr>
<td>Minimum particle radius:</td>
<td>0.5mm</td>
</tr>
<tr>
<td>Particle density:</td>
<td>2620 kg/m$^3$</td>
</tr>
<tr>
<td>Friction coefficient of platen ($\tan \phi_w$):</td>
<td>0.5</td>
</tr>
<tr>
<td>Poisson’s Ratio of platen ($\nu_w$):</td>
<td>0.3</td>
</tr>
<tr>
<td>Young’s modulus of platen ($E_w$):</td>
<td>200GPa</td>
</tr>
<tr>
<td>Friction coefficient of particle ($\tan \phi_p$):</td>
<td>0.3</td>
</tr>
<tr>
<td>Poisson’s Ratio of particle ($\nu_p$):</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TUNING PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of particle ($E_p$):</td>
<td>87.0 (GPa)</td>
</tr>
<tr>
<td>Shear/normal spring stiffness ratio ($\alpha$):</td>
<td>0.54</td>
</tr>
<tr>
<td>Shear strength of bonding ($\tau_b$):</td>
<td>235.0 (MPa)</td>
</tr>
<tr>
<td>Tensile strength of bonding ($\sigma_b$):</td>
<td>26.0 (MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CALIBRATION RESULTS</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS of rock model (MPa):</td>
<td>200</td>
<td>200.63</td>
</tr>
<tr>
<td>Young’s modulus of rock model (GPa):</td>
<td>70.0</td>
<td>69.25</td>
</tr>
<tr>
<td>Poisson’s Ratio of rock model:</td>
<td>0.250</td>
<td>0.244</td>
</tr>
<tr>
<td>Tensile strength of rock model (MPa):</td>
<td>10.0</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Fig. 4 Stress-strain curves.
4. Simulation Results

4.1 Stress-strain curves

Figure 4 shows the stress-strain curves obtained from the DEM simulation. Though actual deformation is three-dimensional, this simulation is two-dimensional, and the strain in the direction of depth is not considered. Therefore, the volumetric strain $\varepsilon_v$ in this simulation is defined by using the axial strain $\varepsilon_1$ and lateral strain $\varepsilon_2$, as below. The stress and the strain are positive in compression.

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_2$$  \hspace{1cm} (16)

Figure 5 shows the relation between the axial stress and the number of AE events. The solid line in Fig. 5 shows the evolution of the axial stress. The open, closed and hatched bar diagrams in the figure express the number of tensile AE, shear AE and slip AE, respectively. As shown in Fig. 5, the number of total AE event increases gradually as the axial stress increases. This result agrees well the typical tendency observed in actual rock fracture under compression [16].

![Fig. 5 Transition of the number of cracks and slip with the evolution of the axial stress.](image)

Figure 6 shows the close-up view of the dotted rectangle in Fig. 5 to clarify the activities of shear and tensile AE. The solid line in Fig. 6 shows evolution of the volumetric strain. The volumetric strain increases (volume of the model decreases) constantly in the initial stage of the loading, and gradually changes into nonlinear behavior as the axial stress increases. It is known that the dilatancy in an actual rock is caused by the growth and opening of microcracks. When a shear crack is generated and slip occurs at the existing crack surface, the tensile cracks develop from both ends of the shear crack with large opening of tensile cracks [17, 18]. Then, the volume of the model increases, and the dilatancy occurs. As shown in Fig. 6, the volumetric strain curve begins to change when the generation of shear AE begins, and decreases (volume of the model increases) with an increase in shear AE and slip AE. This result indicates that occurrence of the
Fig. 6 Close-up view of the transition of the number of shear and tensile AE (dotted rectangle in Fig. 5) with the evolution of the volume strain.

Fig. 7 Spatial distribution of the tensile and the shear AE events generated in each phase. Tensile and shear cracks are expressed as closed and open circles, respectively. The diameters of each circle correspond to their respective magnitudes of energy. (a) Phase I [Step 1-190 ($\times 10^4$)]. (b) Phase II [Step 190-320 ($\times 10^4$)]. (c) Phase III [Step 320-360 ($\times 10^4$)]
dilatancy observed in an actual uniaxial compression test can be appropriately reproduced by the DEM simulation.

4.2 Transition of the number of AE events and AE source mechanism

As shown in Figs. 5 and 6, the rock fracture process under uniaxial compression can be divided into three phases (Phase I, II and III) according to the AE activities [5]. Figure 7(a), (b) and (c) show the spatial distribution of the tensile and the shear AE events in each phase, respectively. The tensile and shear AE are classified and expressed as closed and open circles, respectively. The diameters of each circle correspond to respective magnitude of tensile and shear AE obtained by equation (12). On the other hand, Fig. 8(a), (b) and (c) show the spatial distribution of the slip AE in each phase, respectively. The diameters of the circle correspond to respective magnitude of slip AE obtained by equation (13). The AE activities in each phase are described as follows.

Fig. 8 Spatial distribution of the slip AE events generated in each phase. The diameters of each circle correspond to their respective magnitudes of energy. (a) Phase I [Step 1-190 (×10⁴)]. (b) Phase II [Step 190-320 (×10⁴)]. (c) Phase III [Step 320-360 (×10⁴)]

In Phase I, tensile AE initiated at a stress level about 35% of the uniaxial strength. As the axial stress increases, the number of AE events increases gradually and shear AE also initiated. As shown in the bar diagram in Figs. 5 and 6, dominant mechanism of the AE events in low stress level was new tensile microcrack generation.

As shown in Fig. 7(a), the energy of AE events generated in Phase I was very small. Although these AE events were widely distributed over the whole model, the density of AE events decreased from the center toward the loaded ends of the rock model. On the other hand, no slip AE was generated in Phase I as shown in Fig. 8(a). After the tensile crack generation, these
tensile cracks opened immediately due to the tensile stress acting perpendicular to the loading axis. Thus, the surface of open crack never touched mutually, and slip did not occur.

In Phase II, in addition to the tensile and shear AE, the slip AE began to be generated. As the axial stress increases, the number of slip AE increased further. This result suggests that the dominant mechanism of the AE occurrence changes from new crack generation to slip occurrence. As shown in Fig. 6, burst of microcracking observed temporarily in this phase, and the number of microcracks decreased substantially after each burst of AE. By comparing the AE magnitudes and location shown in Fig. 7(b) and Fig. 8(b), it is found that a few shear AE events that release comparatively large energy were generated in this phase. The slip AE events were generated at the same positions where the strong shear AE occurred.

In Phase III, the number of AE events increased rapidly. A macroscopic fracture was formed in a very short time, and the model resulted in collapse. The macroscopic fracture grew toward upper left and right from the center of the model. At this stage, 95% of AE events were due to the slip occurrence. As shown in Fig. 7(c), the shear and tensile AE concentrated near the center of the model and they progressed to both the upper left and upper right of the model along the macroscopic fracture. These AE events were the shear AE and released large energy compared with other AE. Moreover, the slip AE events that released large energy were also generated along the macroscopic fracture path as shown in Fig. 8(c).

### 4.3 b-value

The b-value is defined as the log-linear slope of the frequency–magnitude distribution of AE [19, 20]. It represents the scaling of magnitude distribution of AE, and is a measure of the relative numbers of small and large AE, which are signatures of localized failures in materials under stress. A high b-value arises due to relatively large number of small AE events comparing to the number of AE events that have relatively large amplitude. A low b-value arises in the contrary case. The b-value is calculated by the Gutenberg–Richter relationship [21], which is widely used in seismology. The equation is as follows.

\[
\log_{10} n = a - bM
\]  

(17)

where \( M \) is the magnitude of AE event, \( n \) is the number of AE events of magnitude \( M \) or greater, \( a \) is a constant and \( b \) is the seismic b-value.

In this simulation, the magnitude \( M \) of an AE event is calculated using equation (18) as logarithm of the energy obtained by equations (12) and (13), and the b-value was calculated by the maximum likelihood method using equation (19) [22, 23].

\[
M = \log_{10} \left( E_h \right)
\]

(18)

\[
b = \frac{n \cdot \log_{10} e}{\sum_{i=1}^{n} M_i - nM_m}
\]

(19)

where \( M_m \) is the minimum magnitude of AE event.

In this simulation, AE events with extremely small magnitude can be observed. However, such small AE events are hardly observed in an actual AE experiment due to the influence of noise. For this reason, AE events having magnitude of less than \( M_m = -3.2 \) were excluded from the calculation of the b-value in this study.
Figure 9 shows the relation between the magnitude of AE events and cumulative AE events with corresponding b-value in each phase. In Phase I, b-value is relatively high at 1.46, since all AE events generated are small. In Phase III, b-value decreased to 0.68 as the axial stress increased. This result agrees well with the trend of actual AE measurements conducted by Lei et al. [4, 5, 24].

The strain energy given by equation (12) or (13) is released from the model when a bond breaks or a slip occurs. This produces a force imbalance, and subsequent stress redistribution induces an AE event. Therefore, logarithm of the energy given by equation (18) does not directly express the magnitude of AE event. However, as shown in Fig. 9, the relation between the magnitudes calculated from equation (18) and the number of AE events appropriately represents the tendency of actual AE. This finding suggests that the strain energy given by equation (12) or (13) is at least qualitatively valid as a value that corresponds to the magnitude of AE. Moreover, several researchers pointed out that the b-value depends on the heterogeneity of rock [4, 5, 19, 20]. Therefore, the DEM simulations with various heterogeneous rock models are effective to discuss the influence of the heterogeneity on the fracturing process of rock that is difficult to examine in experiment.

5. Discussion

5.1 Generation of tensile AE at lower stress level

During Phase I, tensile AE events were dominant and widely distributed over the whole model. This result is in agreement with experiment that shows the major mechanism of the AE events at lower stress level being the tensile cracks associated with the initial rupture of pre-existing flaws [4, 5]. This indicates that the DEM can successfully represent the grain-scale microstructures such as pores, microcracks and grain boundaries directly by considering each grain as a DEM particle.

Figures 10(a-c) show the distribution of maximum principal stress, the minimum principal stress, and the maximum shear stress in the model at time step, $71 \times 10^4$. The stress is positive in
compression. The arrows in Fig. 10(a) and (b) indicate the direction of maximum and minimum principal stress, respectively. As shown in the figure, the stress distribution in the rock model is non-uniform. This is because the stresses that act between particles are evaluated by using the radii of the particles. The radius and position of a particle are generated by random numbers, while the microscopic parameters, such as Young’s modulus and strength of the spring, are constant. Therefore, the transmission of force becomes irregular, and the stress distribution in the rock models is heterogeneous.

As shown in Fig. 10(b), there are some regions where a relatively large tensile stress exists. The tensile AE events were predominantly generated in such regions because the tensile strength of the spring that connects between particles is small compared with the shear strength as shown in Table 1. Thus, the tensile micro-cracks are widely distributed in the rock model. However, the number of macro-cracks is few in Phase I as the tensile micro-cracks do not influence each other and did not grow further. Figure 10(b) also indicates that the tensile stress at the loaded ends of the rock model is lower than elsewhere. According to this stress distribution, the density of AE events decreases from the center toward the loaded ends. This is due to the frictional restraints between the rock model and the loading platen interfaces [25].

Fig. 10  Stress distribution at time step $71 \times 10^4$. Cracks initiate at this time step. (a) Maximum principal stress, (b) Minimum principal stress, (c) Maximum shear stress.

5.2 AE clustering

Phase II produced a few shear AE events that released comparatively large energy. Additionally, slip AE events were generated at the same position as shear events, previously shown in Fig. 7(b) and Fig. 8(b). Figure 11 shows the cumulative distribution of all AE events (tensile, shear and slip AE) in Phase II. The size of each symbol corresponds to the number of the overlapping AE events. We find that occurrence of AE events became active at several points of the model in this phase. Such concentration of AE events is usually called “clustering”.

The total number of micro-cracks increases, intensifying the interaction between micro-cracks in Phase II. Once the interaction becomes strong enough within a certain region, enhancing the local stress concentration, new micro-cracks are generated one after another in the same region and an AE cluster is formed [2, 26]. Such a concentration of micro-crack generation relieves local stress. When stress has been sufficiently relieved in the region, a new micro-crack stops forming.
After the AE cluster formation, microcracking activity migrates to other clustering regions, and many small AE clusters are formed [2]. Thus, many small AE clusters form in Phase II, with attendant reduction in the number of microcracking after each clustering as shown in Fig. 6.

Fig. 11 Spatial distribution of AE events (tensile, shear and slip AE) in Phase II. The size of each symbol corresponds to the number of the overlapping AE events.

Fig. 12 Propagation processes of the macroscopic fracture in four periods of Phase III. Small microcracks are ignored. (a) Step $339 \times 10^4$, (b) Step $340 \times 10^4$, (c) Step $341 \times 10^4$, (d) Step $342 \times 10^4$.

5.3 Formation of the catastrophic fracture

In Phase III, a catastrophic fracture was formed and the model resulted in collapse within a very short time. Figure 12 shows the propagation processes of the macroscopic fracture in four periods of Phase III preceding the collapse. The solid lines express the fracture, which is represented by the connection of large opened microcracks. To clarify the macroscopic fracture, small
dispersed microcracks are ignored in these figures. Figure 12(a) shows microcracks initially concentrated near the center of the model in the region surrounded by the dotted ellipse. Most of the microcracks are tensile cracks, and stably propagated in the direction of loading axis.

At the next time step \((340 \times 10^4)\), shown in Fig. 12(b), microcracks were repeatedly connected by sliding and the fracture grew rapidly toward top left (see arrow). Next, at time step \(341 \times 10^4\), fracture also grew toward top right from the center (arrow in Fig. 12(c)). Most of the microcracks generated in these two time steps were shear cracks, and these cracks released large energy, as shown in Fig. 7(c). Such concentration of shear cracks is called “shear band”. This result suggests that the formation of shear bands is guided by the development of a process zone where the tensile microcracks have coalesced into dominant shear cracks in this phase [2, 5].

Finally, as shown in Fig. 12(d), a large wedge-shaped block is separated from the rock model by the formation of shear bands. The wedge-shaped block moves downward by the loading in the direction as shown by an open arrow, and many tensile fractures propagate toward the bottom of the rock model in the region surrounded by the dotted circle in Fig. 12(d).

Numerous slips occurred at the existing crack surfaces due to the impact from the formation of shear band, and many slip AE events occurred. Moreover, strong slip AE events were generated at the wedge-shaped block in the rock model (cf. Fig. 8(c)). This suggests that the burst of AE events when the rock model collapsed was governed by the slip of pre-existing cracks.

Numerous microcracks developed in Phase III. The interaction among the microcracks is strong and the local stress concentration is very intense compared with the previous two phases. Therefore, Phase III is unstable and once a catastrophic fracture initiated at one location, microcracks joined catastrophically until completely collapse [2, 4, 5]. This process is similar to the AE clustering process in Phase II, but the stress level is lower and the interaction among microcracks is less. Therefore, each AE cluster cannot sufficiently grow.

Since many strong AE events occurs during the formation of shear bands, weaker AE waves may be hidden, making it is difficult to locate the sources of all AE correctly in experiment. On the other hand, the forming processes of the cluster and the shear band are difficult to evaluate in experiment, but the DEM reveals details of such processes.

5.4 Comparison of the energy

The conventional theories suggest that tensile cracks cause AE events because the number of accumulated AE events is positively related to the amount of the dilatancy, and the tensile strength of rock is obviously small compared with compressive strength [27]. The microscopic observations also revealed that many tensile cracks exist in the rock specimen under uniaxial compression, and the shear crack is few [28], indicating the dominant mechanism of AE to be tensile crack. Figure 13(a) expresses the spatial distribution of all the cracks generated during this simulation. Many tensile cracks were generated. 72% of all the cracks generated in Phases I and II were tensile. This is in accord with the conventional theories and microscopic observations. In AE experiment, many of observed AE events originated in the generation of shear cracks [2, 29, 30]. Thus, there is an inconsistency between the conventional theory and the AE results. This simulation resolves the inconsistency by considering the energy of AE as discussed below.
Fig. 13  Spatial distribution of all the cracks obtained during this simulation. (a) A tensile crack is shown with a closed circle of same size, (b) a shear crack is expressed with an open circle. Its diameter indicates energy magnitude.

Figure 13(b) expresses the spatial distribution of the shear cracks, and the diameters of the circles correspond to their respective magnitudes of energy. It turns out that the energy released from a tensile crack is small compared with that of a shear crack. Theoretically, the same result has been predicted [31, 32]. The present simulation results are consistent with the theory.

Although a large number of the tensile cracks are generated in the simulation, the energy released from the tensile cracks is small because the tensile strength of rock is low. Such weak AE is easily buried in noise and hard to detect in experiment. Lei at al. [2] recorded several thousands AE events with waveforms and more than 50% of the recorded events were located appropriately. However, only 10% among the located events have clear P-wave first motions and reliable focal mechanism solutions can be obtained from their radiation pattern. It is difficult to make clear assignments of the focal mechanisms for other events since some of polarities of the first motions cannot be determined due to their vague first motions. In AE experiment, energetic shear AE events that can be recorded with clear waveforms are observed predominantly.

6. Conclusion

We simulated the uniaxial compression test of rock using a self-programmed DEM code considering AE events generated by the slip at crack surfaces. The findings are as follow:

1. The volumetric strain increases constantly in the first stage of the loading, and gradually changes into nonlinear behavior as the axial stress increases. The volumetric strain curve began to change when the generation of shear AE begins. Our research indicates that occur-
2. Since extremely energetic AE events occur during the formation of shear bands, weaker AE waves of tensile microcracks may be hidden. Therefore, it is difficult to locate all AE sources correctly in experiment. The formation of cluster and shear band is difficult to follow in experiment, but can be evaluated in detail using the DEM simulation.

3. Initially, dominant mechanism of AE events under low stress level in a uniaxial compression test was tensile microcracking. As the axial stress increases, the dominant mechanism of AE changed to the slip occurrence at the existing crack surface.

4. The burst of AE events when the rock model resulted in collapse was governed by the slip occurrence at the existing crack surface.

5. The simulation result indicates that the rock fracturing process proceeds in three phases. In Phase I, tensile microcracks are dominant. The microstructures of rock such as pores, microcracks and grain boundaries govern this process. In Phase II, the number of cracks increases and the interaction between the cracks becomes stronger. This induces the coalescence of neighboring microcracks and results in clustering of microcracks. In Phase III, once a catastrophic fracture initiated at one location, it grows rapidly within a very short time. The catastrophic fracturing is guided by the development of a process zone encompassing tensile cracks.

6. The b-value at the beginning of loading (Phase I) is high because all AE events generated in this phase were weak. The b-value decreases as the axial stress increases, and becomes the lowest at the collapse stage of Phase III. This result agrees with the tendency of actual AE experiment. Since the b-value depends on the heterogeneity of the rock, the DEM simulations are effective to examine the influence of the heterogeneity on the fracturing process of the various heterogeneous rocks. This is difficult to do in experiment.

7. The conventional theories and the microscopic observations suggest that tensile cracks cause AE events. Many tensile cracks are generated during the rock fracturing under uniaxial compression, but the energy released from the tensile microcracks is small because the tensile strength of rock is small. The weak AE is easily buried in noise and may be missed in experiment. In AE experiment, many AE originated from the generation of shear cracks. This inconsistency can be resolved by considering the energy of AE and shear AE with large energy is dominantly observed.

The results of our simulation can explain time-space distribution of AE activity in the course of a uniaxial compression test, and agree well with the fracturing process deduced from previous AE measurements in laboratory. This indicates that DEM is an effective numerical analysis technique for studying the dynamics of microcracking in brittle materials like rock.

References