IDENTIFYING QUASI-BRITTLE FRACTURE BY AE AND DIGITAL IMAGING

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Abstract

Fracture of rock and other quasi-brittle material exhibits significant microcracking, so the identification of crack growth is well suited for AE, although interpretation of detailed features often relies on supplemental measurements or analyses. By recording the time histories and determining the arrival time of the P-wave in each waveform, it is possible to locate an AE event in a laboratory specimen with an error of a few mm. Opening and mixed-mode fracture experiments were performed within a closed-loop, servo-controlled testing system using a three-point bend specimen with a central or an eccentric notch, resulting in a combination of Mode I and II stress intensity factors ($K_I$ and $K_{II}$). The experimental setup included AE monitoring and digital image correlation (DIC), a particle tracking method that can be used to determine displacements of a speckle pattern in a digital image. The matching process is the identification, between two images, of a small area called a subset, which has a unique intensity pattern. Cross-correlation with a fast Fourier transform method was used to search the intensity values in a region of interest such that the displacement vector was obtained for each subset. A cluster of microcrack locations clearly identified the fracture and its growth. Complimentary analyses from DIC provided (1) a detailed position of the crack tip, and (2) a zone of increased tangential displacement but no differential movement near the crack tip, suggesting that the process zone in “slight mixed-mode loading ($K_{II}/K_I = 5\%$) is governed by Mode I opening.

Keywords: Digital image correlation (DIC), mixed-mode fracture, process zone, quasi-brittle material.

Introduction

Fracture of a quasi-brittle material such as rock generates microcracking, clearly identified by AE [1]. Indeed, the existence of a process zone is often directly related to the locations of AE, and crack growth can be monitored by tracking the event hypocenters. Furthermore, numerical analyses of fracture experiments suggest that AE locations are associated with the process zone [2].

Opening and mixed-mode fracture tests were performed using a three-point bend specimen with either a center or an off-center (eccentric) notch within a closed-loop, servo-controlled load frame. A mixed-mode loading condition was achieved by changing the notch position along the tensile region of the beam, resulting in different combinations of Mode I and II stress intensity factors. The experimental setup included AE monitoring with eight sensors at known positions, and digital image correlation (DIC) with a charge-coupled device (CCD) camera and a corresponding image acquisition system. The resolution of measurements from AE is on the order of mm for the laboratory experiment, and the locations are three-dimensional. In contrast to AE, DIC, with displacement accuracy of a few microns, provides a detailed picture of the...
fracture process by determining the displacement fields. However, a limitation of DIC, at least with the use of surface photographs, is the two-dimensional nature of the observations, whereas AE gives information through the volume. Thus, the two techniques are complimentary in the study of rock fracture.

Background

**AE locations**

A common type of source location algorithm involves the arrival time of the P-wave [3]. Microseismic activity due to a change in stress or environment is recorded by each sensor with a known position at a given time. From the relative arrival times of the P-wave and a known (separately measured) P-wave velocity of the material, the event hypocenter can be determined with a minimum of five sensors. The problem contains four unknowns: the spatial coordinates \(x, y, z\) of the event and the time \(t\), at which the event occurred, but a fifth sensor (or other information) is needed to remove ambiguities arising from the quadratic nature of the distance equation. Because some error is associated with arrival-time detection of low amplitude signals and with the P-wave velocity model, the number of sensors should be increased so that the location problem becomes over-determined. Then an algorithm can be developed whereby the error is minimized to obtain a best-fit type of solution, and statistical methods can be used to evaluate the goodness of the fit.

The distance \(r_i\) between the source and the \(i^{th}\) sensor is related to the P-wave velocity \(c_p\) by

\[
r_i = c_p(t_i - t) + \varepsilon_i
\]

where \(t\) is time at which the event occurs, \(t_i\) is arrival time at the \(i^{th}\) sensor, and \(\varepsilon_i\) is residual of computed distances. From equation 1, it can be seen that a time shift does not affect the source location, so an arbitrary time base can be selected. The travel distance \(r_i\) can be expressed by the unknown source coordinates \((x, y, z)\) and the known sensor coordinates \((x_i, y_i, z_i)\) by

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}
\]

The sum of the squares of the residuals \(\varepsilon_i\) can be written

\[
I = \sum_{i=1}^{N} \varepsilon_i^2
\]

where \(N\) is the number of sensors. The unknowns \(x, y, z\) and \(t\) can be determined using a least-squares method by minimizing \(I\). However, the equations are nonlinear in the source coordinates \(x, y, z\), so the minimization is carried out numerically using the Levenberg-Marquardt algorithm. The first estimate of \(x, y, z\) is obtained by a linearization of the equations [4].

**Digital Image Correlation (DIC)**

DIC refers to a group of non-contacting approaches that can extract full-field displacements through imaging analysis, based on the acquired digital photograph of an object [5, 6]. The determination of displacement relies on matching a subregion that has a unique intensity pattern between the two images. This subregion, named a subset, acts as a target during the matching process. Generally, a subset is small as compared to the corresponding specimen surface, such that it is not practical to search the whole field to find a subset. Also, the displacements involved in the measurement are generally small too, so it is not necessary to examine the whole image to find it. Thus, a relatively larger area surrounding a subset, named the region of interest (ROI), can serve as a domain for the identification of the subset.
For DIC processing, two digital images are selected, where the image before deformation is called the reference image and the image of the deformed surface is called the objective or current. Based on the prepared speckle pattern of the specimen surface or simply the natural surface characteristics, the size of the subset and corresponding ROI are selected, as shown in Fig. 1. The purpose of DIC is to correlate the original and displaced subsets through the best matching of the intensity patterns. In other words, if the intensity patterns of the two subsets in the reference and objective images are best matched, the two subsets are considered the same, before and after deformation.

![Figure 1. Displacement (u and v) of a subset in the region of interest.](image)

Figure 1 shows the subset and ROI within the reference image, and the original subset is at the center of ROI at position \( P(X, Y) \). The intensity values that are located at position \( P \) can be represented as

\[
I(P) = I(X, Y)
\]  

(4)

After deformation, point \( P \) is displaced to position \( p \) and the intensity values are changed to

\[
I'(p) = I'(X + u, Y + v)
\]  

(5)

To find the best match of the two subsets, cross-correlation with a fast Fourier transform (FFT) method is used to search the intensity values in the ROI. The cross-correlation function \( R \) is defined as the two-dimensional spatial convolution of \( I \) and \( I' \) with the separation vector \( s \) in the correlation plane:

\[
R(s) = \int I(P)I'(P + s) dP
\]  

(6)

There is a sharp peak of the cross-correlation function on the correlation plane. For a given subset of a pattern involving several speckles, the peak of the cross-correlation \( R(s) \) will reach a maximum in the ROI under the condition \( s = \bar{u} \). Actually, this highest peak in the correlation plane represents the most probable match of the subset between reference and current images. Note that the principle of correlation-based DIC determines the average displacement of a small group of speckles that form a unique speckle pattern within the subset, rather than determination of the displacement of an individual speckle [7]. The DIC processing divides a digital image into subsets. Then, the cross correlation is calculated over all subsets, such that one displacement vector is obtained for each subset. A full-field displacement within the image is determined through assembly of the displacement vectors.
Experimental Procedure

A simple testing approach to achieve opening or mixed-mode fracture is a beam under three-point bending (Fig. 2), with a center (β = 0%) or off-center (β = 30%) notch, where the eccentricity factor β is the normalized distance, with respect to the half-span, from the centerline. The nominal size of the beams were height $H = 60$ mm, span $S = 147$ mm, and thickness $B = 26$ mm; one notch length was used, $a/H = 0.1$, with $\beta = 0$ or 30%. Crack initiation and propagation were controlled with a closed-loop, servo-hydraulic system and crack mouth opening displacement (CMOD) as the feedback signal. The sedimentary rock used for testing, Berea sandstone, consists of uniformly sized grains ranging from 0.1 – 0.8 mm, with an average grain size of 0.2 mm. For the material tested, the P-wave velocity measured perpendicular to bedding (natural layering) was 2160 m/s and parallel to bedding it was 2290 m/s; the rock is anisotropic, but only slightly (less than 10%), meaning that the ray direction of the P-wave can be approximated by the wave front normal, or that the group and phase velocities are approximately the same.

![Fig. 2. Experimental configuration.](image)

Eight piezoelectric transducers (Physical Acoustics model S9225), four on each side, were glued to a beam to cover a region of approximately 40 mm in radius surrounding the notch (Fig. 2). The AE signals were recorded by a computer automated measurement and control data acquisition system, equipped with four DAQ cards (National Instruments model PCI-5112). Each card has two individual 8-bit analog-to-digital converters, and a sampling rate of 20 MHz was set to record AE signals that are conditioned by bandpass filters from 0.1 – 1.2 MHz and 40dB gain. The transducers have a diameter of approximately 3 mm, with the frequency response from 0.1 – 1 MHz. All channels were triggered when the signal amplitude exceeded a threshold on the anchor sensor. A program coded with LabView controls the signal acquisition over a 200-µs window, with a 100-µs pre-trigger.

An isotropic velocity model was assumed, even though the rock possesses slight anisotropy. An “average” velocity measurement was performed on the specimen before it was tested, so that the sensors were attached in exactly the same way as for the experiment. An artificial AE event was produced on the surface of the specimen by breaking a 0.5-mm pencil lead at a marked location, and this was repeated a number of times. The first arrivals were then determined for these signals and the velocity was varied until the event locations matched, with minimum error, the known coordinates of the artificial events.
One surface was selected to produce the speckle pattern for the image matching (Fig. 3). To achieve a random pattern of speckles, the general procedure of preparation is (1) lightly coat the specimen surface with white paint; (2) after the white paint is dry, overspray the coated surface with a dark mist by a spray paint; (3) continue misting and re-misting until the speckle pattern is unique. A charge-coupled device (CCD) digital camera is used to acquire the digital images (Fig. 2), specifically a Unibrain (San Ramon, CA) Fire-I 810 IEEE-1394 CCD camera with 1600 × 1200 effective square pixels in combination with a Computar lens (Model M3Z1228MP, Commack NY) with manual control of aperture, focus, and zoom. The fastest rate the camera could acquire digital images is 15 frames/s. The camera is connected to the computer by an IEEE-1394 or “firewire” cable, which is a standardized specification that allows for data transfer speed of up to 400 Mb/s. The image acquisition toolbox inside Matlab was used as the control software to acquire the digital images.

![Figure 3. Digital image of the speckle pattern.](image)

The matching process was performed on intensity values of the digital images, and the unit of measure from the computational analysis is pixel-based. Thus, a magnification factor \( M \) is introduced to transform the results from the digital image to the physical dimension on the specimen surface:

\[
u = Mu^D \quad \text{and} \quad v = Mv^D
\]

(7)

where \( M \) is a constant magnification factor associated with the experimental setup, which is largely dependent on the digital camera; \( u^D \) and \( v^D \) are the pixel-based displacements from the image analysis. The magnification factor relates a physical dimension on the specimen surface to a corresponding dimension on the digital image. Different magnification factors can be obtained by the adjustment of lens magnification and/or the camera position. However, since the CCD array of the digital camera is fixed (1200 × 1600), there are limitations in setting \( M \) because of the importance of the field of view. Since the height of the specimen was about 60 mm, \( M = 30.0 \ \mu \text{m/pixel} \) was selected, with the observation area of 36 × 48 mm (Fig. 3).

The other important parameter for the image matching process is the subset size, where it is optimal to have 8 – 10 speckles inside the subset and a balance of intensity distribution between
the speckles and background [6]. With the surface preparation producing speckles 100 – 200 µm in diameter and \( M = 30 \, \mu m/pixel \), the speckle size on the image ranged from \( 3 \times 3 \) to \( 7 \times 7 \) pixels. Thus, the subset was selected to be \( 20 \times 20 \) pixels or a 0.6 mm square.

Identification of Fracture

Mode I fracture

Using CMOD as the feedback signal, fracture propagation can be well controlled by the servo-hydraulic system even though crack growth is unstable. The results are presented at increments of post-peak load, both for AE locations and incremental displacement contours (AE or displacement measured between two load levels).

![Figure 4](image)

Fig. 4. Mode I fracture test of specimen CN-1: 95-90% of peak load. (a) AE locations. (b) Incremental horizontal displacement contours \( \Delta u \).

Figure 4a shows the post-peak AE locations from 95–90% of peak load during crack propagation for specimen CN-1 with a center notch. A clustering of AE is clearly identified along the entire fracture developed by Mode I loading; AE extend from the notch \( y = 6 \, mm \) to the position \( y = 20 \, mm \). The AE locations are 3D, representing damage processes through the thickness, but for convenience, the locations are projected on the viewing plane; the zone width is about 5 mm, although it is influenced by crack tortuosity. Figure 4b displays the incremental horizontal displacement \( \Delta u \) contours in the same post-peak stage of loading (95 – 90% of peak load). Because of the loading configuration, the horizontal displacement exhibits a symmetric pattern and the center vertical cross section \( x = 0 \, mm \) represents the line of symmetry. The \( \Delta u \) contours merge at \( x = 0 \, mm, \, y = 20 \, mm \), which indicates the tip of a displacement discontinuity. For example, a few mm below the tip, the incremental opening displacement is about 8 µm, i.e. -4 µm to the left and +4 µm to the right. As indicated by the AE activity, it can be interpreted that cohesive traction was acting in this region [2].
Fig. 5. Mode I fracture test of specimen CN-1: 80-70% of peak load. (a) AE locations. (b) Incremental horizontal displacement contours $\Delta u$. (c) Incremental vertical displacement contours $\Delta v$.

With an increase of CMOD, the crack propagated as the applied load reduced. Figure 5 shows the AE locations and the incremental displacement contours for 80 – 70% of post-peak load. Note that 85% of AE locations were from a region between $y = 10$ mm and $y = 25$ mm along the symmetry line, and the width of the region is about 5 mm (Fig. 5a). Very few AE events were observed at positions of $y < 10$ mm, suggesting that this region is traction-free [2]. The incremental horizontal displacement contours indicate the tip at position $x = 0$ mm, $y = 25$
mm (Fig. 5b). An interesting observation concerns an island of incremental vertical displacement of \( \Delta v = -13 \) \( \mu m \) near the region of the crack tip, which may be associated with the process zone (Fig. 5c); this vertical displacement can be thought of as tangential displacement of the crack, which displays no differential movement, meaning that \( K_{II} = 0 \), an obvious result for the symmetric loading.

**Mixed-mode fracture**

Specimen EN-1 with an eccentric notch of \( a/H = 0.1 \) and \( \beta = 30\% \) exhibits mixed-mode fracture, where sliding displacement occurs along (a portion of) the crack and symmetry is not maintained. A linear elastic stress analysis based on a boundary element code [8] gives a combination of Mode I and II stress intensity factors with \( K_{II}/K_{I} = 5.4\% \).

Fig. 6. Mixed-mode fracture test of specimen EN-1 from 100–90% of peak load. (a) AE locations. (b) Incremental normal displacement contours \( \Delta u \).

For 100–90% post-peak load, the AE locations (Fig. 6a) cluster within a zone that is slightly longer than that shown for Mode I loading (Fig. 4a), and the fracture is inclined at 20° from the vertical. Thus, it is convenient to establish a new coordinate system \((x_1, y_1)\) positioned at the notch tip and inclined at 20° along the fracture direction (Fig. 6b), such that the normal and tangential (and possibly sliding) displacements can be identified from the measured horizontal and vertical displacement fields. The incremental displacements \( \Delta u_1, \Delta v_1 \) based on the new coordinate system can be computed through a transformation of the incremental horizontal and vertical displacements \( \Delta u, \Delta v \). Note that the contours of transformed displacements are still plotted in \( x, y \) coordinates, in order to compare to the AE locations; the merging of the contours identifies the tip, which coincides well with the AE.

For 90–80% of post-peak load, the AE locations (Fig. 7a) cluster within the range from \( y = 10 \) mm and \( y = 26 \) mm, with a traction-free length suggested where very few (< 10%) AE are located. Figure 7b shows the incremental normal displacement \( \Delta u_1 \) along the fracture, and the tip is at \( x = -14 \) mm, \( y = 26 \) mm. The incremental tangential displacement contours again show
Fig. 7. Mixed-mode fracture test of specimen EN-1 from 90-80% of peak load. (a) AE locations. (b) Incremental normal displacement contours $\Delta u_1$. (c) Incremental tangential displacement contours $\Delta v_1$.

an island (Fig. 7c), similar to the opening mode behavior, but it is important to notice that the island in the mixed-mode experiment shows no differential tangential displacement – no sliding – and consequently $K_{II} = 0$ in the island. Furthermore, sliding displacement is displayed along a portion of the crack, as displayed by the contours varying in color between the upper and lower parts of the fracture. Thus, the island in mixed-mode loading is an opening region where no sliding occurs. Indeed, the observation of slip below the position $y_1 = 20$ mm confirms an assumption concerning the process zone in mixed-mode loading [9], where slip is absent until a critical amount of opening displacement takes place. It appears that the process zone, for a
mode-mixity ($K_{II}/K_I$) of approximately 5%, is associated with opening, and AE activity is concentrated in this zone.

**Summary**

Two techniques, AE and digital image correlation (DIC), were used to identify fracture of rock under opening and mixed-mode loading conditions. Experimental results demonstrated that both techniques provided similar measurements on the length and character of the fracture. A grouping of microcrack locations clearly identified the cohesive-nature of a portion of the fracture, and crack growth. DIC provided a position of the crack tip, and a zone of increased tangential displacement but no differential movement near the crack tip, suggesting that the process zone for slight mixed-mode deformation is governed by Mode I opening.

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**References**